Novel Inertial Heterogeneous High Strain Rate Test for Non-Linear Constitutive Model Identification with the Virtual Fields Method

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Abstract. The study of the mechanical behaviour of materials at high strain rates is of growing interest in many engineering applications such as automotive crash-tests, high speed machining and defence. With the development of modern high speed cameras, full-field deformation of solids at high strain rates can be recorded with techniques such as Digital Image Correlation (DIC). Current high strain rate techniques such as the Split Hopkinson Pressure Bar (SPHB) fail to provide entirely reliable experimental data for material model identification. Recently, the Virtual Fields Method (VFM) has enabled the identification of linear elastic parameters from full-field strain measurements at high strain rates with ultra-high speed cameras with remarkable accuracy. The idea is to use fields of acceleration as a load cell, whose virtual work is balanced out with the virtual work of stresses in the principle of virtual work, in the novel test configuration proposed. This promising approach is presently taken to identify non-linear model parameters at high strain rate with the VFM. Elasto-plastic and elasto-viscoplastic models for metals and polymers are considered. The procedure developed for the identification of simple elasto-plastic and elasto-viscoplastic models at high strain rates will be applied on a novel inertial test recorded with an ultra-high speed camera.

Introduction

Engineering applications at high rates of strain are an area of growing interest, especially in dynamic material behaviour modelling. A current high strain rate testing technique is the Split Hopkinson Pressure Bar, a widely used one dimensional stress wave propagation apparatus that has shown for many years experimental limitations due to the constraining requirements on the system and specimen geometries.

The development of modern high speed cameras has opened the way to the recording of full-field deformation of solids at high strain rates, with techniques such as Digital Image Correlation (DIC) or the grid method. DIC is a method based on camera framing of grey level speckle patterns deposited at the surface of test specimens. The grid method is another method for strain measurement using deposited grids with grey level contrast. Where the current high strain rate testing techniques fail at providing reliable experimental data for detailed material modelling, the use of the Virtual Fields Method [1] enables mechanical model identification directly from full-field strain measurements.

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Methodology

As recently realised in elasticity [2], the identification of the stiffness components of composite materials has been realised at strain rates up to 3000s-1 using the grid method with the experimental set-up of Fig. 1. The principle used was to reconstruct stresses from the acceleration field, differentiated from the time-resolved displacement measured with DIC or the grid method. No external force measurement such as in the SPHB is necessary, the camera recording sufficient information to identify the two stiffness components.

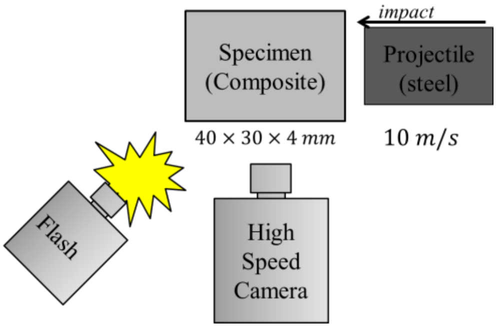

Figure 1: Schematic of previous work in linear elasticity [2].

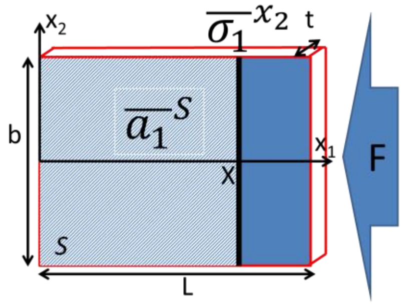

Figure 2: Specimen schematic

Stress reconstruction from the acceleration field. The Virtual Fields Method is applied to the specimen represented Fig. 2 with the virtual field described in Eq. 1. The principle of virtual work leads to the expression Eq. 2 where the external impact force F is not involved, and only the stress and virtual strain tensors **σ** and **ε***

are related to the vector fields of acceleration a and virtual displacement u*. ρ is the mass density of the material. After simplification by the constant thickness t, the integrals are simplified to the mean of stress σ_1 over the x_1=X line and the mean of acceleration a_1 over the surface S, given the discrete and regular form of the fields measured (see [2] for more developement details). It gives the final expression Eq. 3. The bar symbol denotes averages over domains in subscript. The expression Eq. 3 is valid for all sections x_1=X.

