

# $C^n$ continuity in Digital Image Correlation

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**Abstract.** In most measurements performed by Digital Image Correlation, a  $C^1$ -continuous deformation field is expected. Hereby, a measurement producing a smooth deformation field can be considered as more accurate than a non-smooth one. In this context, it can be assumed that a digital image correlation algorithm producing a  $C^1$ -continuous deformation field is more accurate than current DIC approaches containing  $C^{-1}$  or  $C^0$ -continuity. In the present work  $C^{-1}$ ,  $C^0$  and  $C^1$  DIC algorithms are investigated to determine whether a continuous DIC approach indeed outperform the  $C^{-1}$  and  $C^0$  approaches. The  $C^{-1}$  approach is the traditional subset method, while the  $C^0$  approach is a self adaptive higher order global approach (AdaptID). The  $C^1$  algorithm is a novel global algorithm, using a  $p^{\text{th}}$  order triangular finite element mesh. It is concluded that the higher order global  $C^1$ -continuous DIC approach is competitive against the other approaches, but does not outperform them. With this investigation it is made clear that including  $C^1$ -continuity does not necessarily provide as much benefits as could be instinctively expected. Due to the automatic refinement and great performance, the  $C^0$  approach (AdaptID) is considered as more valuable.

**The Digital Image Correlation Algorithms.** Before the comparison is made, the three different method are briefly introduced. The advantages and disadvantages are listed.

**$C^{-1}$  Approach.** As  $C^{-1}$  approach, the subset method is chosen [1]. The subset method is the most popular approach for DIC and is used in almost all correlation software. In the subset method each pixel is tracked from the reference to the deformed image using a group of pixels surrounding the considered pixel, also named the subset. Because of the independent tracking, no continuity is taken into account and robustness to noise is very dependent on the subset size. Also related with the subset size is the spatial resolution of the measurement.

**$C^0$  Approach.** As  $C^0$  approach, the AdaptID method is chosen [2]. This method is a global approach, using adaptable higher order elements in its mesh. The adaptive procedure becomes possible by using hierarchical shape functions, which can be adapted during the correlation. The change of element order is, in comparison to most global approaches, not based on user experience but on element convergence norms [3]. The use of the self adaptive algorithm results in less user dependent measurement, as the spatial resolution of the method is not limited by initial user settings. As all global approaches, the  $C^0$ -continuity of the displacement field increases the robustness against noise [4]. An extra advantage of the AdaptID method is the reconstruction of very low spatial signals by using higher order shape functions. It has been shown that increasing the element order is more beneficial than reducing the element size.

**$C^1$  Approach.** As  $C^1$  approach, a novel global algorithm is used [5]. The algorithm uses a continuous triangular finite element mesh, where the triangular elements within the mesh have a generic  $p^{\text{th}}$  order description. In this way elements can vary both in element size as element order while remaining  $C^1$ -continuity. Because of the generic higher order elements a self adaptive scheme, similar as in AdaptID, could be implemented. This would introduce the advantages of AdaptID with the extra advantage of a continuous strain field.

**Implementation.** It is worth noting that all algorithms presented above where implemented in specific developed platforms. For the subset method, the platform MatchID is used [6]. The  $C^0$  and  $C^1$  approaches are implemented in the platform AdaptID. These platforms use the same libraries for the key calculations such as interpolation, matrix calculations, filtering, etc. In this way a more honest comparison is achieved.

Algorithm	Elements	Order	Refinement	Platform
$C^{-1}$	Square Subset	Affine, Quadratic	Manual	MatchID
$C^0$	Curved Quad	Generic $p^{\text{th}}$ order	Automatic	AdaptID
$C^1$	Linear Triangle	Generic $p^{\text{th}}$ order	Manual/Automatic	AdaptID

Table 1: Overview of different correlation algorithms.

**Comparison.** In present work, the following quantities are used to compare the different algorithms: Resolution: The resolution is determined by using a so-called self correlation test. This test implies the correlation between two images where no deformation is performed. Due to noise and other influences, a deformation field between both images is measured. The resolution is defined as the global standard deviation of the biased measured field. Spatial resolution: The spatial resolution will be evaluated as the lowest period (i.e. highest frequency) of a sinusoidal deformation that a method is able to reproduce before losing a certain percentage of the amplitude. In this way, a “poor” spatial resolution is a high value and an optimum value is a low one, similar as for the resolution.

