Integration of DIC into VFM: how do measurements contribute to identified material properties?

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Motivation

Full-Field measurements

- Grid
- Photoelasticity
- Interferometry
- Digital Image Correlation
- ...

Measurement errors

Mechanical properties of materials

- Finite element updating
- Constitutive equation gap
- Equilibrium gap
- Reciprocity gap
- Virtual Fields Method
- ...

Identification errors
DIC is a complex and non-linear process

... results depend on many parameters:

- Correlation criterion
- Interpolation routines
- Shape functions
- Regularization parameters:
  - Subset
  - Virtual strain gauge size
- Speckle pattern
- ...
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Mechanical properties of materials

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Measurement errors

Any realistic UQ on the mechanical parameters requires to take into account the measurement process

Identification errors
How?

Simulation of Experiment
- Known material parameters
- As close as possible to experimental conditions

Virtual Field Method

Identified material

Statistics
- Confidence margins for the determined material parameters
- Optimization of test design (geometry of specimen, smoothing, ...)

KU LEUVEN
Simulate the whole identification chain

**Input data:**
- CCD camera characteristics
- Design variables
- Stiffness components

**Comparison**

Parameter identification by VFM

**Full-field measurement by DIC**

**FE model**

Interpolation of the displacement field at the pixel coordinates

**Generation of the deformed image from a reference image**

**Introduction of artificial noise**
Case study: unnotched Iosipescu test

- Glass/epoxy unidirectional composite
- Linear elastic orthotropic

Stress state is composition of compression, bending and shear

- Design variables:
  - Free length L
  - Fibre orientation $\alpha$

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Input data

Design Variables

- Free Length L:
  \(10 \rightarrow 60\) mm; \(L = 2\) mm

- Fibre orientation \(\alpha\):
  \(0^\circ \rightarrow 90^\circ\); \(\alpha = 10^\circ\)

Stiffness components

<table>
<thead>
<tr>
<th>Stiffness</th>
<th>Maximum stress</th>
</tr>
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<tbody>
<tr>
<td>(Q_{xx}) (MPa)</td>
<td>40920</td>
</tr>
<tr>
<td>(Q_{yy}) (MPa)</td>
<td>10230</td>
</tr>
<tr>
<td>(Q_{xy}) (MPa)</td>
<td>3069</td>
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<tr>
<td>(Q_{ss}) (MPa)</td>
<td>4000</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Camera characteristics

- Spatial resolution: 1320 x 1024
- Dynamic range: 8-bit
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- Full-field measurement by DIC

**Interpolation of the displacement field at the pixel coordinates**
- Generation of the deformed image from a reference image
- Introduction of artificial noise
FE generated displacement map + image of real speckle pattern

SPECKLE DEFORMATION
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Generation of the deformed image from a reference image

Interpolation of the displacement field at the pixel coordinates
Synthetic image generation

Numerical deformation errors should be as low as possible: SUB-SAMPLING

HR reference image (5x5 sub-pixels) → Image deformation → Final reference image

HR deformed image (5x5 sub-pixels) → pixel sub-sampling → Final deformed image

DIC analysis
Synthetic image generation: validation

P. Reu, Experimental and numerical methods for exact subpixel shifting, Experimental Mechanics 51 (2011) 443-452

Rossi M., Lava P., Pierron F., Debruyne D. and Sasso M. Effect of DIC spatial resolution, noise and interpolation error on identification results with the VFM, submitted to Strain (2014)
Simulate the whole identification chain

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**Full-field measurement by DIC**
Intensity/color fluctuation around the “actual” image intensity/color.

- Gaussian
- Poisson-Gaussian
- Extreme Value

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Introduction of artificial noise
Full-field measurement by DIC

Exx

Eyy

Exy
Simulate the whole identification chain

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- **FE model**

- **Comparison**
  - Parameter identification by VFM
  - Full-field measurement by DIC

- **Interpolation of the displacement field at the pixel coordinates**
- **Generation of the deformed image from a reference image**
- **Introduction of artificial noise**
Virtual Fields Method

Based on the principle of virtual work:

$$- \int \sigma : \varepsilon^* dV + \int \bar{T} \cdot u^* dS + \int b \cdot u^* dV = \int \rho a \cdot u^* dV \quad \forall u^* K$$

**$$W^*_\text{int}$$**  

**$$W^*_\text{ext}$$**  

**$$W^*_\text{acc}$$**

Homogeneous linear elastic orthotropic materials

$$Q_{xx} \int_S \varepsilon_x \varepsilon_x^* dS + Q_{yy} \int_S \varepsilon_y \varepsilon_y^* dS + Q_{xy} \int_S (\varepsilon_x \varepsilon_y^* + \varepsilon_y \varepsilon_x^*) dS +$$

$$Q_{ss} \int_S \varepsilon_s \varepsilon_s^* dS = \int_{\partial S} T_x u_x^* dl + \int_{\partial S} T_y u_y^* dl$$
Simulate the whole identification chain

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Full-field measurement by DIC
The **identification error** is defined as

\[
Err = \sqrt{\sum_{ij} w_{ij} \left( 1 - \frac{Q_{ij}}{Q_{ij}^0} \right)^2}
\]

with \( ij = [xx, yy, xy, ss] \)

\( Q_{ij}^0 \) are the reference parameters introduced in the FE model

\( Q_{ij} \) are the parameters identified with the VFM

\( w_{ij} \) is a weighting parameter

Noise introduction (random): **several** simulated experiments are repeated
Optimization in view of TEST DESIGN

SS = 21, VSG = 201 are FIXED
(1360x1024; noise: 2 grey levels; 8-bit; 20 repetitions; ΔL = 2mm; Δα = 10°)

Optimum L = 30 mm and α = 50-60°

Rossi M., Lava P., Pierron F., Debruyne D. and Sasso M. Effect of DIC spatial resolution, noise and interpolation error on identification results with the VFM, submitted to Strain (2014)
Results

Parameter $Q_{xx} w_{ij} = [1\ 0\ 0\ 0]$

Parameter $Q_{yy} w_{ij} = [0\ 1\ 0\ 0]$

Parameter $Q_{xy} w_{ij} = [0\ 0\ 1\ 0]$

Parameter $Q_{zz} w_{ij} = [0\ 0\ 0\ 1]$
Optimization in view of Regularization

- Shear strain maps obtained by image deformation plus different levels of smoothing

VSG=5 pixels (no smoothing)  VSG=60 pixels (local polynomial)

Which one is ‘the best’?
Results

$L = 30\,\text{mm}, \alpha = 55^\circ$ are FIXED: optimization in view of DATA ANALYSIS

Smoothing ... but not too much
Heterogeneities ... but not too much
Conclusions

• Simulator for material identification combining DIC and VFM

• Design specimen geometries which maximize performance of DIC and VFM

• Select the optimum regularization parameters (subset, VSG, …) that minimize the error on the identified properties in function of your experimental setup (noise, lighting, …)

• Provided realistic confidence margins for the identified stiffnesses

• Applied to any material model
Demo of direct integration of VFM and DIC

- Glass/epoxy unidirectional composite
- Linear elastic orthotropic
- Fibre orientation: 80 degrees
- Load: 3736N
- Thickness: 1mm
- Width = length = 20mm

Reference values:

Q11 = 41000 MPa
Q22 = 10300 MPa
Q12 = 3090 MPa
Q66 = 4000 MPa
DIC course
Metrology beyond colors
January 12-16, 2015 - Ghent, Belgium

http://diccourse.matchid.org