Mode I-III decomposition of the *J*-integral to calculate the stress intensity factors from digital image correlation displacement data

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Abstract. Displacement fields resulting from cracking are widely available due to the success of full field optical methods such as digital image correlation (DIC). Several methods exist to quantify cracks, mainly in terms of stress intensity factors (SIFs) or energy release rates; however, simplicity of use is a key factor as to whether such methods are adopted in practice. The proposed method provides direct access to decoupled energy release rates, and SIFs in linear-elastic materials. This is achieved by the mode separation of displacement fields based on symmetry about the crack plane, as originally proposed by Ishikawa et al. [1]. The results show that reliable estimates are obtainable from specimens under mixed loading. The method is recommended for the experimentalist who requires simple and efficient calculation of SIFs from mixed mode displacement data, where displacement fields are not limited to those resolved by the DIC technique.

Introduction

In many applications, stress intensity factors (SIFs) provide valuable insight into crack propagation and the prediction of failure. Several methods are available that allow for the measurement of mixed-mode SIFs from full-field displacement data. A prevalent method for obtaining full-field displacement data is digital image correlation (DIC), which resolves displacement fields from sequences of digitally stored images captured during various material deformation stages. However, DIC provides poor resolution close to the crack faces and the crack tip. Therefore, the J-integral approach is of particular interest as it is able to calculate strain energy release rates through a single integration contour path in elastic or elastic-plastic materials; and may be converted to equivalent SIFs if a linear-elastic material is assumed. Furthermore, it is possible to select contours far from the crack tip where DIC matching achieves the highest accuracy.

The classical J-integral returns the total strain energy release rate as the sum of mode I-III components.

$$J = J_I + J_{II} + J_{III} \tag{1}$$

The decomposition method, originally proposed by Ishikawa et al [1], directly separates the crack tip fields based on their distinct symmetry characteristics about the crack plane.

Methodology and results

Provided a prior knowledge of the crack plane, DIC displacement fields (u_{DIC}) may be decomposed directly into separate mode I-III components (M = I-III):

$$u_{DIC} = \sum_{M=I}^{III} u_M = \{u_I\} + \{u_{II}\} + \{u_{III}\} = \frac{1}{2}\begin{Bmatrix} u_1 + u_1' \\ u_2 - u_2' \\ 0 \end{Bmatrix} + \frac{1}{2}\begin{Bmatrix} u_1 - u_1' \\ u_2 + u_2' \\ 0 \end{Bmatrix} + \frac{1}{2}\begin{Bmatrix} 0 \\ 0 \\ u_3 - u_3' \end{Bmatrix} \tag{2}$$

where, the notation u_i' represents a displacement field reflected about the crack plane ($u_i' = u(x_1, -x_2)_i$), in the local co-ordinate system of the crack front (Fig. 1a). Subsequently the J-integral may be calculated on separate mode I-III displacement fields to yield decoupled J_{I-III} results;

$$J_M = \int_\Gamma \left(W_M n_1 - \sigma_{M_{ij}} \frac{\partial u_{M_i}}{\partial x_1} n_j \right) ds \quad (i,j = 1,2; M = 1,2), \tag{3}$$

$$J_{III} = \int_\Gamma \left(W_{III} n_1 - \sigma_{III_{3j}} \frac{\partial u_{III_3}}{\partial x_1} n_j \right) ds \quad (j = 1,2) \tag{4}$$

Mode separated strain fields ($\varepsilon_{M_{ij}}$) were evaluated by differentiation of the mode I-III displacement fields, $\varepsilon_{M_{ij}} = \partial u_{M_i}/\partial x_j$. As DIC displacements are measured on the surface, and only contours remote from the crack tip were considered, stress ($\sigma_{M_{ij}}$) and strain energy density (W_M) fields could be calculated assuming plane stress and linear elasticity. Note that mode III anti-plane shear loading requires that in-plane tractions are zero, resulting in Eq. 4. Subsequent K_{I-III} values could be obtained by the relation: $J_{I-II} = (K_{I-II})^2/E$, and $J_{III} = (K_{III})^2/2G$, where G is the shear modulus.

