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Young Stress and Vibration Analyst Competition Finalists

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A validated point-wise approach to the analysis of stresses and strains around complex composite geometries using Digital Image Correlation and Thermoelastic Stress Analysis

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Introduction
Composite materials are weak normal to the plane of the laminate due to a lack of through-thickness reinforcement, with strength dictated by the brittle epoxy matrix. Therefore it is important to evaluate the through-thickness load transfer in composite bonded joints because of their discontinuous nature to improve confidence in the join and inform more efficient joint designs.

Joint structure
A double butt strap joint (DBSJ), Figure 1, was constructed with 800g/m² unidirectional and 450g/m² chopped strand matt glass fibre in a [CSM₈ 90₄₄]₈ sequence using Gurit Prime 20lv epoxy resin using the resin infusion process. Araldite 2015 epoxy adhesive was used to bond the adherends.

Meso scale analysis
Component strains within the DBSJ were evaluated using 2D DIC. A Canon mp-e65 macro lens connected to a 5Mp LaVision 5Mp E-Lite camera was used to image an area of 3.1 mm x 2.6 mm around the discontinuity between adherends. The specimen was mounted in an Instron 5569 test
machine and loaded at 1mm/min up to failure. Complex localised strain distributions were revealed, shown in Figures 2 and 3. Detailed analysis of the strain fields identifies small, yet critical, through-thickness and shear strains evolving within the joint. To validate the DIC strain values, Thermoelastic Stress Analysis (TSA) was conducted. TSA provides a fast and accurate experimental technique, to capture the complex behaviour around the join at similar length scales as the DIC. The two independent experimental data sets provide sufficient detail to fully inform numerical models.

![Figure 2: Peel strain distribution at 18kN](image1)

![Figure 3: Shear strain distribution at 18kN](image2)

**Validation of DIC strains using Thermoelastic Stress Analysis**

When a material experiences a stress change it is accompanied by a small temperature change $\Delta T$. The relationship between $\Delta T$ and the change in the sum of principal stresses, $\sigma_1$, $\sigma_2$ is as follows for an orthotropic material

$$\Delta T = -\frac{T}{\rho C_p} \Delta(\alpha_1 \sigma_1 + \alpha_2 \sigma_2)$$  \hspace{1cm} (1)$$

where $\alpha_1$ and $\alpha_2$ are the coefficients of thermal expansion in the principal stress directions, $\rho$ is the material density, $C_p$ is the specific heat at constant pressure, and $T$ is the specimen temperature. These constants form the ‘thermoelastic constants’ in the principal stress directions $K_1$, $K_2$, providing the relationship between the sum of the principal stresses,
\[
\frac{\Delta T}{T} = -(K_1 \Delta \sigma_1 + K_2 \Delta \sigma_2)
\]  

(2)

\(T\) and \(\Delta T\) are scalar quantities. This equation can be rearranged into a form that provides a calibrated stress metric, but this changes equation (2) into the following tensorial quantity:

\[
\text{calibrated stress metric} = \frac{\Delta T}{K_1 T} = \Delta \sigma_1 + \frac{K_2}{K_1} \Delta \sigma_2
\]  

(3)

In [2] and [3] methodologies were established to obtain the thermoelastic constant in the principal material directions parallel and transverse to the fibre direction, \(K_P\) and \(K_T\). These were used in subsequent joint analysis making the assumption that the material directions and the principal stress directions were coincident. This assumption worked well away from the region of the discontinuity. In the current work detailed analysis takes place close to the root of the join between adherends. Due to the geometric discontinuity there is significant variation in the principal stress directions, hence the thermoelastic constants in the principal stress directions are required.

The DBSJ was loaded at 20 Hz with a mean load of 6 kN and loading amplitude of 3 kN in an Instron 8800 test machine. TSA was conducted by imaging the root of the join using a Flir Silver SC5000 infra-red detector fitted with a G1 high resolution macro lens.