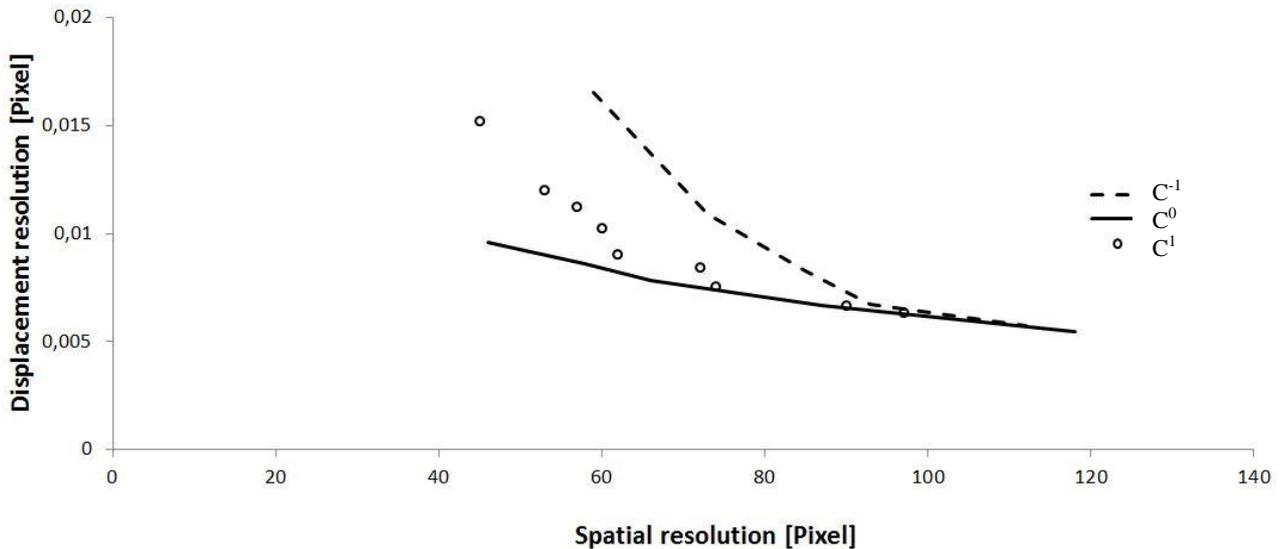


Fig 1: Displacement resolution versus spatial resolution for  $C^{-1}$ ,  $C^0$  and  $C^1$ .

Fig 1 clearly indicates the relation between resolution and spatial resolution. It is seen that a global procedure is indeed more robust to noise than the local approach. Further, Fig 1 also indicates that having strain continuity did not outperform the  $C^0$  approach. The extra constraint of  $C^1$ -continuity improves the noise robustness but also limits the elements deformation flexibility. Because of this limitation, the  $C^0$  approach has still the best resolution to spatial resolution relation.

## Conclusion.

In this work it is validated whether continuity of the displacement field ( $C^0$  or  $C^1$ ) increases the performance of DIC approaches. This statement is derived from the intuitive feeling that a smooth strain field is more accurate than a non-smooth field. This is investigated by comparing three in-house developed DIC approaches ( $C^{-1}$ ,  $C^0$  and  $C^1$ ). It is concluded that all DIC approaches are competitive against the other approaches. Using a global approach ( $C^0$  and  $C^1$ ) increases the robustness to noise towards the local approach ( $C^{-1}$ ). The introduction of strain continuity however, did not improve the performance. The constraint of  $C^1$ -continuity limits the elements deformation flexibility more than the gain in noise robustness. With this investigation it is made clear that including  $C^1$ -continuity does not provide as much benefits as could be instinctively expected. Due to the automatic refinement and great performance, the adaptive higher order global approach (AdaptID) is considered as more valuable. As the results were competitive though, the  $C^1$  approach can still be a valuable alternative when  $C^1$ -continuity is explicitly requested.

## References

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