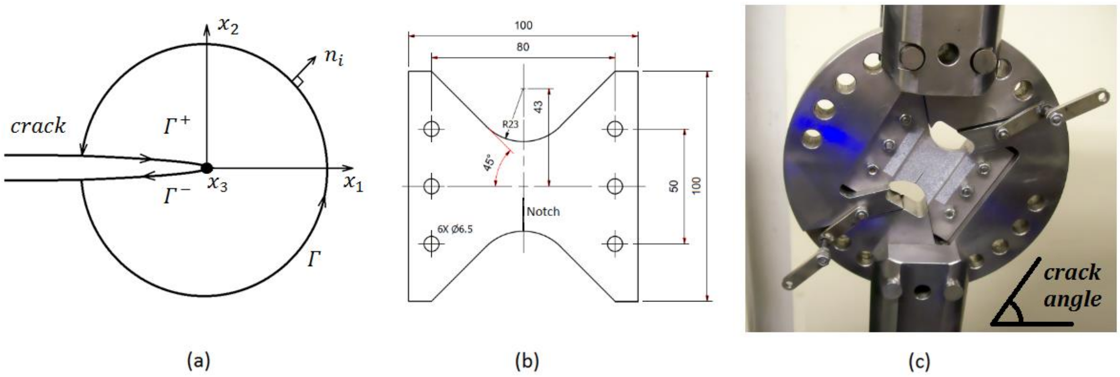

Figure 1: J line integral and crack tip local co-ordinates (a), Arcan specimen (b) and modified Arcan setup (c)

Mixed loading at 15 degrees in-plane rotation and fixed 3mm anti-plane displacement was achieved with a modified Arcan setup (Fig. 1c) using 6 mm PMMA (Polymethylmethacrylate) specimens as shown in Fig. 1b. Resulting decomposition throughout the sequence of images taken during loading resulted in increasing K_{I-II}, and constant K_{III} estimates, normalised to the values for K_{I-III} measured at fracture (Fig. 2). In addition, K_I and K_{II} estimates are compared with analytical solutions from Banks-Sills and Arcan [3], and SENT specimens respectively (mode III Arcan solutions were not available). Material properties (Young's modulus, $\mathrm{E} = 3\,\mathrm{GPa}$; Poisson's ratio, $\upsilon = 0.36$) and K_{Ic} $(1.7\,\mathrm{MPa}\sqrt{\mathrm{m}})$ values were ascertained from ASTM tests, and showed close agreement with mode I results using the Arcan specimen geometry.

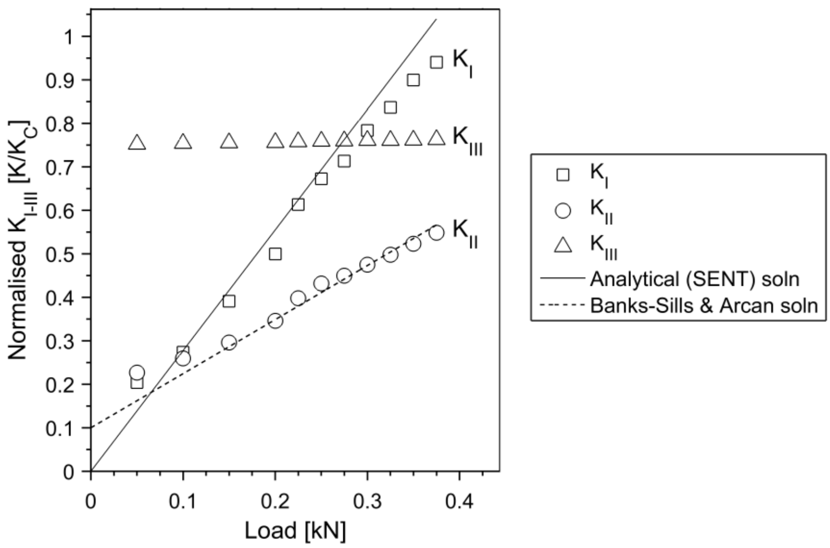

Figure 2: Normalised K_{I-III} results compared to analytical SENT and Arcan solutions

Conclusion

This work demonstrates that displacement data obtained from DIC may be used to decompose mode I-III of the J-integral to obtain equivalent SIFs. Agreement between the proposed method and analytical solutions demonstrates that the method also provides accurate separation of the failure modes.

References

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