Preliminary evaluation of validation metrics for computational mechanics models.

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Introduction

Computational models are widely used to assess and predict future behaviour of engineering systems. Due to the increasing computational capabilities, it is now possible to simulate a large variety of processes. However, it is desirable that simulation does not just correctly compute the underlying mathematics used to model the physics but represents the real situation of the system`s intended use. The provision of the credibility of obtained results becomes vital and can be achieved through the Verification and Validation process. In the scope of this study it will be assumed that verification procedures, concerned with the accurate representation of the underlying mathematics, have been performed with certain level of confidence and thus validation, i.e. ‘the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model’ [1], can be applied.

A number of guides and approaches for validation in the area of mechanics are currently available (see e.g. [1, 2]). A similar template for the main processes is apparent throughout these guides; nonetheless, there is no single validation methodology that is widely accepted. Usually, for model validation comparison between experimental and computational data sets is performed, with the experimental data used as a referent, and the outcome is a decision on whether the model is valid or not. This outcome may be sufficient for some cases, but ultimately it is desired to quantify the quality of the model with respect to the validation criteria. Another important aspect motivating this research on validation techniques is modelling of the materials and structures for which only limited knowledge about their mechanical behaviour is available; in turn, this presents opportunities for model updating through parameter identification.

The objective of this work is to investigate and compare some of the existing validation metrics and evaluate their application to the computational model of an indented rubber block (Fig. 1a).

