International Seminar on Metal Plasticity
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Organizers: Marco Rossi, Sam Coppieters

THEORETICAL MODELING OF PLASTICITY

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June 19, 2017

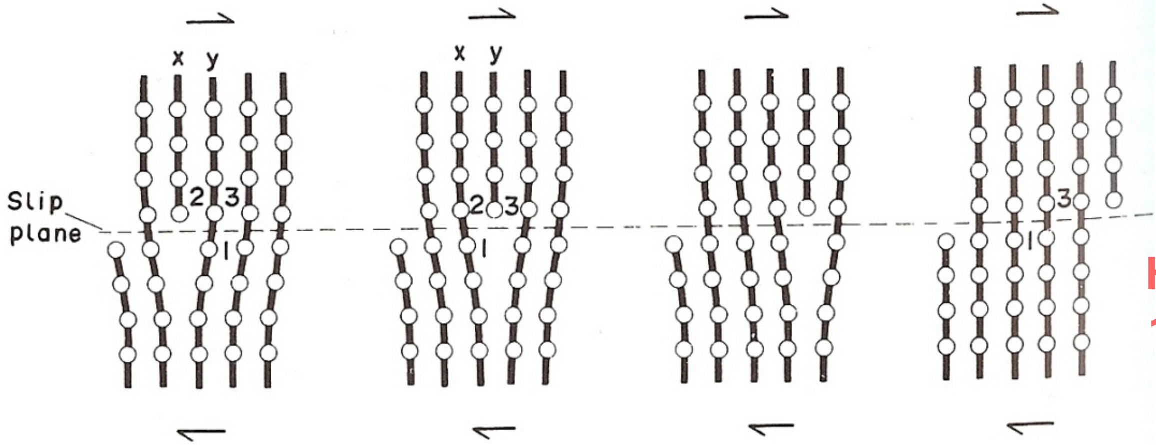
Outline

1. Introduction
2. Plasticity modeling
3. Isotropic hardening
4. Anisotropic hardening
5. Final remarks

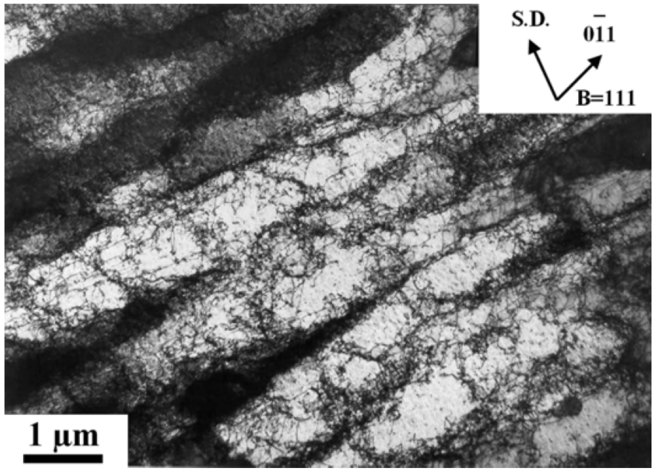
1. Introduction – Deformation mechanisms

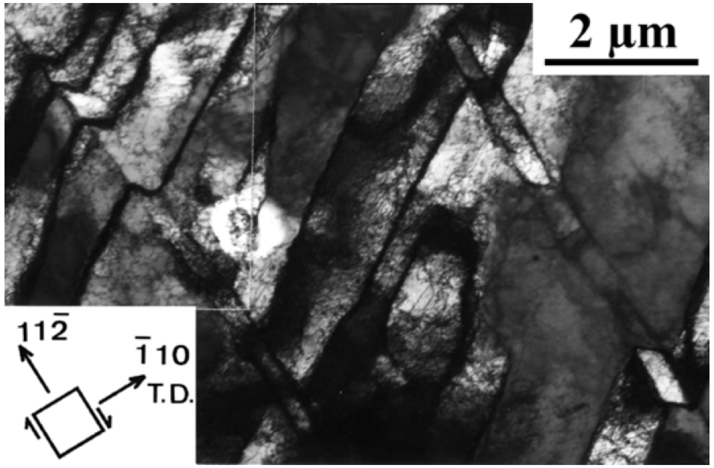
Plastic behavior in metals

1. Dislocation glide (slip)



Hull, 1983





Rauch et al., 2011

2. Other mechanisms

1. Introduction – Approaches

Continuum scale plasticity

Crystal plasticity (slip, twinning and homogenization schemes)

Dislocation dynamics (dislocation network with interaction rules)

Multiphase material modeling (unit cells)

Atomistic (lattice with interatomic potentials) and ab-initio

This presentation is about continuum scale plasticity

2. Plasticity modeling – Approaches

Variables

- External variables (elastic strain, plastic strain, strain rate, temperature)
- Thermodynamically conjugate variables through the expression of the free energy (stress, entropy)
- State variables assumed to represent deformation mechanisms (explicitly or implicitly)

Context of this presentation

- Rate and temperature-independent behavior (mostly)
- Isotropic hardening (applicable for monotonic loading)
- Anisotropic hardening (applicable for non-monotonic loading)

2. Material modeling – Approaches

Total strain increment

- Elastic–plastic decomposition

$d\boldsymbol{\varepsilon}^{tot} = d\boldsymbol{\varepsilon}^{ela} + d\boldsymbol{\varepsilon}^{pla}$ ($d\boldsymbol{\varepsilon}^{pla} = d\boldsymbol{\varepsilon}$ in this presentation)

Plasticity concepts

- Yield condition (applied stress equal to yield stress in uniaxial tension)
- Hardening model (stress-strain curve in uniaxial tension)
- Flow rule $d\boldsymbol{\varepsilon}$ ($d\varepsilon_{trans} = -{}^{1}/_{2}\, d\varepsilon_{long}$ for isotropic material in uniaxial tension)

2. Material modeling – Plasticity concepts

Yield condition

$\Phi(\boldsymbol{\sigma}) = 0$ for instance $\bar{\sigma}(\sigma_{ij}) - \sigma_y = 0$

- Effective stress and yield stress
- Yield condition defines yield surface

Hardening rule

- Same yield condition but with evolving state variables (microstructure)

$\Phi(\boldsymbol{\sigma}, \Theta, x) = 0$ for instance $\bar{\sigma}(\boldsymbol{\sigma}) - \sigma_R(\Theta, x) = 0$

- x represents (scalar or tensorial) state variables

2. Material modeling – Plasticity concepts

Flow rule

- Associated or non-associated. For metal, associated flow rule is consistent with plastic deformation mechanisms

- Argument based on crystal plasticity by Bishop and Hill (1951) general approach (not restricted to specific boundary conditions)

$$d\boldsymbol{\varepsilon} = d\lambda \frac{\partial \Phi}{\partial \boldsymbol{\sigma}}$$

- Work-equivalent effective strain $\bar{\sigma} d\bar{\varepsilon} = \boldsymbol{\sigma} : d\boldsymbol{\varepsilon}$ defines a possible state variable (accumulated deformation or accumulated dislocations)

- Associated flow rule reduces to $d\boldsymbol{\varepsilon} = d\bar{\varepsilon} \frac{\partial \bar{\sigma}}{\partial \boldsymbol{\sigma}}$

Choice of the yield condition fully defines the material behavior

3. Isotropic hardening – Isotropic material

Isotropic yield conditions

$$\bar{\sigma}(\boldsymbol{\sigma}) - \sigma_R(\bar{\varepsilon}) = 0$$

- The effective stress is based on invariants such as von-Mises, Tresca, Hershey, etc.

$$\bar{\sigma} = \left\{\frac{|\sigma_1-\sigma_2|^a+|\sigma_2-\sigma_3|^a+|\sigma_3-\sigma_1|^a}{2}\right\}^{1/a} - \sigma_R(\bar{\varepsilon}) = 0 \quad \text{(Hershey, 1954)}$$

- Principal stresses are invariants
- Non-quadratic and convex yield function
- Reduces to Tresca or von-Mises for specific values of a
- Identification of $\sigma_R(\bar{\varepsilon})$, e.g., using least square approximation. Issue for extrapolation

