159 An Experimental and Theoretical Study of Longitudinal Bulk Strain Waves Generated by Fracture

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Abstract. From a dipole lattice model, we derive a Boussinesq-type equation that governs the propagation of a longitudinal strain wave in a uniform pre-strained bar with constant cross section. We then describe an experiment based on high-speed, single-point photoelasticity to measure strain waves in a Polymethylmethacrylate (PMMA) bar generated when it breaks whilst undergoing a tensile test. The experimental results are finally compared to the model predictions, showing a good correspondence in terms of wave propagation, strain rate at the leading edge and amplitude and frequency of the following ripple.

Introduction

Understanding of the dynamic properties of plastics is a current area of important research, largely due to their wide range of uses in a variety of different applications [1,2]. Recently, researchers at the Ioffe institute in Russia have been investigating the propagation of longitudinal strain waves in PMMA bars that arise after one end of the bar is hit by a sudden pulse. In this experiment, the pulse is provided from a shock wave propagating through a water tank which arises after a high-power laser pulse evaporates a thin foil target. The bar behaves effectively as a waveguide for the strain wave, which is observed using holographic interferometry [3]. This very specialist technique has produced some intriguing results. It was noticed that the strain wave would propagate for a long distance at a constant speed without significantly losing amplitude, giving strong evidence to suggest that the generated localised strain waves are solitons [4].

The focus of the research presented here differs from the experiments detailed above as we are investigating propagation of longitudinal bulk strain waves naturally generated by fracture (lower strain rates), and propagating into a pre-strained PMMA bar. We also derive the equation that governs the propagation of a longitudinal strain wave in a pre-strained uniform bar of rectangular cross-section by using a dipole lattice model, and compare the numerical results to the experimental measurements.

Mathematical Model

The derivation of the evolution equation is based on a dipole lattice model and comparison with the equation for a strain free bar obtained from nonlinear elasticity [5,6]. Each particle is of mass m and interacts with the neighbouring particles by forces containing linear and quadratic terms with respective pairs of interaction coefficients shown in Fig. 1. The constants a, d and $2l$ are distances between these pairs of particles in equilibrium. A dipole is formed of particles $(\mathrm{P_n}, \overline{\mathrm{P_n}})$. Each dipole has four degrees of freedom, namely the horizontal and vertical displacements of the dipole's geometric centre, described by variables u_1^n and u_2^n respectively, rotation of the dipole about the normal (vertical) dipole axis by an angle of $\Delta\phi^\mathrm{n}$ and a change in distance between the particles of a dipole themselves given by $2\mathrm{u}_4^\mathrm{n}$.

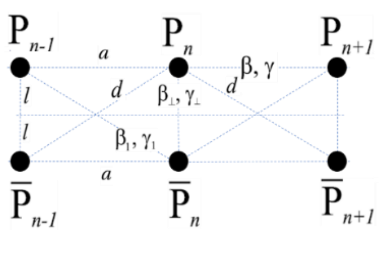

Fig 1. The dipole lattice.

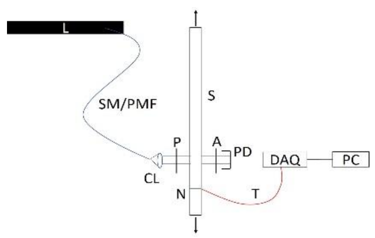

Fig *2*. Experimental setup.

Using the equations of motion for this dipole lattice, waves propagating on a pre-strained background are considered and it can be shown that the equation for the long weakly-nonlinear longitudinal waves asymptotically uncouples from other degrees of freedom. The evolution of the leading order longitudinal strain, $e = e(x,t)$, (derivative of the longitudinal displacement) is described by the model equation

$$e_{tt} - \frac{E + \kappa\beta}{\rho} e_{xx} = \frac{\beta}{2\rho}(e^2)_{xx} + \frac{(b_1^2 + b_2^2)(c_0^2 - c_1^2)\nu^2}{3c_0^2} e_{ttxx}, \qquad (1)$$

where e_{tt} denotes the 2nd partial derivative of e with respect to t, etc. The constant E is Young's modulus of the material of the bar, ρ is the density, κ is the longitudinal pre-strain, ν is Poisson's ratio, $c_0 = \sqrt{E/\rho}$ is the linear longitudinal wave velocity, $c_1 = c_0/\sqrt{2(1+\nu)}$ is the linear shear wave velocity and $2b_1$ and $2b_2$ are the height and width of the rectangular cross section. The value of β is dependent on Young's modulus, Poisson's ratio and Murnaghan's 3rd order elastic moduli [6].

