AFM, Frequency Analysis and Modelling Techniques for the Size Effect in Beam Bending

C. Liebold[1] and W.H. Müller[1a]

[1]Technische Universität Berlin,
Lehrstuhl für Kontinuumsmechanik und Materialtheorie,
Institut für Mechanik – LKM,
Einsteinufer 5, 10587 Berlin, Germany

[a]Wolfgang.H.Mueller@TU-Berlin.DE

Abstract

In context with the static theory of elasticity for isotropic and nonsimple materials, different approaches of generalized continua are presented. These allow modeling a size-dependent deformation behavior, which has been experimentally observed in very small quantities of matter or in materials with an intrinsic microstructure. (Modified) Strain gradient-, micropolar-, couple stress- and surface theories are used and constitutive relations are derived by following a cascade of principles of rational mechanics. Analytical solutions following Euler-Bernoulli beam theory, as well as numerical implementations in terms of finite element variational formulations of higher-order theories are presented and solved with the help of the open-source Finite Element code FEniCS. Atomic Force Microscopy (AFM) investigations of the engineering polymer materials epoxy, SU-8, aluminum foams, aluminum with artificial heterogeneities and nanoporous nickel alloy were performed and higher-order material parameters, i.e., material length scale parameters are obtained by a method of least squares from analytical and experimental data.

Introduction

Experimental methods, such as Atomic Force Microscopy (AFM) and frequency analysis are presented for experimental assessment of the bending behavior of materials with real micro- or macrostructure. It is well known that mechanical properties of materials may depend on the intrinsic morphology of its structure and that the deformation behavior can be size dependent, reflected in a stiffer or softer elastic response, if the dimensions of a body approach the intrinsic length scale. We investigate the size dependent bending behavior of beams made of the materials epoxy, SU-8, aluminum foams, aluminum with artificial heterogeneities and nanoporous nickel alloy corresponding to the method of size effect described by LAKES (1995) [1].

Higher-order continua

Higher-order theories of elasticity, such as the strain gradient theory of MINDLIN's 2nd type (cf., [2]), the modified strain gradient theory (cf., [6]), the micropolar theory (cf., [3]), the Couple Stress theory (CS) [5], as well as the Surface Elasticity theory (SE) [4] are presented for the purpose of quantitative modeling of the bending rigidities of slender beams. When taking the assumptions of EULER and BERNOULLI for beams into account, the size dependent elastic modulus of bending structures in extended continuum theories can be derived analytically, e.g.:

$$E^*_{\mathrm{CS}} = E\left(1 + 6\frac{\ell^2}{T^2}\right) \text{, or } E^*_{\mathrm{SE}} = E + E^{\mathrm{SE}}\left(\frac{6}{T} + \frac{2}{W}\right), \tag{1}$$

where E denotes the conventional Young's modulus, T the thickness and W the width of beams for (exclusively) rectangular cross sections. E^{SE} and ℓ represent additional material coefficients, which need to be experimentally assessed.

Experimental results

Size dependent bending behavior is investigated for beams made of the materials epoxy, SU-8, aluminum foams, aluminum with artificial heterogeneities and nanoporous nickel alloy corresponding to the "*method of size effect*" described by LAKES (1995) [1]. In particular, by using an AFM, deflection measurements were performed and force data were recorded for micro-beams. Static bending tests are performed on freestanding microbeam structures. A load of $0.5 < F < 250$ µN is applied by using an off-axis laser-reflective AFM and deflections of 40 nm $< w <$ 10.0 µm were recorded. Moreover, by using the frequency analysis method (FREQ) flexural vibration frequencies were extracted by Fast-FOURIER-Transformation (FFT) from the acoustic signal of macro-beams with decreasing thicknesses. By assuming rectangular cross-sections of the specimens, the following classical relations between the AFM measures (F/w), the measure of the frequency analysis method f_1 and the elastic modulus E can be used:

$$E_{\text{AFM}} = \frac{4L^3}{WT^3}\frac{F}{w} \quad , \quad E_{\text{FREQ}} = 0{,}95\frac{\rho L^4 f_1^2}{T^2} \tag{2}$$

The results of the experiments show a clear variation of nearly twice the value of conventional elastic modulus (cf., Fig. 1). The corresponding additional material coefficients were fitted to the afore-mentioned theoretical models (cf., Table 1 and Eq. 1).