3. Isotropic hardening – Anisotropic material

Anisotropic yield conditions

- Same yield condition as for isotropic case but stress components must be expressed in material symmetry axes (eg., RD, TD, ND)

$$\bar{\sigma}(\boldsymbol{\sigma}) - \sigma_R(\bar{\varepsilon}) = 0$$

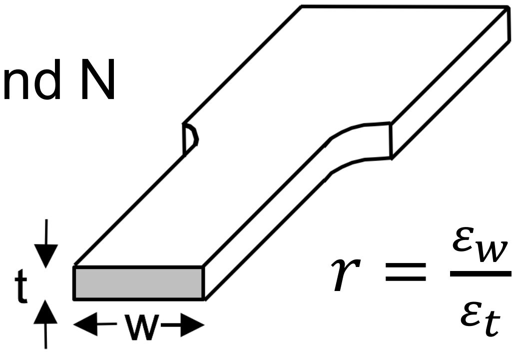
Effective stress based on Hill (1948)

- Plane stress case

$$\bar{\sigma} = \left\{F\left(\sigma_{yy} - \sigma_{zz}\right)^2 + G(\sigma_{zz} - \sigma_{xx})^2 + H\left(\sigma_{xx} - \sigma_{yy}\right)^2 + 2N\sigma_{xy}^2\right\}^{1/2} = \sigma_R(\bar{\varepsilon})$$

- Reduces to von Mises for specific values of F, G, H and N

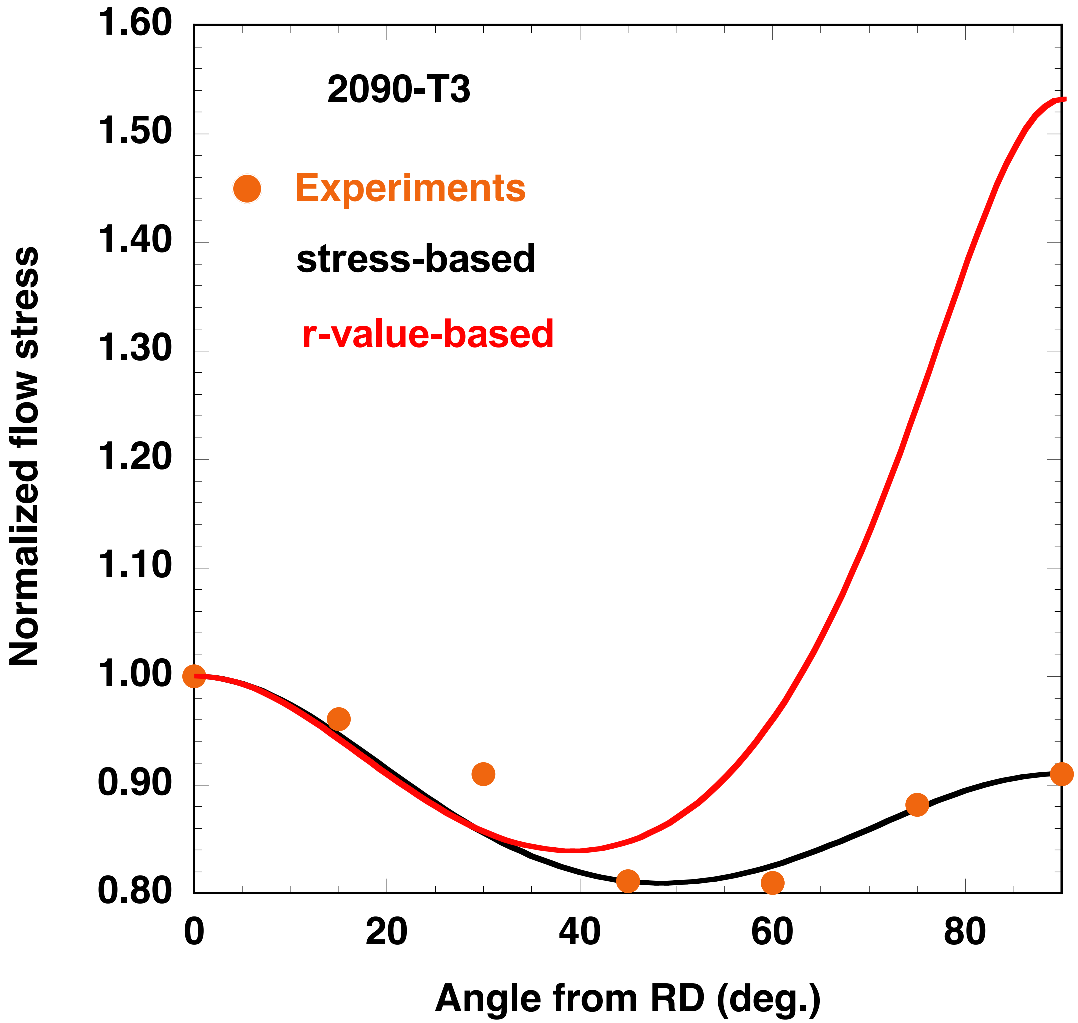
- Issue for identification F, G, H and N: Based on flow stresses or r -value in uniaxial tension?



$$r = \frac{\varepsilon_w}{\varepsilon_t}$$

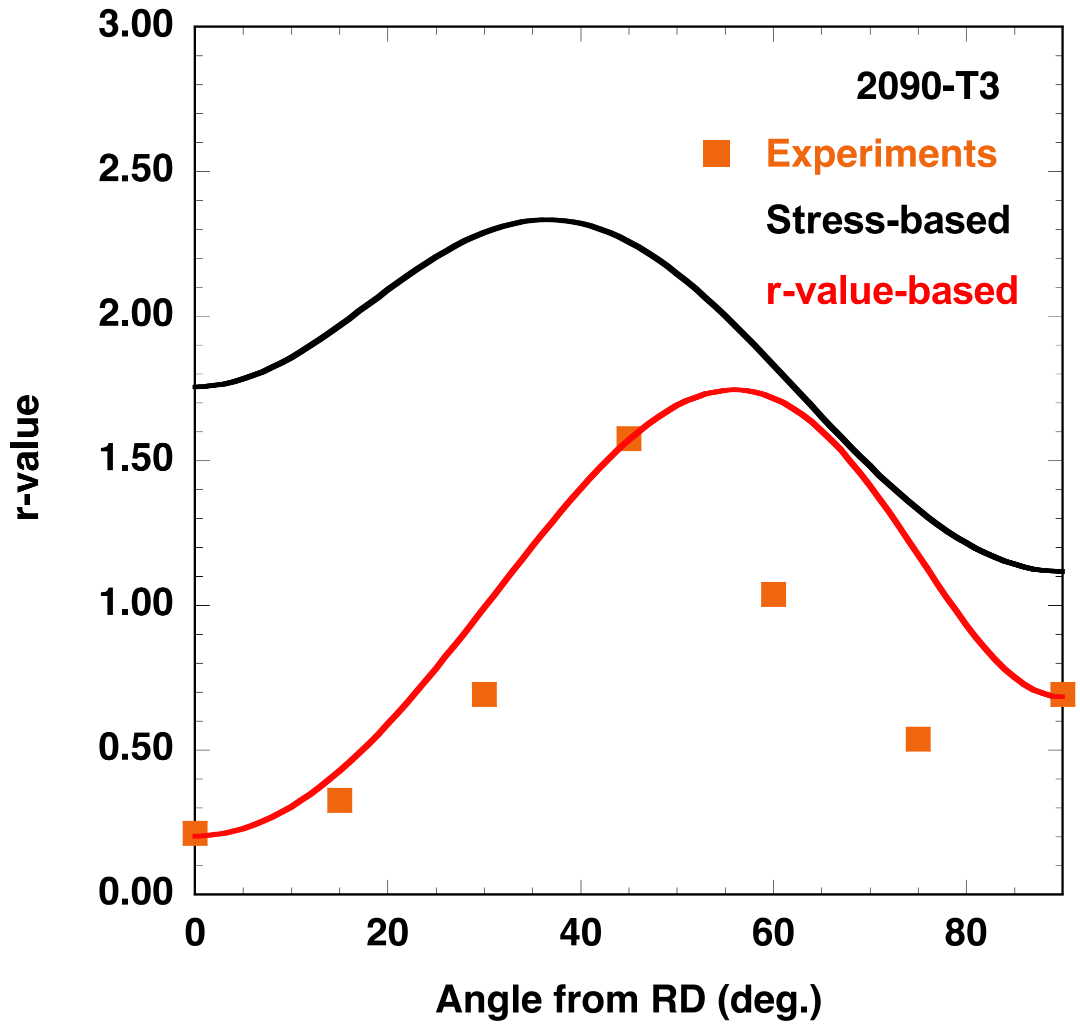
3. Isotropic hardening – Anisotropic material