The DIC strain and TSA stress data were manipulated into the form of the calibrated stress sum in equation (3) for direct comparison. The DIC data showed that the principal strain (stress) directions are not coincident with the principal material directions, and rotate significantly in the vicinity of the discontinuity. This means that the \(K_P\) and \(K_T\) values obtained using the calibration specimens in [2] and [3] are not valid. Therefore they must be transformed into values associated with the principal stress directions using data from the DIC point by point around the geometric discontinuity for each TSA data point. Likewise, it is necessary to transform the DIC component strains to principal strains and use transformed material stiffness matrices to obtain the principal stresses, which are substituted into equation (3), along with the transformed thermoelastic constants. The TSA/DIC transformation methodology is shown in Figure 4a and 4b.
Figures 5 and 6 show the two comparable DIC and TSA data sets. There is very good agreement showing both techniques provide accurate measurements of the complex joint behaviour as well as validating the two experimental techniques. The position of the stress concentration corresponds with the failure initiation sites observed from experiments. The combination of high peel strain and stress concentrations at the root of the discontinuity in the inner adhered lead to the initiation of cracks, before propagating along the preferential high strain, low stress, region at the adhesive/adherend interface in the outer adhered.
Conclusions

The principal strain directions obtained from DIC studies have been used to enable TSA of a complex composite structural component using a point by point calibration. For these complex geometry situations the assumption that the principal material and principal stress axes are coincident cannot be made. Analysis using the two experimental techniques has shown how the heterogeneous stress and strain fields are intricately linked to the initiation and propagation of damage, in addition to final failure behaviour of the joint. Validation of these two experimental techniques adds a greater depth to the analysis of the joint under load, allowing evaluation of both stresses and strains within the structure.


Assessment of Corneal Deformation Using Optical Coherence
Tomography and Digital Volume Correlation

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Introduction
The study of the mechanical behavior of the cornea under intraocular pressure is important for the assessment of corneal biomechanics for instance for pathology assessment. Recent advances in optical coherence tomography (OCT) \cite{1, 2} enable the non-invasive and non-destructive reconstruction of the volume microstructure of semitransparent inhomogeneous samples such as corneas. Swept Source Optical Coherence Tomography (SS-OCT) systems are able to provide sample microstructure volumes with high sensitivity and at video rates. Digital image correlation can be applied to measure surface full-field deformations of the cornea \cite{3}. However, 3D full-field deformation measurements have rarely been obtained yet. In the present work, digital volume correlation is applied to measure the volume displacement and strain fields from the volume images generated through SS-OCT of phantom and porcine cornea samples under posterior inflation, which is more representative of the physiological state than the more standard uniaxial tests \cite{4}. An important objective of this work is to study in detail the metrological performances that can be expected from the digital volume correlation procedure when using SS-OCT volume images.

Methods
A schematic and photo of the experimental set-up is shown in Fig 1. The specimen was mounted and fixed on the artificial anterior chamber (AAC) which has inlet and outlet for the fluid and a pressure transducer. The simulated intraocular pressure was achieved by adjusting a 1 ml micro-syringe. At each pressure increment, a 3D volume image sequence of the specimen was acquired using a Swept Source Optical Coherence Tomography system (Thorlabs OCS1300SS). Both silicon resin phantom (seeded with titanium oxide particles) and porcine cornea were used as the specimen in the inflation test. The outer edge diameter and thickness of the phantom are 15.6 mm and 0.58 mm, respectively. Regarding the porcine cornea, the diameters of the maximum and minimum meridians are 13 mm and 10 mm respectively, while the average cornea thickness for the central part of the cornea is 1.2 mm. The cornea and phantom samples were inflated from 2 to 2.25 kPa as illustrated in Fig. 1. The reconstructed volumes at both load steps were recorded for digital volume correlation and displacement and strain fields computed. Fig 2 shows a central transverse slice of the phantom and porcine cornea, respectively.
Due to the less adequate speckle contrast in the bottom regions, geometrical mask was added to mask out the regions with low quality, as illustrated in Fig 2 (b). Then three dimensional digital volume correlation was performed on the volume images using the Davis (LaVision) commercial package based on a fast Fourier transform algorithm.

![Fig 1. Schematic diagram and photo of the experimental set-up](image)

**Fig 1. Schematic diagram and photo of the experimental set-up**

Fig 2. Central transverse slice of the phantom and porcine cornea generated through the SS-OCT system, scales show dimensions in the phantom and porcine cornea specimens

### Noise study

The effect of noise and reconstruction uncertainties are evaluated through correlation of two subsequent reconstructed volumes of the stationary phantom. The displacement and strain resolutions of the DVC are evaluated after calculating their mean and standard deviation values. Then several reconstructed volumes are recorded one after the other, introducing a rigid body translation of 10 microns between successive images to examine the performance of the DVC algorithm in producing displacement and strain fields. Their mean and standard deviation values are then calculated and related to the quality of image recording. From Fig 3 (a) it can be observed that for the stationary test the mean values of all strain components are very small and close to zero, while their standard deviations are between 0.05% and 0.1%. Similar result is found for the rigid body translation test in Fig 3 (b) of which the strain standard deviation is slightly larger.

