Image-based Stress Field Reconstruction
R. Seghir\textsuperscript{1,a}, F. Pierron\textsuperscript{2}, L. Fletcher\textsuperscript{2}
\textsuperscript{1}Research Institute in Civil and Mechanical Engineering (GeM), Ecole Centrale de Nantes, Université de Nantes, UMR 6183 CNRS, F-44 321 Nantes,
\textsuperscript{2}Engineering and the Environment, University of Southampton, Highfield campus, University Road, Southampton SO17 1BJ, UK
\textsuperscript{a}rian.seghir@ec-nantes.fr

Abstract. In many instances in life, materials are subject to deformation at high rates, for example: impact, crash, metal forming or pulsed welding. In this context, the transient and inhomogeneous nature of such loading as well as the strong multi-physic couplings induced by quasi-adiabatic conditions make: the experimental capture of the mechanical response very challenging. Additionally, assumptions regarding the constitutive relation of the deforming material are generally required. To overcome both issues, we demonstrate that experimental full-field measurements of acceleration fields can be directly used to invert the local equilibrium equation and reconstruct fields of the stress tensor with no assumption on the constitutive relation and its spatial and temporal variations. We also demonstrate that both experimental stress and strain fields can be recombined to eventually identify the local tangent stiffness tensor of the material. This study constitutes a first step in the field of "direct model identification", as opposed to standard parametric model identification.

Introduction
When localised deformation occurs in a material due to the loading e.g. dynamic waves, to instabilities e.g. necking, adiabatic shear-bands or simply when cracking, assumptions or models are required to connect experimental measurements at the VER scale to the true local (thermo-) mechanical state. In the last 30 years there has been a massive expansion in the use of full-field measurement techniques for model validation. However, such models still cannot be validated at the scale they are formulated since local stresses are still inaccessible. Furthermore, in many cases only energy quantities are sought as in fracture mechanics where the fracture energy could be simply derived from a local energy balance, regardless of the constitutive equation, if only the stress and strain fields were accessible. Recent developments in image-based techniques have shown that acceleration fields can be experimentally captured during an impact event. Using ultra-high speed imaging (greater than 1MHz), the acceleration fields can be used as an embedded load cell if the material density is known \cite{1}. This allows for a contactless measurement of both averaged strain and stress in the microsecond regime. In this paper, we extend the concept to full-field stress tensor reconstruction from the acceleration fields. This new methodology is termed image-based stress reconstruction (IBSR). Explicit dynamic simulations are used to validate this methodology. Using simulation data, each component of the stress field is recovered from acceleration fields under plane stress assumptions. The material stiffness maps are then back calculated by recombining the "measured" strain and the identified stress. In the future this methodology will be applied to experimental data for full-field stress reconstruction.

Theoretical Framework
The main idea behind the proposed IBSR methodology is to replace a standard a priori parametrisation of the material “law” by a piecewise spatial parametrisation of the acceleration and by extension of the stress components through dynamic equilibrium. This is accomplished using conservation laws, linear and angular momentum, boundary conditions and continuity. Here, the acceleration fields are expanded using a bi-linear element formulation leading to a quadratic form of the piecewise components of the stress tensor field. Combining equilibrium, continuity and boundary equations an overdetermined linear system of equation is assembled and can be solved in a least-square sense if acceleration fields, material density and boundary conditions are measured/known. In the following, the material density is considered homogeneous, acceleration fields are obtained by double differentiation of displacements output from explicit dynamic simulations and the boundary conditions are partially known due an inertial impact test configuration \cite{1}. Indeed, in the latter the free boundary conditions impose the normal and the tangent components of the stress tensor to be zero at all times at the specimen borders.
Simulation
The model consists in a 54 mm × 34 mm rectangular specimen, meshed with 136 × 216 4-nodes quadrilateral plan stress elements. The loading is introduced by explicitly simulating the impact between the specimen and a 10 mm long steel projectile at a speed of 45 m/s. The specimen material is isotropic linear elastic and homogeneous. The explicit dynamic simulation is performed using LS-Dyna and results are output with a time-step of 200 ns (equivalent to an acquisition rate of 5 MHz). It is worth noting that while the material is homogeneous, stress fields are neither uniform nor constant. Indeed, the dynamic loading produces waves that propagate back and forth in the specimen interfering each other. For the stress reconstruction, a mesh size of 5×5 FE elements is chosen.

Results
Figure 1 a-c) present the converged stress fields (at t=16µs). Fig 1 b) shows the comparison between FE and reconstructed stresses over time at a specific point (marked with a white dot) and Fig 1 e) shows the variability over time and space of the identified material properties (mean and standard deviation). The stress fields are highly heterogeneous and Fig 1 d) clearly shows the ability of the proposed methodology to accurately capture the variation of the stress field in space and time. Fig 1 e) shows that the material properties are reasonably well captured over time. Note that some time steps lead to a higher error due to the decrease in the input signal (i.e. low acceleration).

Conclusion
The preliminary results of this study validate the IBSR methodology. This shows that it is possible to reconstruct the full stress tensor from the acceleration field and an appropriate test configuration. Experimental data, for isotropic and orthotropic material, are currently being processed with the IBSR method and will confirm the use of the method for real experimental data. The IBSR method will be especially useful for accessing strength in an off-axis impact test on a composite specimen as it will allow for the full stress tensor to be reconstructed and rotated into the material co-ordinate system.

References