Experiment

A tensile test was performed on PMMA samples of dimension 3 mm × 10 mm × 750 mm. Each sample was notched with a sharp blade 150 mm from one of the ends to initiate fracture at a known location. A bright-field single-point circular polariscope was used to measure the strain as a function of time after fracture. Fig. 2 shows a schematic of the experimental setup which consists of: He-Ne laser L; single mode polarization maintaining optical fiber SM/PMF; collimation lens CL that produces a beam with a diameter of 1 mm; polarizer P; analyzer A; notch N; sample S; photodetector PD with a bandwidth of 1MHz; triggering circuit T; acquisition board DAQ running at 1MS/s and personal computer PC. The polariscope was placed at 50 mm increments from the notch for different samples in order to study the evolution of the strain wave. The stress-optic and the biaxial Hooke's laws were used to evaluate the longitudinal strain from the measured intensity vs time data.

Results

Fig. 3 shows the experimental and numerical strain profiles from the time of fracture for different samples. Each profile represents a 50 mm increase in distance from the notch, from 50 mm (bottom profile) up to 500 mm (top profile). On closer inspection of the profiles, we notice an oscillation that grows with distance immediately after the release wave passes through the point of measurement. This evidence, and the model, suggest that given enough propagation distance, this wave will develop into an undular bore. A slower shear wave is observed in the measurements, but it is not modelled in this study.

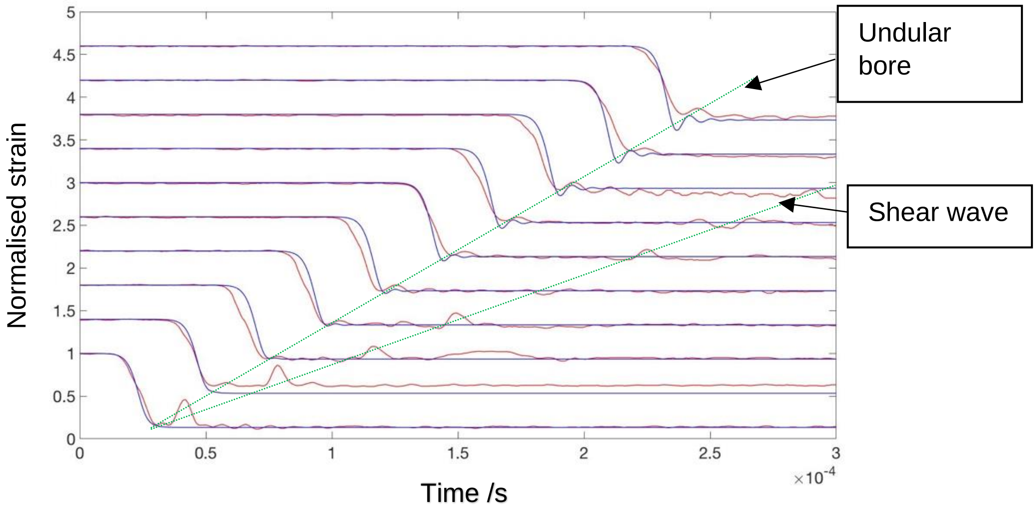

Fig 3. Experimental (red) and model (blue) strain profiles recorded every 50 mm from the notch. Each profile has been normalised by the strain present in the bar at the time of fracture. A vertical offset is given between each plot for clarity. The two observed waves are tracked by the green (dashed) lines.

References

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