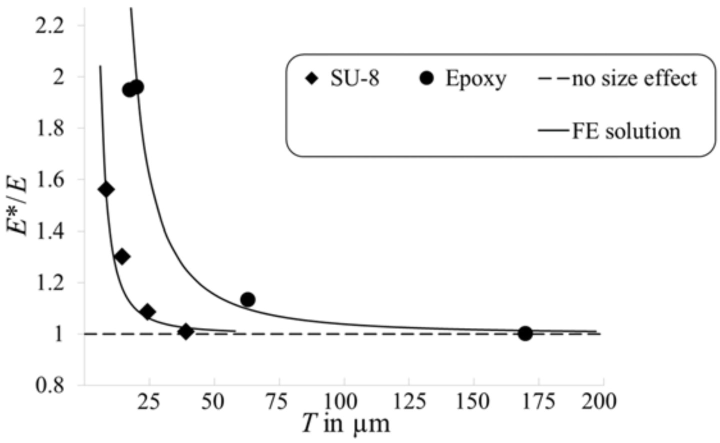

Figure 1: Mean-values of experiments (after Eq. (2)) on epoxy and SU-8, including smoothed FE results.

Material	Theory	Coefficient-1	Coefficient-2
Epoxy	Couple stress theory	E=3.93 GPa	ℓ=7.75 µm
	Mod. strain gradient theory	E=3.93 GPa	ℓ=4.35 µm
	Surface elasticity theory	E=3.22 GPa	E^{SE}=14.1 kN/m
	Finite Element solution	E=3.9 GPa	ℓ=7.8 µm
SU-8	Couple stress theory	E=4.13 GPa	ℓ=2.50 µm
	Mod. strain gradient theory	E=4.14 GPa	ℓ=1.39 µm
	Surface elasticity theory	E=3.36 GPa	E^{SE}=3.95 kN/m
	Finite Element solution	E=4.2 GPa	ℓ=2.5 µm

Table 1: Exemplary material parameters

Conclusion

The value of the material length scale parameter for epoxy that has been measured in the present work, can be compared to the literature value from [3], where there is a deviation of about 17%. This deviation may be attributed to a different manufacturing processes, to a difference in the base materials for the epoxy resin and to different technical equipment. As reported for epoxy, there is no size effect in *tensile* testing with specimens of the same thicknesses that were tested here, cf., [3], which is reproduced in the higher-gradient models.

Acknowledgements

The present work is supported by (Deutsche Forschungsgemeinschaft) DFG under Grant MU 1752/33-1. The author like to thank the Fraunhofer Institute for Reliability and Microintegration Berlin for sample preparation and PTB-Braunschweig for AFM calibration.

References

[1] Lakes R.: *Experimental methods for study of Cosserat elastic solids and other generalized elastic continua*. Ed. H. Mühlhaus, J. Wiley, N.Y., Ch. 1 (1995) pp. 01–22.

[2] Mindlin R.D., Eshel N.N.: *On first strain-gradient theories in linear elasticity.* International Journal of Solids and Structures, **4**, 1968, pp. 109–124.

[3] Eringen A.C.: *Nonlocal Continuum field theories*. Springer-Verlag, 2010, New York.

[4] Gurtin M.E., Murdoch A.I.: *A continuum theory of elastic material surfaces*. ARMA, **57**(4), (1975) pp. 291–323.

[5] Toupin R.A.: *Elastic materials with couple-stresses*. ARMA, **11**, (1962) pp. 385–414.

[6] Lam D.C.C., Yang F., Chong C.M., Wang J., Tong P.: *Experiments and theory in strain gradient elasticity*. J. of the Mechanics and Physics of Solids **51**(8), (2003) pp. 1477–1508.