$$x_1 = X, \begin{cases} u_1{}^* = x_1 \\ u_2{}^* = 0 \end{cases} \Rightarrow \begin{cases} \varepsilon_1{}^* = 1 \\ \varepsilon_2{}^* = 0 \\ \varepsilon_6{}^* = 0 \end{cases} \quad \left(x_1 < X, \begin{cases} u_1{}^* = X \\ u_2{}^* = 0 \end{cases}, X < x_1 < L, \begin{cases} u_1{}^* = 0 \\ u_2{}^* = 0 \end{cases} \right) \Rightarrow \begin{cases} \varepsilon_1{}^* = 0 \\ \varepsilon_2{}^* = 0 \\ \varepsilon_6{}^* = 0 \end{cases} \tag{1}$$

$$\int_V \boldsymbol{\sigma}.\boldsymbol{\varepsilon}^* dV = \rho \int_V a.u^* dV \tag{2}$$

$$\overline{\sigma_1(x_1 = X)}^{x_2} = \rho X \overline{a_1}^S \tag{3}$$

Stress reconstruction from the strains. Stresses can also be reconstructed from strains, given non-linear models, describing the elastic-plastic and elasto-viscoplastic behaviour of metals and polymers at high strain rate. A bilinear isotropic hardening elasto-plastic model, of parameters the elastic limit σ_0 and the hardening modulus E_t is first considered. The impact at a speed of 20m.s^{-1} of a 50x40x5mm rectangular coupon with purely elastic properties on a metal specimen of same dimensions as in Fig. 1 is simulated in Abaqus. The strain and acceleration data of the specimen are exported into 2D-maps at different loading times in Matlab®, likewise DIC data. An iterative routine for stress calculation from strains, called Return-Mapping Algorithm, is used to reconstruct stress maps from the two parameter values, based on [3].

Identification by cost function. By difference between the reconstructed stress from the strain field and the stress reconstructed from the acceleration, both calculated from the displacement measured in DIC, the cost function C_f in Eq. 4 can be built. A minimization algorithm such as described in [4,5] seeks the best matching set of sought parameters σ_0 and E_t, closest solution of the non-linear model identification problem.

$$C_f(\sigma_0, E_t) = \sum_{i=1}^{n} \sum_{t=t_0}^{t_f} \left[\overline{\sigma_1(\sigma_0, E_t)}^{x_2} - \rho X_i \overline{a_1}^{S_1} \right]^2 \tag{4}$$

with t the discrete time steps, X_i the discrete abscissa of each vertical section on the maps, and σ_1 and a_1 the axial stress and acceleration maps. The FEA strain and acceleration data are used to validate the procedure by identifying the initial values inputted in the FEA model.

Results and discussion

Both parameters are identified with accuracy below the percent (Fig. 3), and are to be compared with high speed DIC (or grid method) experiments soon. The possible sources of errors in identification have been investigated, such as the resolution and interpolation used to project the FE data onto regular strain maps, the inaccuracy in stress reconstruction coming from the iterative procedure and also the time resolution. Experimental conditions are expected to increase errors so test conditions will require modelling and optimization.

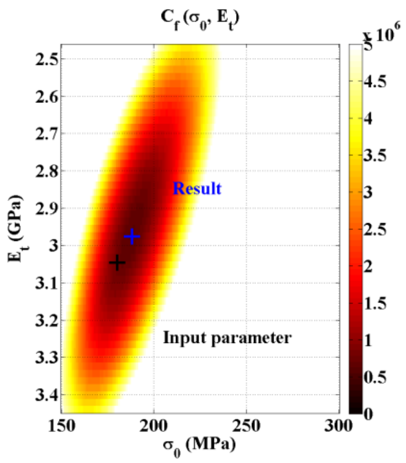

Figure 3: Cost function values

Conclusions and future work

The computer routines used to identify two non-linear elasto-plastic material model parameters at high strain rates have been validated on finite element data. Future experimental implementation on metal and polymer specimens will be presented. Identification results of elasto-plastic and elasto-visco plastic model parameters for metals and polymers should be presented.

References

[1] F. Pierron, and M. Grediac: *The Virtual Fields Method: Extracting Constitutive Mechanical Parameters from Full-field Deformation Measurements*, Springer New York (2012).
[2] F. Pierron., H. Zhu and C. Siviour: *Beyond Hopkinson's bar*. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, Vol. 372(2014), p. 1471-2962.
[3] M.A. Sutton, X. Deng, J. Liu and L. Yang: Experimental Mechanics, Vol. 36 (1996), p.99-112.
[4] S. Avril, F. Pierron, Y. Pannier, and R. Rotinat: Experimental Mechanics, Vol. 48 (2008), p. 403-419.
[5] M. Grediac, and F. Pierron: International Journal of Plasticity, Vol. 22(2006), p. 602-627.

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