An Innovative Method for Measuring Young's Modulus of a Flexible Circular Ring (Own-weight Circular Ring Method)

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Abstract. This report describes a development of an innovative mechanical testing method (Own-weight Circular Ring Method) for measuring Young's modulus of a flexible ring material in taking into account large deformation behaviours. A newly proposed method is based on the large deformation theory. A ring specimen subjected to own weight is deformed greatly. By just measuring the horizontal or the vertical displacement of the ring, Young's modulus can be easily obtained for thin frexible materials. Measurements were carried out on a thin piano wire (SWPA).

Introduction

In recent years, flexible materials with very high performance are widely used. Therefore, Young's modulus of these materials is very important to predict large deformation. This paper describes a new testing method (Own-weight Circular Ring Method) based on a nonlinear large deformation theory. By using this method, Young's modulus of various thin flexible material (plate and wire) can be easily obtained by just measuring the horizontal displacement or the vertical displacement. Measurements were carried out on a thin wire (piano wire). The results reveal that the new method is suitable for thin flexible materials. In the meantime, the new "Own-weight Circular Ring Method" proposed in this paper is quite a promising method and can be extended to measure Young's modulus of every thin layer in a flexible multi-layered material formed by PVD, CVD, Coating, Electrodeposition, Paint, Cladding, Lamination, and others.
Besides the Own-weight Circular Ring Method studied here, the Cantilever Method [1], the Circular Ring Method [2, 3], the Compression Column Method [4] for a single-layered material have already been developed and reported, based on the nonlinear large deformation theory.

Theory

A typical illustration of a deformation shape is given in Fig.1 for a ring (2*L*: full circular length) subjected to own weight. The horizontal displacement is denoted by *x,* vertical displacement by *y*, and θ is the deflection angle. Moreover, an arc length is denoted by *s*, the radius of curvature by *R* and the bending moment by *M*. The relationship among *R*, *M*, *s*, x, y and θ are given by:

$$1/R1/R = M/(EI) = -d\theta/ds\,, dx = ds\cdot\cos\theta, dy = ds\cdot\sin\theta \tag{1}$$

where *EI* is the flexural rigidity.
From an equilibrium of shearing force at an arbitrary point B(x,y),

$$Q = w(L-s)\cos\theta - F_A\sin\theta \tag{2}$$

The basic equation is derived in the form of:

$$EI\left(d^2\theta/ds^2\right) = -w(L-s)\cos\theta + F_A\sin\theta \tag{3}$$

Introducing the following non-dimensional variables,

$$\left.\begin{aligned}&\xi = x/L,\ \eta = \mathrm{y}/L,\ \varsigma = s/L\\ &\gamma = wL^3/(EI),\ A = F_A L^2/(EI)\end{aligned}\right\} \tag{4}$$

Equation 3 reduces to Equation 5.

$$d^2\theta/d\zeta^2 = -\gamma(1-\zeta)\cos\theta + A\sin\theta \tag{5}$$

and From Eq. 1, the following equation is obtained.

$$d\xi = d\zeta\cdot\cos\theta,\ d\eta = d\zeta\cdot\sin\theta \tag{6}$$

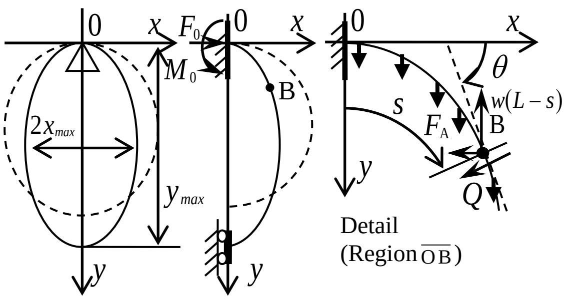

Fig.1 Large deformations of a circular ring subjected to own-weight.

