Vibration Testing Using High Speed Digital Image Correlation
and Adaptive Geometric Moment Descriptor

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Abstract. High-speed optical experimental mechanics enables full-field vibration measurement. In this extended abstract, spatial and temporal signal processing techniques of such measurements in the application of vibration modal testing will be discussed.

Digital image correlation
The application of high-speed cameras with digital image correlation (DIC) algorithms enables full-field vibration measurement of engineering structures. There are numerous advantages of using this optical experimental method, which include no masses added to the structures; applicable to rotating structures; suitable for high temperature specimens and applicable to tiny specimens. However, the full-field captured data are usually spatial information redundant. In the application of modal testing, the estimation of point-wise transfer functions may be inaccurate due to the uncertainties of the DIC method, particularly in the case of using high-speed cameras. The uncertainties may be caused by luminous conditions, calibration procedure, camera noise, the pattern of the specimen surface and facet size etc. The full-field measured displacement at any time instant may contain $10^{3-6}$ data points. To reduce the redundancy of the full-field data, shape decomposition approach may be employed to effectively condense the shape information. Fourier Transform, wavelet transform, image moment descriptors are conventional decomposition approaches [1]. These methods require image domains regularly defined.

Adaptive geometric moment descriptor
The DIC method measures surface responses and the common engineering structures are generally irregular with curved surfaces, so that construction of orthogonal elementary waveforms defined on the measured domain requires rather special treatment. 3D curved surfaces are essentially 2-manifolds and 2D analytical functions may be defined on two curvilinear coordinates of the measurement surface. Surface parameterisation is the technique used to determine the curvilinear coordinates.

A generic approach of determining the shape features from 3D DIC measurement was proposed [2]. It may be divided into two steps: namely (1) surface parameterisation – to determine the 2-parametric-coordinates from 3D surfaces; and (2) orthogonal kernel function construction using Gram-Schmidt orthogonalisation. The geometric moment descriptor (GMD) as proposed by Hu [3] adapts two-dimensional monomials $u^p v^q$, $u, v \in \mathbb{R}$, $p, q \in \mathbb{N}$, for planar domain as the kernel functions. This is extended to parametric domain for 3D surface by the authors [2] and denoted as adaptive geometric moment descriptor (AGMD) and may be expressed as

$$a_i(\zeta) = \int_{x \in \Omega} S(x, \zeta) \mathcal{H}_i(x, \zeta) dx \quad (1)$$

where $S(x, \zeta)$ denote the full-field displacement or excitation (e.g. acoustic wave), $x \in \{\text{measured surface domain}\}$ denotes the spatial coordinates, $\zeta$ is the temporal parameter, and $\mathcal{H}_i(x, \zeta)$ is the $i^{th}$ kernel function adaptively constructed from 2D monomials [2]. It is found that only a small number of shape features are significant and need to be retained to reconstruct the full-field displacement. The dimensionality of the retained shape descriptors is usually $10^{0-2}$. In the same time, the noise of the spatial data may be reduced since the retained shape descriptors are usually associated with the decomposition kernel functions having long wavelengths. Therefore, the modal properties identification could then be carried out using the information efficient shape features.

Transfer functions
Identifying modal properties from estimated transfer functions is one of the common ways [7]. For instance, the transfer function between force excitation and displacement of a proportionally damped system may be expressed as

$$h_{j,k}(\omega) = \sum_{r=1}^{N} \frac{\phi_{rj} \phi_{rk}}{\omega_r^2 - \omega^2 + i\omega \eta_r} \quad (2)$$

where $\omega$, $\eta$, and $\phi_{rj} \phi_{rk}$ are the $r^{th}$natural frequency, damping factor and modal constant, respectively. As mentioned in the previous section, the AGMD is spatial information efficient and noise-robustness. It is possible to construct transfer functions in the shape feature space formed by the AGMDs; written as [2]

$$h_{t,n}(\omega) = \sum_{r=1}^{N} \frac{\theta_{t} \theta_{rn}}{\omega_r^2 - \omega^2 + i\omega \eta_r} \quad (3)$$
where $\gamma_r = \int_{D(x)} \mathcal{W}_r(x) \mathcal{R}_r(x) \, dx$ denotes the $r$th shape feature of the $r$th mode shape $\mathcal{W}_r(x)$. Therefore, modal identification may be carried out more effectively by using Eq. (3) than Eq. (2).