Hill (1948) plane stress



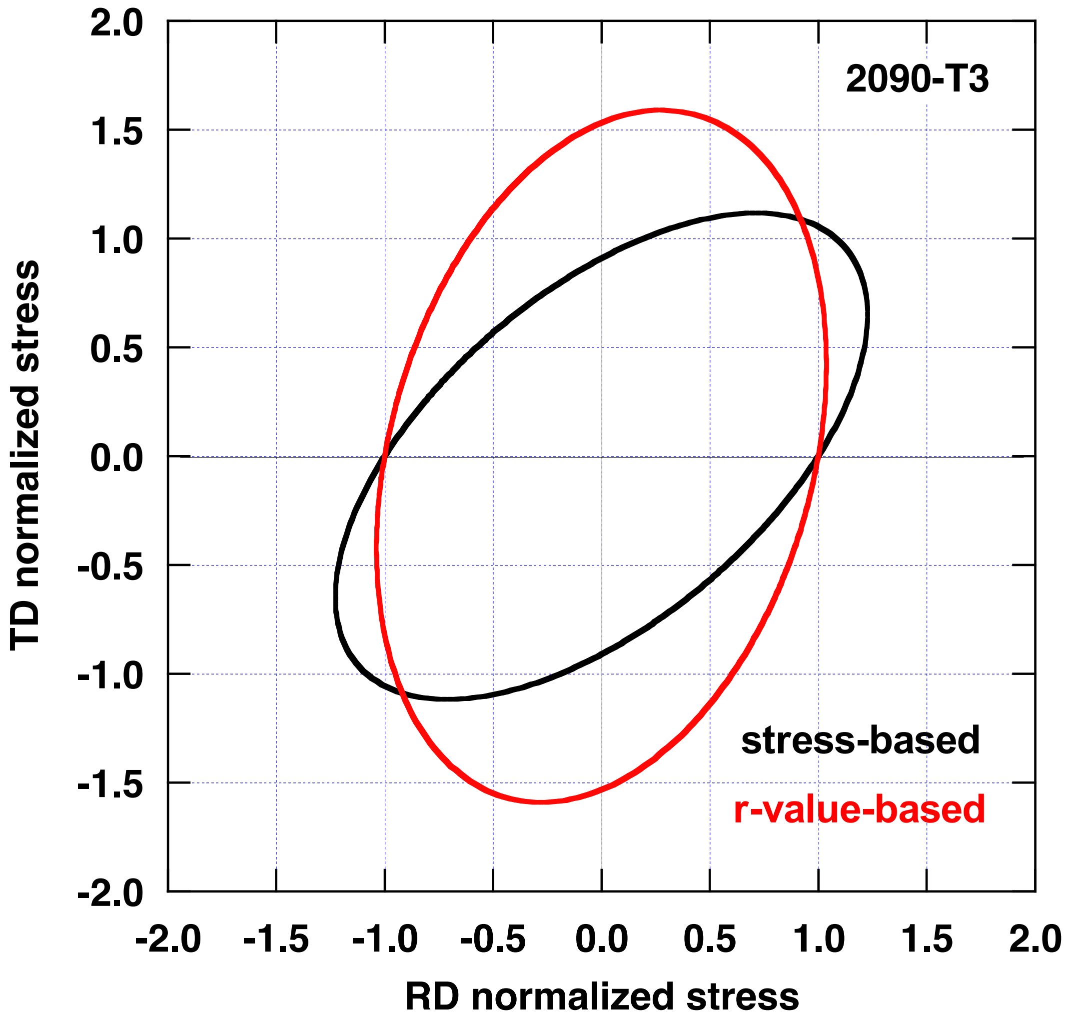
3. Isotropic hardening – Anisotropic material

Hill (1948) plane stress



3. Isotropic hardening – Anisotropic material

Hill (1948) plane stress



3. Isotropic hardening – Anisotropic material

Hill (1948) limitations

- Cannot, in general, model uniaxial tension properly
- Use average behavior (still inaccurate) or non-associated flow rule with strain potential (not based on the physics of slip)

Non-quadratic yield functions and isotropic hardening

- Note that Hill (1948) cannot be generalized directly, i.e.,

$$\cancel{\bar{\sigma} = \left\{F\left(\sigma_{yy} - \sigma_{zz}\right)^a + G\left(\sigma_{zz} - \sigma_{xx}\right)^a + H\left(\sigma_{xx} - \sigma_{yy}\right)^a + 2N\sigma_{xy}^a\right\}^{1/a} = \sigma_R(\bar{\varepsilon})}$$

- This formulation does not work because it is component-based, not invariant based

3. Isotropic hardening – Anisotropic material

Non-quadratic yield functions

- Linear stress transformation approach
- For instance, with two transformations $\boldsymbol{\sigma}'^{(t)} = \mathbf{C}^{(t)} : \boldsymbol{\sigma}' \ (t = 1,2)$
- Plane stress case: Yld2000-2d

$$\bar{\sigma} = \left\{ \frac{\left|\tilde{\sigma}_1'^{(1)} - \tilde{\sigma}_2'^{(1)}\right|^a + \left|2\tilde{\sigma}_2'^{(2)} + \tilde{\sigma}_1'^{(2)}\right|^a + \left|2\tilde{\sigma}_1'^{(2)} + \tilde{\sigma}_2'^{(2)}\right|^a}{2} \right\}^{1/a} = \sigma_R(\bar{\varepsilon})$$

- Total of eight anisotropy coefficients in $\mathbf{C}^{(1)}$ and $\mathbf{C}^{(2)}$
- Reduces to isotropic Hershey (1954) when $\mathbf{C}^{(1)}$ and $\mathbf{C}^{(2)}$ are the identity