![Fig 2. Central transverse slice of the phantom and porcine cornea generated through the SS-OCT system, scales show dimensions in the phantom and porcine cornea specimens](image)
The influence of sub-volume size on the correlation precision is analyzed quantitatively then. 4 different sub-volume sizes were chosen for the stationary test. The strain resolutions are compared in Fig 4 for $\varepsilon_{xx}$ and $\varepsilon_{yy}$ only, for the sake of legibility. Among different sub-volume sizes, 24x24x24-voxel sub-volume is found to be a good compromise between the strain resolution and spatial resolution. Further increasing it gives slight improvement for the resolution, which, however, degrades the spatial resolution. This is particularly critical here because of the small thickness of the phantom and cornea compared to the spatial resolution of the SS-OCT system. As a consequence, a sub-volume size of 24x24x24 will be kept for the rest of this study.

Results and Discussion

Typical displacement fields along the X, Y and Z axes for a central z-slice for both phantom and porcine cornea specimens under inflation conditions are shown in Fig 5 and compared with an elastic finite element (FE) model ($E=0.9$ MPa, $\nu=0.48$) of the porcine cornea. The displacement fields of the phantom are close to that of the porcine cornea and both of them match the results of the FE model, which is consistent with the inflation condition. The difference observed between the z-displacement maps for the phantom and porcine cornea is due to the z-slice chosen in each case. For the porcine
cornea, z-slice is located in the front section, while that of the phantom is in the back section.

The strain fields for the same transverse central slices of phantom and porcine cornea were calculated from the displacement data and are shown in Fig 6. As expected from the displacement fields, the strain maps of the phantom and porcine cornea are also consistent with those from the FE model. The results show that the major deformation occurs in the inner central part of the cornea, while the outer part is less deformed. Comparing the strain values from the inflation tests to the strain resolution, it can be observed that the strain values are much greater, which verifies that these distributions are the results of material behavior rather than noise artifact. Although there are some apparent irregularities such as the negative $\varepsilon_{zz}$ in the central z-slice, generally the strain maps are consistent with expectations for both specimens.

**Conclusions**

In the present study, digital volume correlation was carried out on the volume images generated through swept source optical coherence tomography. A 24x24x24-voxel sub-volume was found to be a good compromise between resolution and spatial resolution. Based on this sub-volume size, reasonable displacement and strain results on the inflation test were obtained. The strain standard deviation is significantly lower than the strain values. Future work will focus on the use of these data to identify the elastic behavior of the cornea using the Virtual Fields Method [5].
Fig 6. Strain fields for: (a) phantom, (b) porcine cornea and (c) FE model inflated from 2 to 2.25 kPa

References


In a commercial environment, equipment life and availability is of critical commercial importance, but safe operation is vital. In industry, corrosion and erosion damage to pressure vessels and pipework is common, leading to defects such as localised thinning. The thickness in these defect areas are often beyond the allowable limits specified for general thickness in the applicable design codes, but general thickness requirements may be over conservative for local thinning. If adequate justification could be provided to allow continued safe operation in the damaged condition, equipment availability could be improved. One method of justification is to model the defects using finite element analysis (FEA) to determine the resulting stress state and hence, margin before failure. Validation of the results to ensure they are reliable and accurate must be carried out before it can be used with confidence. The aim of this project was therefore to determine a conservative methodology of creating and analyzing simplified models of locally thinned pressure equipment using a specified finite element package, Solidworks that could be employed to confidently prolong equipment life at INEOS ChlorVinyls and to validate it through experimental testing. INEOS ChlorVinyls wish to use Solidworks for routine assessment of defective pipework.

Defects were artificially introduced into a new 6” carbon steel, seamless pipe and aligned along the pipe at appropriate intervals following the guidelines of Saints Venant’s Principle [1]. A 40bar analogue pressure gauge was attached to the pipe and water pressurisation used to exert an internal pressure. The flange rating limited the experimental pressure to 21bar.