**Modal testing using AGMDs**

**Single input and multiple outputs (SIMO).** If field loading was adopted to excite the structure, e.g. an uniform acoustic wave with different frequencies, the estimation of the SD-FRF may be considered as SIMO case. That is the outputs are the shape features of the full-field transient responses captured by high-speed systems and the input is the only shape feature of the field excitation pattern - because the acoustic wave is assumed to be uniform and uniform pattern is usually defined as the first kernel function. The modal properties Eq.(3) could then be identified, e.g. using the curve-fitting approach [7].

**Multiple shape feature inputs and shape feature outputs (MIMO).** Using point-wise shakers is common way to excite a structure. The shaker force may be considered as Dirac delta function in the spatial domain. Retaining multiple shape features is usually necessary to construct a descent spatial Dirac delta function. It this case, the identification becomes multiple inputs/outputs problem.

Converting to single spatial input and multiple shape feature outputs [2]

$$\gamma_k(\omega) = \sum_{r=1}^{N} \frac{g_{r \ell} \sum_{m} (C_{m}^r \theta_{rm})}{\omega^2 - \omega^2 + i \omega \eta_r}$$

(4)

where $C_{m}^r$ is the value of the $m$th kernel function evaluated at the shaker’s location. Eq.(4) is the transfer function between the shaker force and the shape descriptors of the full-field displacement. Model properties as shown in the right hand side of Eq.(4) may be determined by the curve fitting techniques. Detailed discussions may be found in [2].

**Output only.** In the case where the input is not available, output-only methods may adopted to determine the modal properties providing that input statistical characteristics is known a priori.

$$g_{yy} = \gamma_k^T G_{xx} \gamma_{xy}$$

(5)

where $G_{xx}$ and $G_{yy}$ are the input and output spectrum density matrices respectively; $\gamma_{xy}$ is the matrix of transfer functions whose elements are defined in Eq.(4). Presuming the input is white noise excitation by a shaker, the output spectrum density function may be written as

$$g_{yy} = \sum_{r=1}^{N} \frac{A_{r \ell} A_{r \ell}^*}{(\omega^2 - \omega^2 + i \omega \eta_r^2)}$$

(6)

where $A_{r \ell} \equiv \hat{\gamma}_r \sum_{p} C_{p} \theta_{rp}$; $A_{r \ell} \equiv \hat{\gamma}_r \sum_{q} C_{q} \theta_{rq}$. The output spectrum density functions may be estimated from experimental data. Thus, the modal properties may be determined by the method of singular value decomposition(SVD)[8]. When the $r$th mode dominates, the $r$th summation term in Eq.(6) is expressed as

$$A_{r \ell} A_{r \ell}^*$$

(7)

where $1/\omega^2 \eta_r^2$ is corresponding to the greatest singular value. Since $\omega_r$, may be picked from the estimated FRFs, i.e. $\omega_r$ is known, $\eta_r$ is then calculated by using the singular value and Eq.(7). In the same time, the residues in the numerator of Eq.(7) may be found out from the singular vector.

**Conclusions**

Full-field measurements consist of very large volumes of data, which can be compressed by using shape descriptors. Measurement noise is concentrated in the high-order shape descriptors and the reconstructed images are robust to measurement noise. AGMDs are suitable of image decomposition of irregular shaped engineering components. Three dimensional curved surfaces can be mapped to planar surfaces by surface parameterisation. Modal identification can be carried out more effectively in the shape descriptor domain.

**References**


