

# Measuring Travelling Waves with Speckle Interferometry

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**Abstract.** Travelling waves can arise due to structural damage in a component and are difficult to measure without capturing the relative vibration phase across multiple points. We have used stroboscopic and high-speed speckle interferometry to measure travelling waves that were produced in a circular disc by applying a frequency modulated signal that excited degenerate modes close to the disc's resonant frequencies. We identified the wave-shape, direction and ratio of standing wave to travelling wave from the measurements. These techniques may be used to identify the presence of travelling waves or the presence on defects in the component under test.

## Introduction

Contact vibration measurement transducers can affect the response of the structure through additional mass and stiffness. Optical techniques such as the laser Doppler vibrometer overcome the effect of mass-loading although the relative vibration phase between separated measurement points will be lost for a single-impact, dynamic test. Full-field interferometers provide high spatial resolution with no mass-loading but usually cannot provide the time-resolved measurements needed for most vibration tests. Stroboscopic illumination and high-speed cameras are two possibilities for increasing the temporal resolution of the interferometer [1, 2]. CMOS high-speed cameras provide additional flexibility by switching between full-field measurements for high spatial resolution time-averaged measurements and then to smaller user-defined regions of interest (ROIs) with higher temporal resolution for transient measurements [3, 4].

The parallel measurement capability of high-speed speckle interferometry has been used to study brake squeal [5, 6]. A travelling wave at 3 kHz that contributed to the acoustic problems associated with brake squeal was visualised in a sequence of captured frames and compared to an analytic model. Full-field measurements of transient deformation have been used to identify defects in a component [7, 8]. In this paper, we use high-speed speckle interferometry to measure the ratio between standing and travelling waves in a structure.

## High-speed speckle interferometer

The interferometer used a diode-pumped, frequency doubled Nd:YVO<sub>4</sub> laser at 532 nm. The light was divided by a polarizing beamsplitter into orthogonally linearly polarised object and reference beams. Each beam was launched into the fast axis of a highly-birefringent optical fibre. The detector was a Photon Focus MVD1024 8-bit CMOS camera that enabled ROIs to be defined: the system could switch from full-field time-averaged and stroboscopic measurements to time-resolved measurements for smaller ROIs at sample rates of up to 70 kHz. Both inter-frame [3] and spatial phase-stepped [4, 9, 10] approaches were used to analyse the high-speed speckle image sequences.

## Travelling waves

A travelling wave can be excited either from a force response using multiple excitation points or by exciting the two degenerate modes that coexist near a structure's natural resonance. The latter approach produces a combination of multiple standing waves that have slightly different wavelengths and frequencies, and interfere with each other to produce a travelling wave. These combined modes produce a response profile that comprises a mixture of standing and travelling wave components. The multiple waves that combine to form a propagating wave can be assumed to have an angular position ( $\theta$  for a circular object) for a single frequency,  $\omega$ . The spatial wave has wavelength  $\lambda_K=2\pi/K$  and the response at a particular point takes the form:

$$\begin{aligned}\omega(\theta, t) &= [A_1 \cos(K\theta) + A_2 \sin(K\theta)]\cos(\omega t) + [B_1 \cos(K\theta) + B_2 \sin(K\theta)]\sin(\omega t) \\ &= \omega_+ + \omega_-\end{aligned}\tag{1}$$

where  $A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$  are the peak values of the spatial wave functions and

$$\begin{aligned}\omega_+ &= [ (A_1 + B_2)\cos(\omega t - K\theta) + (B_1 - A_2)\sin(\omega t - K\theta) ]/2 \\ \omega_- &= [ (A_1 - B_2)\cos(\omega t + K\theta) + (B_1 + A_2)\sin(\omega t + K\theta) ]/2\end{aligned}\tag{2}$$

Eq. 2 represents a propagating wave that includes both standing and travelling components. The terms  $(\omega t - K\theta)$  and  $(\omega t + K\theta)$  in Eq. 2 determine the positive and negative direction of the travelling wave. The real and imaginary parts of Eq. (1) can be plotted on the complex plane and produce an ellipse if both travelling and standing waves are present. The direction of the travelling wave can be determined from the gradient of the phase of the spatial wave across the ROI and the ratio of travelling wave to standing wave is given by the ellipse's major and minor axes:

$$\text{Standing wave ratio} = \frac{|\omega_+| - |\omega_-|}{|\omega_+| + |\omega_-|} \quad (3)$$

## Results

Fig. 1 shows the least squares fit of an ellipse to the measured spatial response data for the circular disc (aluminium alloy, diameter 14 cm, thickness 1 mm) clamped at its centre. The disk was excited using a B&K Type 4810 shaker attached to the disc with a stinger and bolt through a 2 mm hole in the plate, and the force was measured using a B&K Type 8203 force transducer at 241.5 Hz with a 241 Hz carrier frequency to excite the degenerate mode. Both the phase distribution from the interferometer and the gradient of the ellipse major axis in Fig. 1 indicate the combination of travelling and standing wave. Fig. 2 shows excitation at 234 Hz with a 241 Hz carrier frequency which is too large a frequency difference to excite degenerate modes: the straight line fit indicates a standing wave with little or no travelling component.

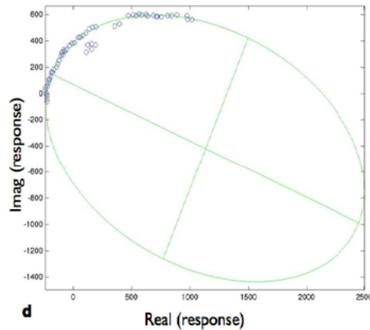


Fig. 1: Travelling wave

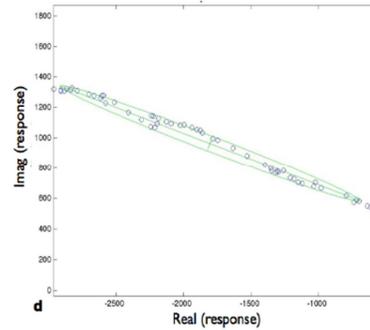


Fig. 2: Standing wave

## Conclusions

Spatial phase-stepped high-speed speckle interferometry has been applied to identify the presence of travelling waves. The fit to the real and complex parts of the vibration data that gave the ratio of standing wave to travelling waves. These parameters can be determined from a single dynamic test due to the full-field, time resolved-measurement capability of the system.

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