3. Isotropic hardening – Anisotropic material

Non-quadratic yield functions

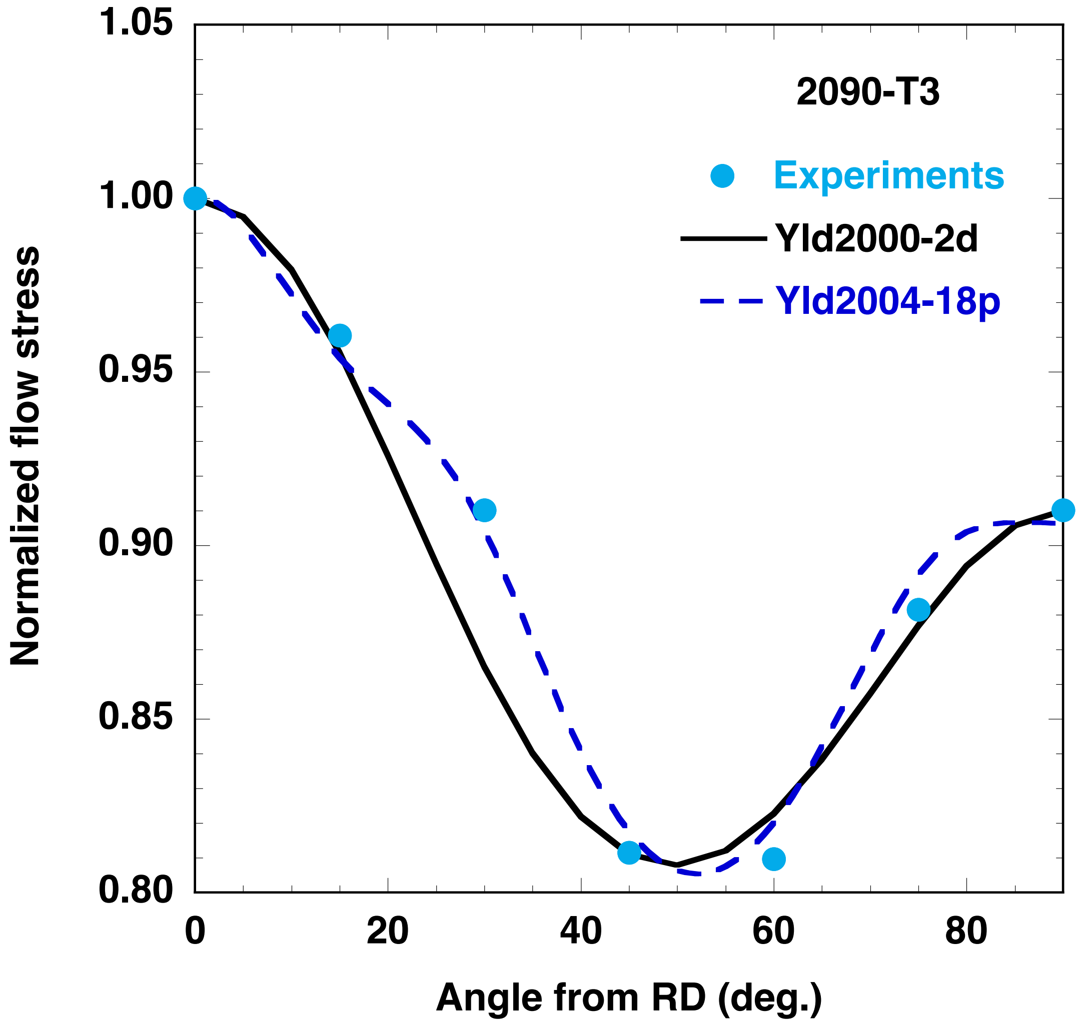
- General stress state Yld2004-18p

$$\bar{\sigma} = \left\{ \frac{1}{4} \sum_{p,q}^{1,3} \left| \tilde{\sigma}_p'^{(1)} - \tilde{\sigma}_q'^{(2)} \right|^a \right\}^{1/a} = \sigma_R(\bar{\varepsilon})$$

- Total of 16 independent anisotropy coefficients in $\mathbf{C}^{(1)}$ and $\mathbf{C}^{(2)}$
- Reduces to isotropic Hershey (1954) when $\mathbf{C}^{(1)}$ and $\mathbf{C}^{(2)}$ are the identity
- Advantage of linear transformations compared to other approaches for plastic anisotropy: Preserve convexity of the isotropic function

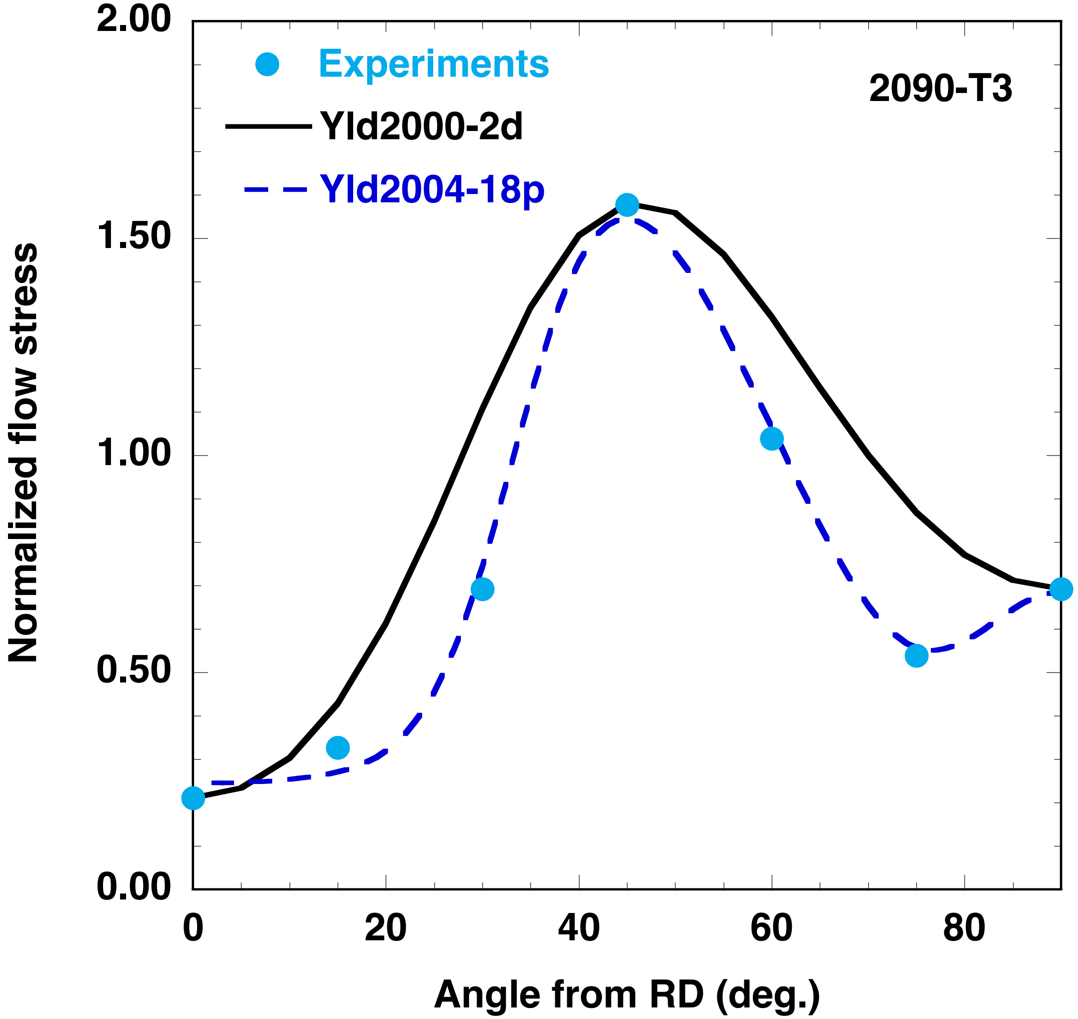
3. Isotropic hardening – Anisotropic material

Non-quadratic yield functions



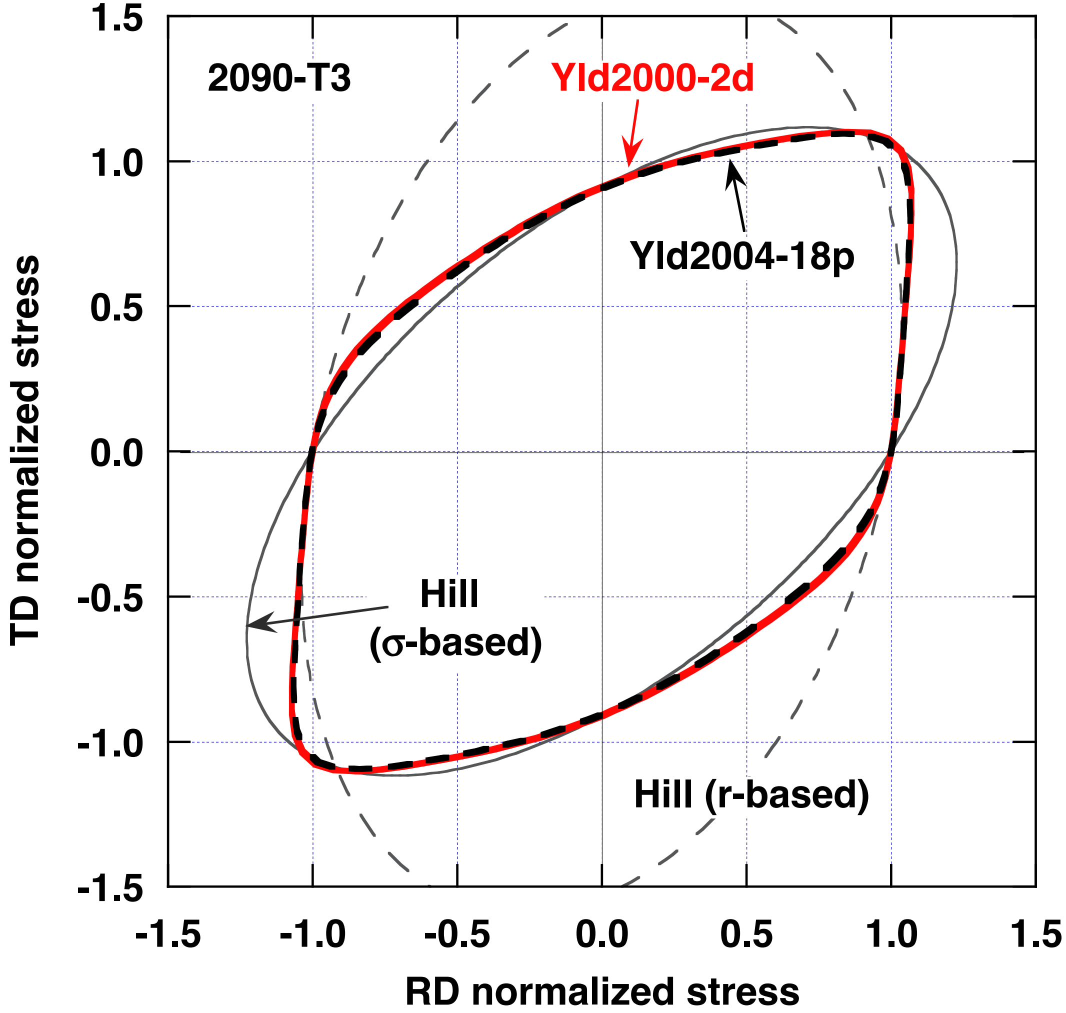
3. Isotropic hardening – Anisotropic material

