Spatial and Temporal Derivatives of Measured Displacements

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Abstract. Moiré, digital image correlation (DIC), electronic speckle pattern interferometry (ESPI), and laser vibrometry are effective means for measuring the in- and out-of-plane displacements. Their temporal and spatial derivatives provide strains and velocities. Strains (hence stresses) or velocities are often determined by physically differentiating recorded displacements; a process which is frequently ill-conditioned and adversely influenced by data noise and quality. This shortcoming can be overcome by processing measured displacements with analytical formulations based on compatibility and equilibrium or using finite element concepts. The presentation will discuss engineering cases involving metals, plastics and composites subjected to static and dynamic loading. Fig. 1 and Fig. 2 involve structures with different cutout geometries, where the individual components of displacement, strain and stress can be provided full-field by processing a single component of DIC recorded displacement at discrete locations with a stress function.

Fig. 1: Strain $\varepsilon_y$ adjacent to the major-axis of an elliptically perforated tensile plate from stress function processed DIC recorded single displacement component, FEA and strip strain gage.

In addition to being technically efficient and fiscally attractive, the ability to obtain both independent components of displacement from a single measured displacement component is advantageous when the recorded orthogonal component is of poor quality or small magnitude. Moreover, to determine both independent displacement components using moiré, holography or ESPI can be appreciably more involved than determining only a single component of displacement by these techniques.
Fig. 2: Contours of $\sigma_{r\theta}/\sigma_{net}$ throughout region adjacent to a circular hole in a loaded plate predicted by FEA (left side) and experimentally with Airy coefficients evaluated from DIC-measured $v$ displacements (right side).

Fig. 3 illustrates the use of time-space elements (based on FEM concepts) to obtain particle (fringe) velocity from moiré measured displacements in an impacted polyurethane ($E = 4.4$ MPa, $\nu = 0.47$) plate. Combining time-space elements with measured displacements is convenient for transient problems in that the technique provides continuous displacement components and their continuous spatial and temporal derivatives throughout a 3D domain (both in-plane coordinates and time).

Fig. 3: Particle motion vs time in impacted half plane determined by processing moiré recorded displacements with space-time-elements.