It is difficult to solve a nonlinear differential equation 5, however, by applying the R-K-G method we can solve Eq. 5 numerically.
In analysing Eq. 5, by introducing the following non-dimensional bending moment μ (Eqation.7), the relation between the deflection angle θ and the non-dimensional bending moment μ is expressed as Eq. 8.

$$\mu = ML/(EI) \qquad (7)$$

$$d\theta/d\zeta = \mu \qquad (8)$$

Finally, the fundamental equation 9 is obtained by transforming variables from Eq. 5.

$$d\mu/d\zeta = -\gamma(1-\zeta)\cos\theta + A\sin\theta \qquad (9)$$

Therefore, Considering the following boundary conditions shown as Eq. 10, the non-dimensional maximum horizontal and vertical displacements ξ_{max}, η_{max} can be obtained.

$$\left.\begin{array}{l} \theta|_{\zeta=0} = \xi|_{\zeta=0} = \eta|_{\zeta=0} = 0 \\ \theta|_{\zeta=\zeta^*} = \pi/2,\ \theta|_{\zeta=1} = \pi,\ \xi|_{\zeta=1} = 0 \end{array}\right\} \qquad (10)$$

In this paper, representative two methods are introduced to measure Young's modulus. The γ-ξ_{max} relation is presented in Fig.2 [Method 1] considering user-friendliness (γ-η_{max} relation [Method 2], omitted here). For the sake of simplicity, the usage of the chart (: Nomograph) is recommended here by the author. Using the chart for Method 1 shown in Fig.2 as an example, Young's modulus for a piano wire (SWPA) with diameter: d=0.3mm, distributed load per unit length: w= 6.8796×10^{-3} N/m is obtained from the following formula 11 based on Eq. 4.

$$E = wL^3/(\gamma I) \qquad (11)$$

When a half length of ring L: 600mm, x_{max}=170.3mm (i.e., $\xi_{max} = x_{max}/L$ =0.2839) is measured and then γ is taken from Fig.2 (γ= 18.2411). Therefore, Young's modulus E is calculated from Eq. 11 as follows.

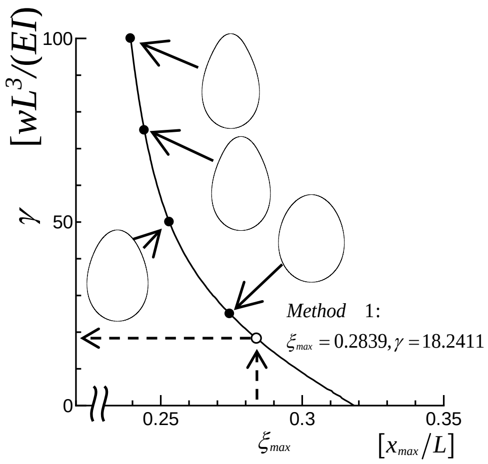

Fig.2 Non-dimensional chart for finding the parameter γ when the deflection x_{max}/L is given.

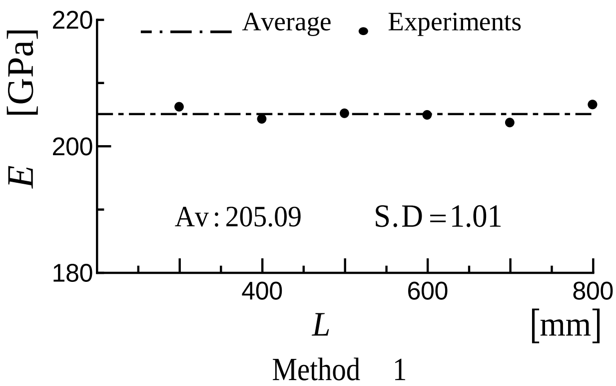

Fig.3 Comparison of Young's moduli for a piano wire (SWPA).

$$E = \frac{wL^3}{\gamma I} = \frac{6.8796\times10^{-3}\times(0.6)^3}{18.2411\times3.9761\times10^{-16}} \fallingdotseq 204.884\times10^9[\mathrm{N/m^2}] \cong 204.9[\mathrm{GPa}] \qquad (12)$$

Experimental investigation

Several experiments were carried out using a piano wire [SWPA, full length: 2L=600.0-1600.0mm, diameter: d=0.3mm]. Young's modulus by applying Method 1 (Method 2, omitted here) is shown in Fig.3. The measured values of Method 1 remain nearly constant for several lengths and the standard deviation (S.D) is very small although Method 1 has a little scattered values.

Conclusion

The Own-weight Circular Ring Method is developed as a new and simpler material testing method. As experimental results, the new method is suitable for measuring Young's modulus in a flexible thin ring material. Based on the assessments, the proposed method is applicable to Young's modulus measurement in various flexible thin materials. Especially, the new method is very effective for initially curved thin materials.

References

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