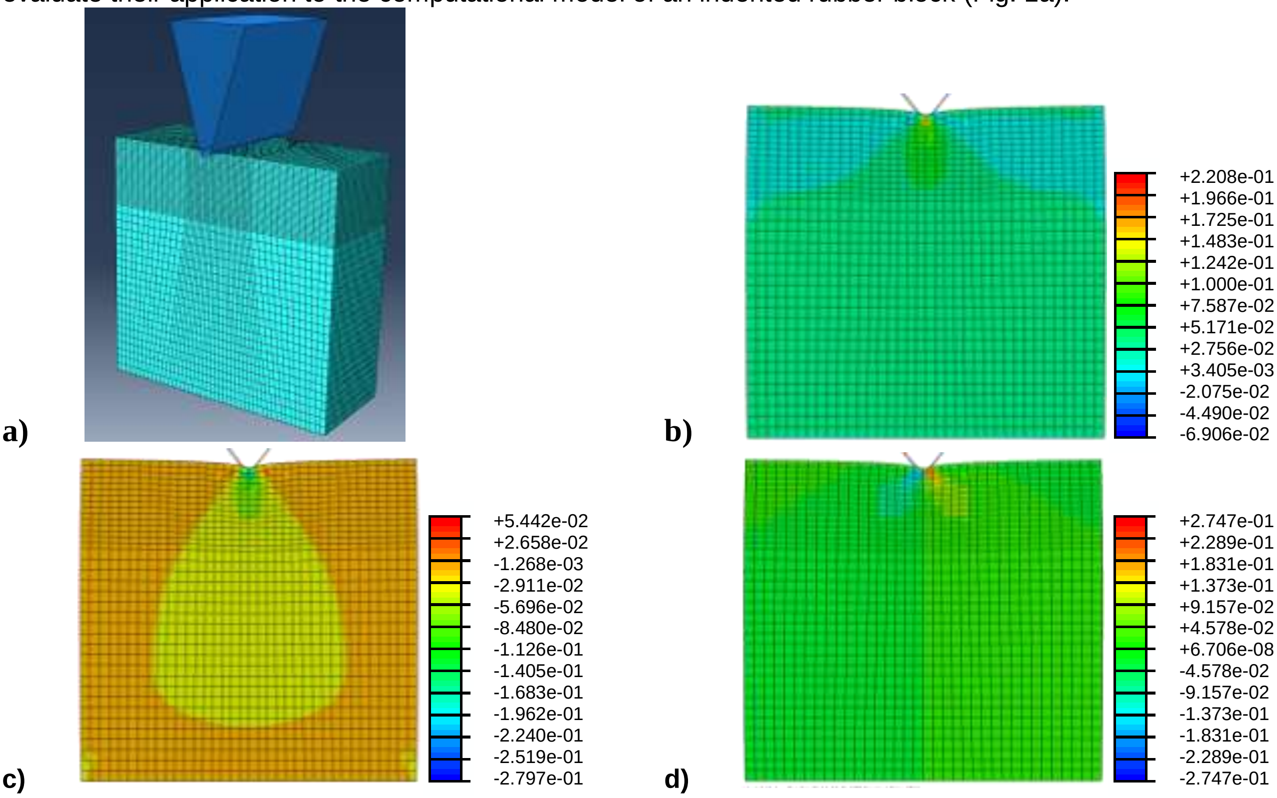

Figure 1: a) Finite element model of the rubber block subject to a 2mm indentation by a rigid wedge and the corresponding strain maps in b) XX, c) YY and d) XY directions.

Methods

Validation can be divided into two stages: application of validation metric, which is the comparison of data sets, and evaluation of this outcome with respect to the accuracy requirements for the intended use of the model. The choice of the validation metric is governed by the data available and by the result required. In this study different techniques from Frequentist approach and Hypothesis testing approach will be compared, in order to distinguish their advantages and disadvantages when considering issues mentioned earlier. In addition, new application of the Frequentist metrics will be investigated.

Hypothesis testing approach. It is common practice to determine simply whether the model is valid or not and this is the essence of the Hypothesis testing approach. An example of such approach has been recently presented in the CEN publication [4] and is based on a technique proposed in [2]. They suggest comparing computational results with experimental data, obtained with the aid of the full-field methods for strain measurement, using the image decomposition technique. Predicted and measured strain fields are treated as images and are decomposed using appropriate polynomial to describe the essential features of the image. The coefficients of these polynomials, i.e. shape descriptors s_E and s_M corresponding to experimental and computational data respectively, are then compared by plotting one against the other. An acceptance band is calculated using the experimental uncertainty $u(s_E)$, such that model validity is established when all of the data points lie within the region defined by

$$s_M = s_E \pm 2u(s_E) \qquad (1)$$

Frequentist approach. This approach comprises of mapping a discrepancy between the computational and the experimental response, and is based on the statistical confidence interval; the outcome includes the degree of the validity of the model. Three different techniques are evaluated here. The first technique is based on Theil's Inequality Coefficient, where the data variance estimates are considered. The second technique is based on a validation benchmark developed by Oberkampf and Barone [4], in which the model validity is established through comparing deterministic computational results with the estimated mean of the experimental measurements. The third technique is based on the Chi-squared test.

Application

Rubber components are commonly used in structures for various applications and usually undergo compression in service, however only a limited understanding of the behaviour of the material is currently available. In this study, a rubber block (60mmx60mmx25mm) indented by 2mm with an aluminium wedge, as shown in Fig.1a, is considered. An experiment was previously conducted [5] and a finite element model has been developed as part of the VANESSA Inter-laboratory study by other researchers. We have modified the model using Abaqus 6.11 software package and then, using their experimental data, performed the validation by the methods described above. Frequentist methods, in comparison to the Hypothesis testing methods, allowed a better understanding of the quality of the current model, through quantifying the differences between the predicted and measured results. Previously these metrics have been applied to point-to-point data; in the present work we have extended their application to a matrix of data, since strain maps formed on the entire surface of the specimen are of the interest (Fig. 1b,c,d).

Conclusions and future work

Validation becomes a significant step as computational models are more frequently used to understand the mechanical behaviour of novel materials and structures. As an outcome of the validation, it is desired to obtain a quantity that would represent not just the validity of the model but also the degree of the quality. Additionally, the validation metric should be applicable to sparse data sets or when only limited knowledge about the behaviour of the material and structure is available. In the present work, we have researched different validation metrics to distinguish their advantages, disadvantages and implementation issues relative to the desired outcome of the validation. A new application of Frequentist metrics has been investigated, including the use of the matrix of data. As the next step, a more complex validation approach such as Bayesian will be investigated for the same application and results will be compared to the Frequentist approach.

Acknowledgments

K. Dvurecenska receives funding from the EPSRC and the National Nuclear Laboratory. E. A. Patterson is a Royal Society Wolfson Research Merit Award holder.

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