Non-quadratic yield functions



3. Isotropic hardening – Anisotropic material

Non-quadratic yield functions



3. Isotropic hardening – Anisotropic material

Strength differential (SD) effect

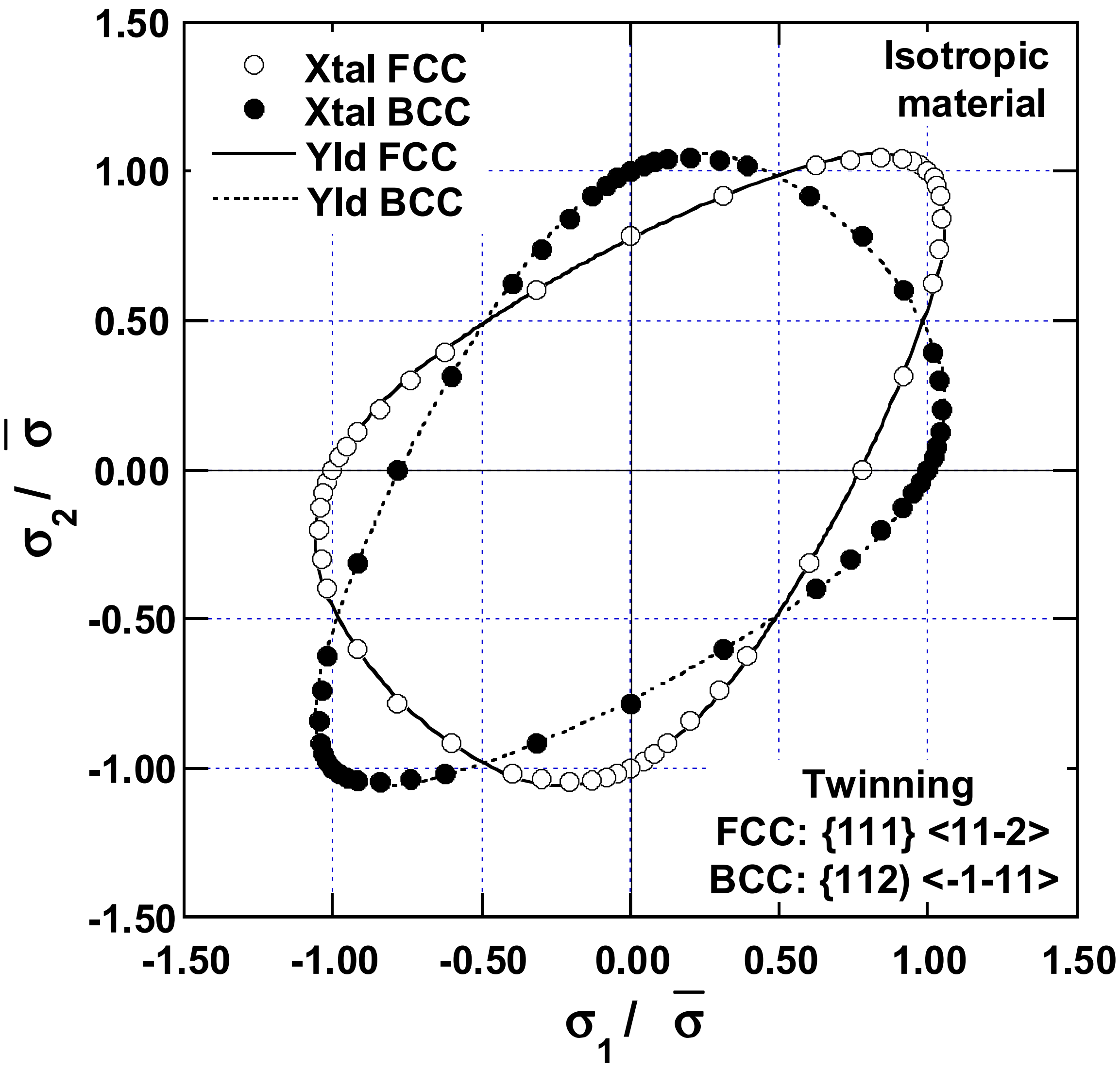
- Isotropic yield function (Cazacu et al., 2006)

$$\bar{\sigma} = \left\{\frac{\left||\sigma_1'|-k\sigma_1'\right|^a+\left||\sigma_2'|-k\sigma_2'\right|^a+\left||\sigma_3'|-k\sigma_3'\right|^a}{K}\right\}^{1/a} - \sigma_R(\bar{\varepsilon}) = 0$$

- Constant coefficient K
- Compression to tension ratio $\frac{\sigma_c}{\sigma_t} = \left\{\frac{2^a(1-k)^a+2(1+k)^a}{2^a(1+k)^a+2(1-k)^a}\right\}^{1/a}$
- Anisotropic yield function using linear transformation