An internally pressurised steel pipe was modelled and analysed using the FEA package Solidworks following the recommendations given in BS7910 [2]. An element size of 13mm was used for the no defect and defect flaw areas following mesh convergence analysis. If INEOS ChlorVinyls adopted the FEA method to assess defective pipes, the equipment being investigated would be in-service, thus any material properties not given on the material test certificate could not be determined through experimental testing. Therefore to follow the procedures that would be adopted by INEOS ChlorVinyls, Poisson’s ratio and Young’s modulus were investigated parametrically to obtain appropriate uncertainties for the simulation results.

The experimental data was obtained using a commercially available digital image correlation (DIC) system (Dantec Dynamics Q-400, Ulm, Germany) and used to validate the displacement and strain results obtained from the FEA model. A 3D DIC technique was used due to the non-planar nature of the samples being tested [3]. The cameras were set-up parallel to the axis of the pipe as shown in figure 1. Prior to the experimental testing a full system calibration was carried out following the procedures given by the SPOTS consortium [4]. A system uncertainty of 3.8% for values of strain in the region of 149µ strain was found; combining this with the experimental uncertainties gave a total DIC uncertainty of 4.6%. In the analysis of the DIC results a facet size of 17 pixels and a grid spacing of 14 pixels were used based on a convergence analysis.

Comparisons of the FEA and DIC displacement maps for a defect are shown in figure 2. The simulation predicts similar behaviour to that obtained using the DIC, with the maximum values of strain in the same location, but the poor quality mesh on the FEA does not allow for detailed analysis of the defect area.

Comparison of the results for the maximum out of plane displacement predicted by the FE model with those found using DIC and thin walled pressure vessel theory were within 8% and 15% respectively for an internal pipe pressured up to 20bar. The corresponding difference for a pipe with a defect was 31% when comparing the FE and DIC (figure 3). This large difference suggests that the FE analysis
provides a conservative result, i.e. it predicts higher deformation and stresses than are experienced in-practice. An attempt was made to calculate values of strain from the DIC results for displacement using the algorithms supplied with the DIC system. However the results were obviously incorrect and unreliable following a comparison with a calculated gross value circumferential strain from the DIC displacements. A difference of 57% was found, with the actual DIC algorithm results being the larger, it was therefore logical to disregard the DIC strain results as the time-scale of the project did not allow this issue to be resolved.

The validation of the results by a finite element analysis software package is an important step in ensuring they are accurate and reliable. Many sources of errors and uncertainties in the DIC experiments and FEA make it difficult to validate the results precisely. Despite this, when the FEA is used more widely in an industrial setting these uncertainties will always be present and therefore it is important to be aware of them and choose appropriate values for the material properties. In addition, the experimental DIC results were obtained in an industrial setting which in itself will introduce more uncertainty than if they had be obtained in a laboratory setting where conditions can be more easily controlled. Also, Solidworks does not allow refinement of the mesh, it only allows the size and not the triangular shape to be altered and hence the results tend to be conservative. Due to the unreliable strain results calculated from the DIC system algorithms it was logical not to use these results in the validation comparison. Despite this, all strains are calculated from displacement and therefore the correlation of the displacement values was deemed to be appropriate to validate the FEA software and demonstrate its accuracy. The FEA results agreed with those from the analytical theory sufficiently for both the displacements and strains for a pipe with no defects. It also produced displacements, which either correlated closely to the appropriate DIC results, or were larger for the defective pipe tests, which means the FEA is producing results which are conservative.

These findings confirm the reliability and give engineers confidence in the results produced from the Solidworks package which could lead to more sustained use of FEA in industry to aid decision making on the prolonging of locally thinned equipment life and be used in routine assessment at INEOS ChlorVinyls.

Figure 1: DIC Experimental Set-up
Figure 2: Comparison of maximum value locations on the displacement maps for both the DIC and FEA for a defective pipe.