3. Isotropic hardening – Anisotropic material

Twinning yield surfaces

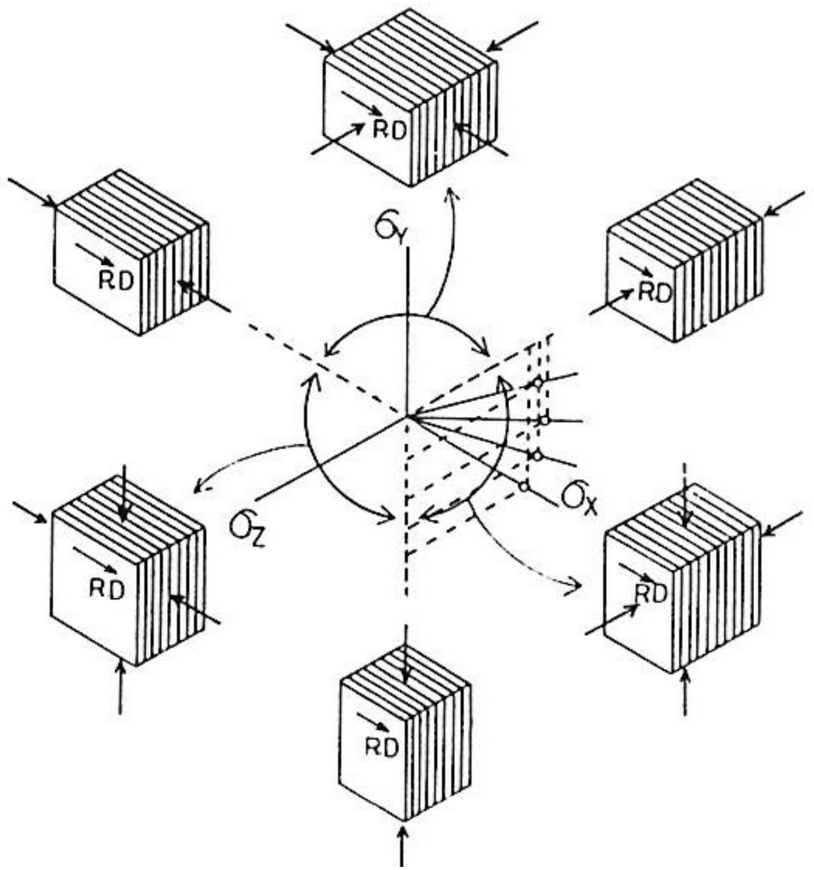


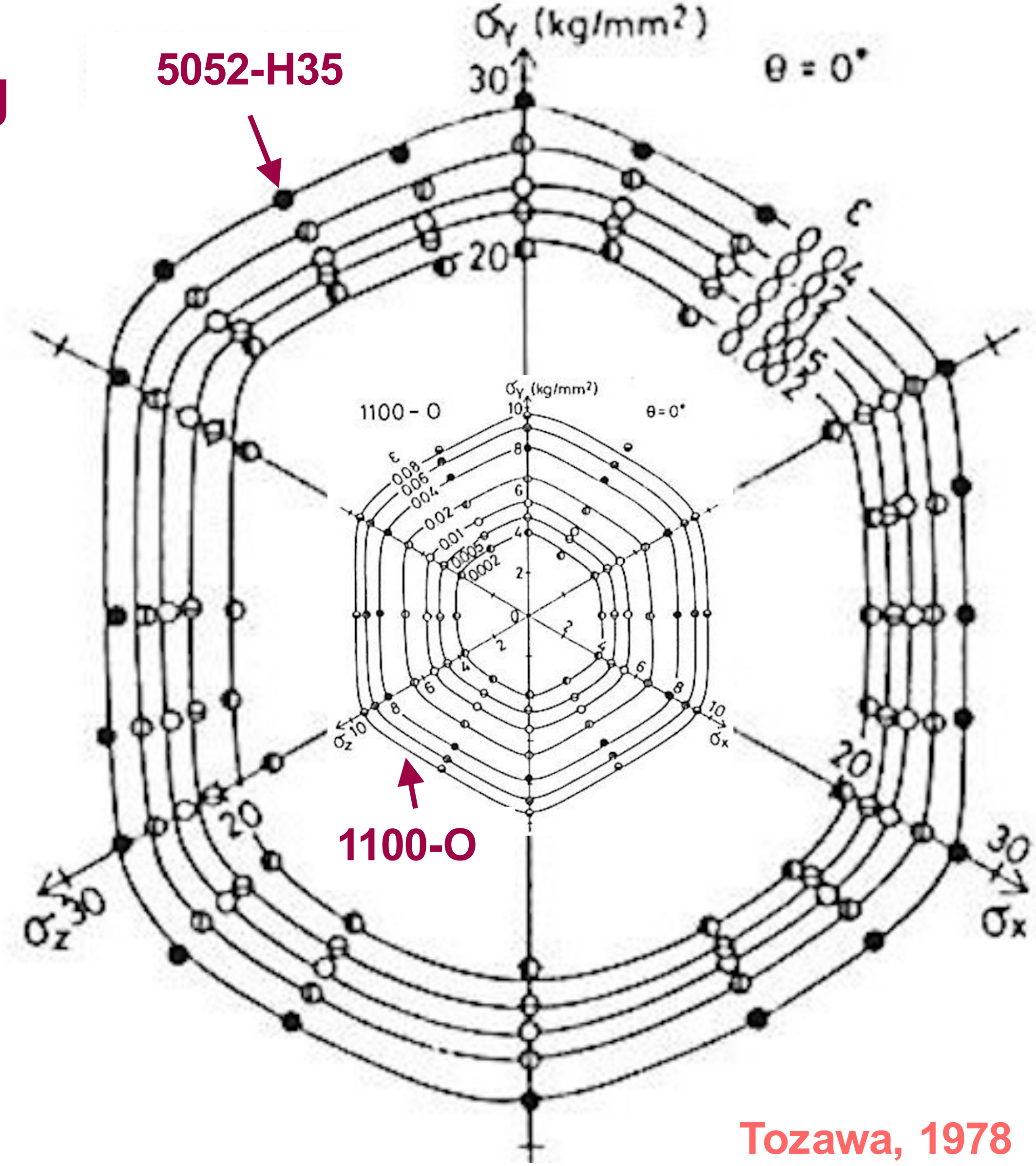
Cazacu et al., 2006

3. Isotropic hardening – Validation

Biaxial compression testing

- stacked sheet specimen

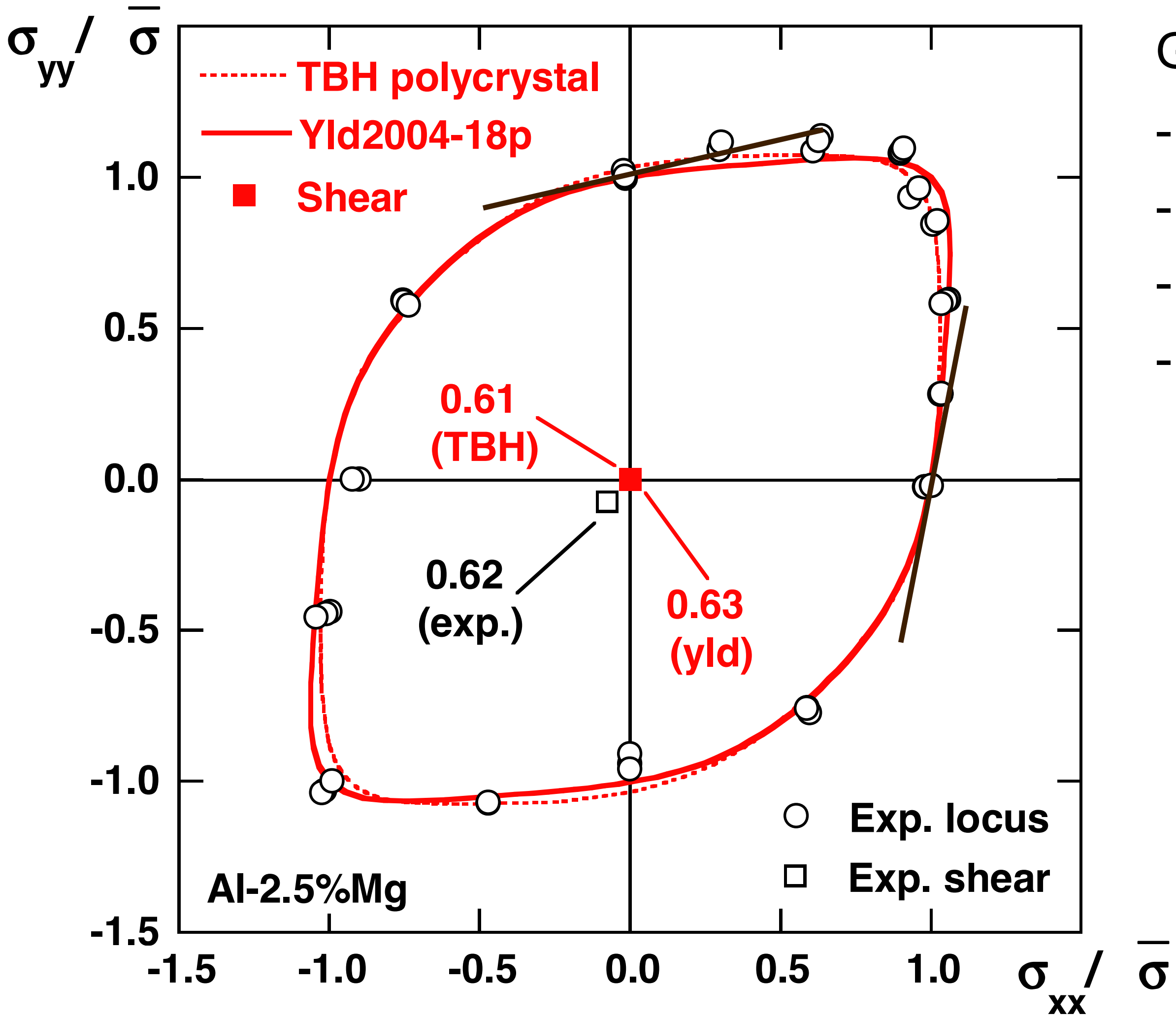




Tozawa, 1978

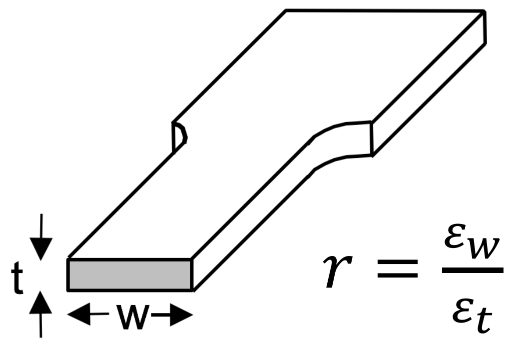
3. Isotropic hardening – Validation

Biaxial compression testing



Good agreement between:

- Experiments
- Crystal plasticity
- Plane stress Yld2000-2d
- General stress Yld2004-18p

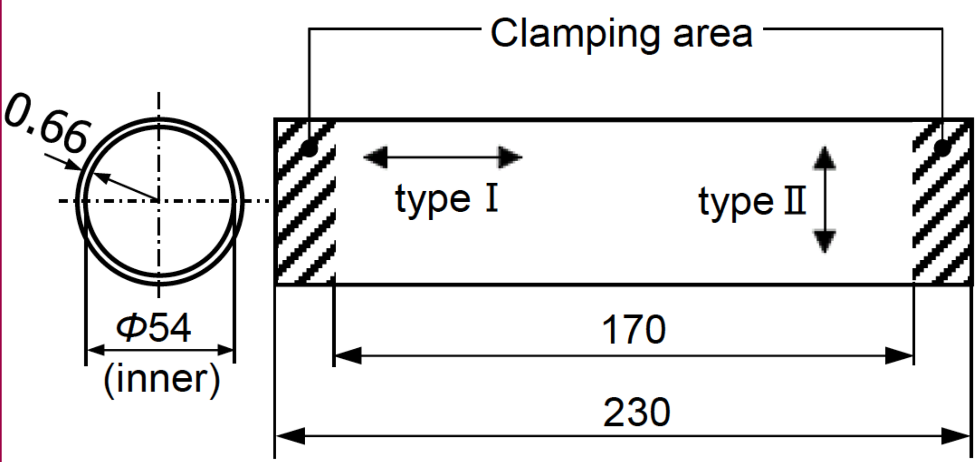


$$r = \frac{\varepsilon_w}{\varepsilon_t}$$

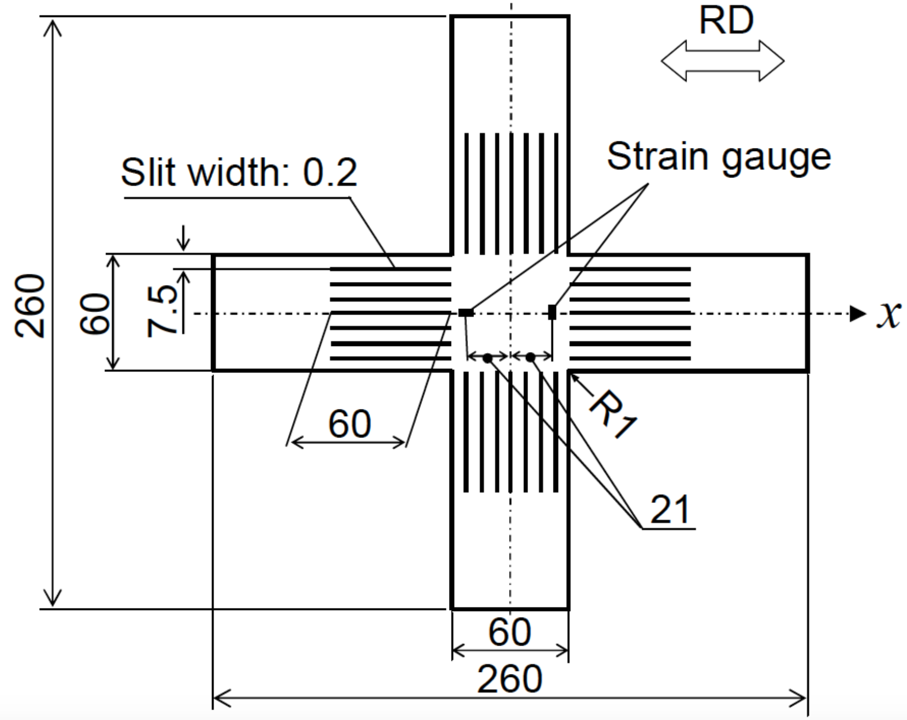
3. Isotropic hardening – Validation

Biaxial tension testing

Tubular specimens



ISO 16842: 2014
Metallic materials –Sheet and strip –Biaxial tensile testing method using a cruciform test piece

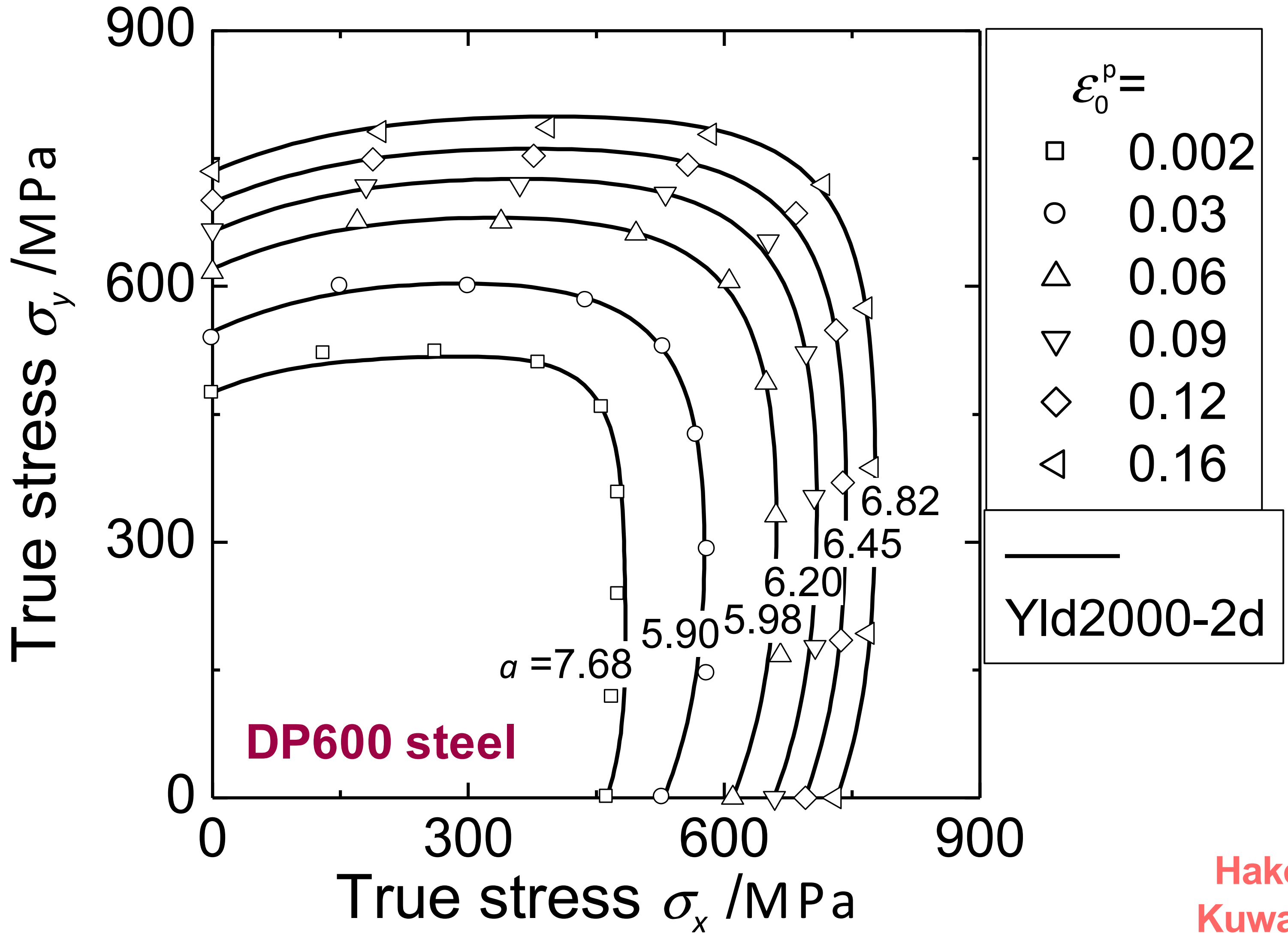


Cruciform specimens

Kuwabara and Sugawara, 2013

3. Isotropic hardening – Validation

Contour of plastic work for DP 600 steel



Hakoyama and Kuwabara, 2015

4. Anisotropic hardening

Approaches

- Differential hardening
- Kinematic hardening
- Combined kinematic - isotropic hardening
- Combined kinematic hardening and distortional plasticity
- Distortional plasticity only