Figure 3: Plot of maximum displacement results at different load pressures for a no defect and defect pipe, comparing FEA (Solidworks), DIC and analytical theory.
References


EXPERIMENTAL STUDY OF MECHANICAL DISTURBANCES CAUSED BY A CYLINDRICAL INCLUSION IN A SOFT MATERIAL SUBJECTED TO CONTACT LOADING

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Introduction

Soft materials, such as rubber, biological tissue, and foam, play an important role in a wide range of technological applications (Hamley, 2007). They are often loaded through transferring forces from other components which often results in large deformation and stress concentrations in vicinity of the contacting areas. It is known that the contact properties of materials can be significantly affected by the presence of heterogeneities (Leroux, 2011). Analysis of the changes induced by material heterogeneities in an elastic medium is a fundamental physical and engineering problem. The first solution to this problem was given by Eshelby (1957), who proposed the equivalent inclusion method to deal with an elliptical heterogeneity in an infinite medium subjected to uniform loading. After that, many researchers have devoted their efforts to this field. Recently, in a numerical study (Chang-Hung, 2007) demonstrated that the contact stress distribution is locally increased by a hard inhomogeneity near the contact region. However, most of the previous works are carried out within the limitations of small strain theory of elasticity, which is not applicable to soft materials as additional inequalities and nonlinearities are introduced. The singular characteristics such as stress concentration within these materials have not been well identified due to their multi-disciplinary nature, complicated physical properties, and variable boundary conditions.

Objective: this work is specifically aimed at developing an experimental technique for measuring and comparing the mechanical differences in a soft material induced by a rigid cylindrical inclusion. Considering the fact that soft materials could be easily deformed by small stresses or thermal fluctuations, a technique has been developed by integrating the digital moiré method with embedded gratings to investigate the mechanical behaviour of a vulcanized silicon rubber under contact loading.

Experiment & specimen

An experiment consisting of a homogeneous bulk material indented by a rigid wedge was performed in the first place; then the same procedure was implemented for a material containing a rigid cylindrical inclusion. Both the specimens, with and without inclusion, were cast from the same room temperature vulcanized silicone rubber and indented by the same rigid wedge (as shown in figure 1). Orthogonal moiré gratings composed of black toner were prefabricated on the plane of symmetry of the specimen, which ensured that the embedded-grating layer would be deformed in plane. In this case, the error caused by out-of-plane deformation could be neglected and which enabled the analysis of the in-plane mechanical behaviour of the soft material under various loads. The specimens were loaded at a constant
rate of 1mm/min. Meanwhile, deformation of the gratings was recorded by a CCD image acquisition system. Afterwards, the images were processed by a digital image processing method as shown in figure 2.

![Image of a soft material indented by a rigid wedge, with the coordinate system located at the apex of indenter (left); a soft material containing a cylindrical inclusion indented by a rigid wedge (middle); Diagrammatic sketch of the specimen with inclusion (right).](image)

**Fig.1** Profile of a soft material indented by a rigid wedge, with the coordinate system located at the apex of indenter (left); a soft material containing a cylindrical inclusion indented by a rigid wedge (middle); Diagrammatic sketch of the specimen with inclusion (right).

![Image of the experimental procedure.](image)

**Fig.2** Schematic diagram of experimental procedure. (a) Embedded moiré grating and specimen under loading; (b) image acquisition system; (c) examples of experimental images; (d) flowchart of digital image processing.

**Discussion & conclusion**

From the displacement data obtained from digital moiré analysis, the corresponding strain fields were calculated within the framework of large strain theory. As can be seen from the strain maps shown in figure 3, two deformation sectors could be observed from the results as predicted theoretically by Gao and Mai (2002). For example, consider the horizontal strain $\varepsilon_x$, there is an Expansion Sector (EX) below the indenter, which is very narrow before deformation and then becomes very wide after deformation. On the contrary, there is a Shrinking Sector (SS) occupying the majority domain on both sides of the rigid wedge before deformation, which is squeezed into a narrow area after deformation. In the presence of the rigid inclusion, the EX was expanded and a symmetric cross area can be observed from figure 3(a2) & (b2), which indicates that the strain outside this area was reduced by the inclusion. As for the shear strain maps, an asymmetrical deformation area can be found below the rigid inclusion.
After the comparison of strain fields, the change in stress near the boundary of cylindrical inclusion caused by the heterogeneity was derived by means of the Mooney-Rivlin model. The plots of Cauchy stresses are illustrated in figure 4. It is obvious that the distributions of contact stresses are locally increased by the hard heterogeneity, and the most significant changes can be found right below the rigid cylinder.

It seems that the proposed experimental approach makes it easy and effective to measure and analyze large deformation and large strains. The obtained strain and stress distributions qualitatively reveal the deformation behaviour and features of the soft material.
Fig. 4  Normalized stress distributions along the boundary of cylindrical inclusion under load of 38N.
Comparison of Cauchy stress component $\sigma_x$ (left), $\sigma_y$ (middle), $\tau$ (right).

References