Differential hardening

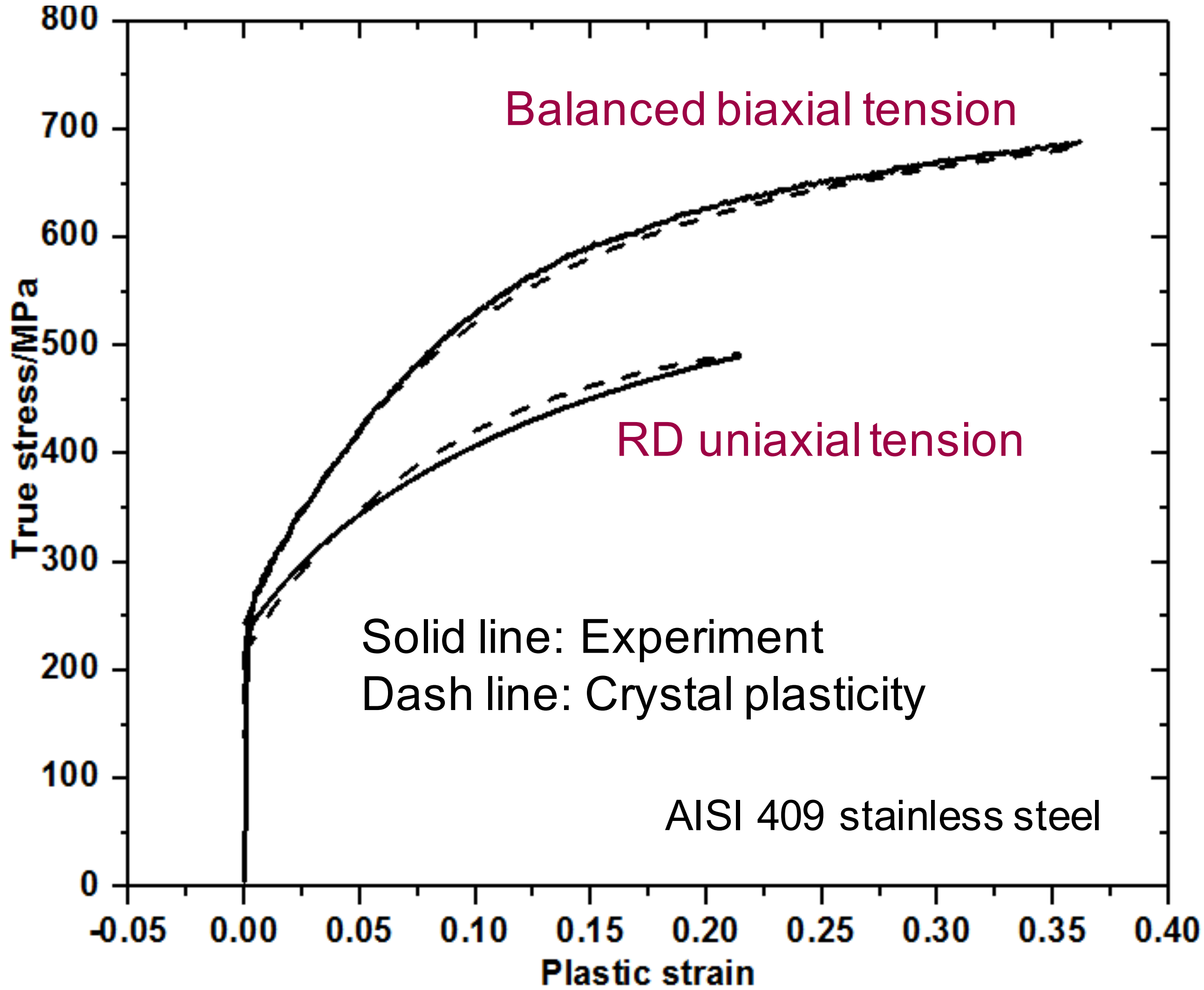
- Hill and Hutchinson (1992)

- Can be modelled by varying the coefficients of an isotropic hardening model

- Relatively simple but based on one given strain path only

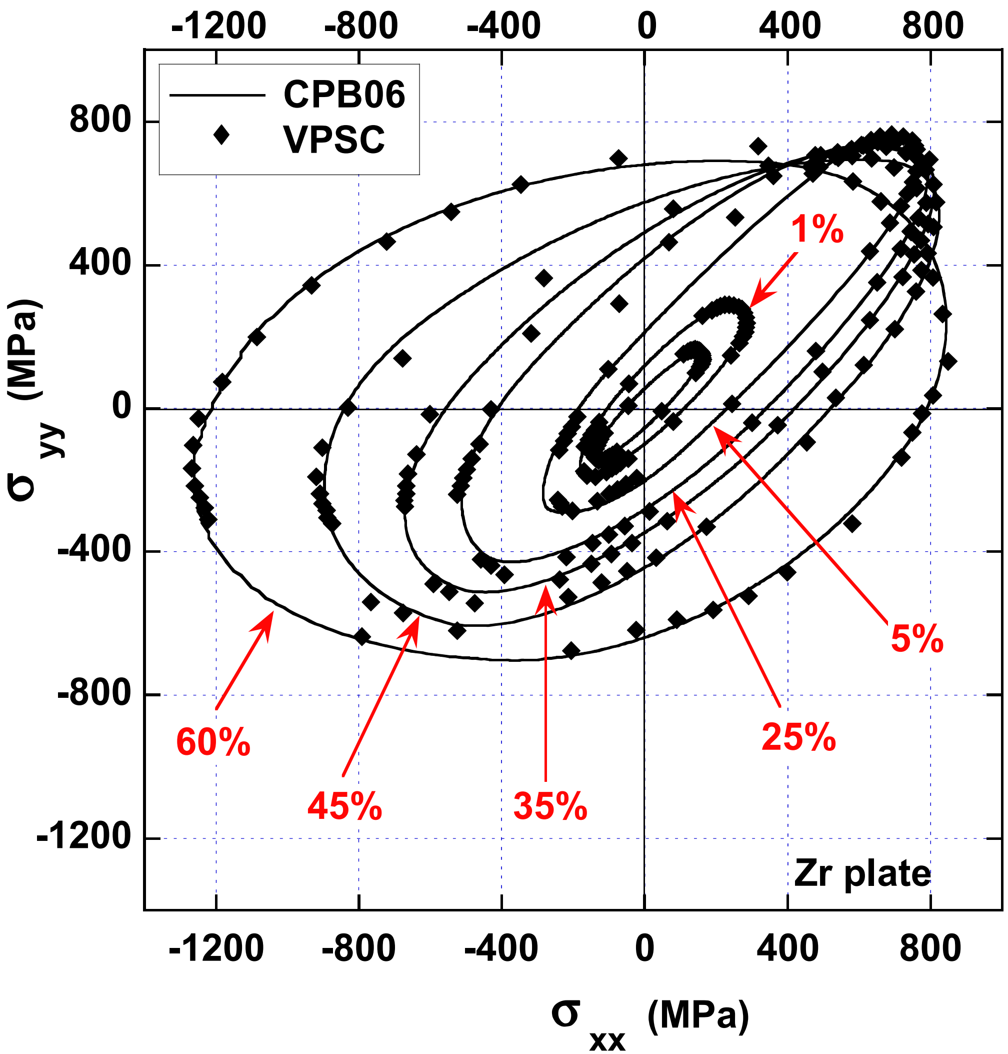
4. Anisotropic hardening – Differential hardening

Example based on experimental data



4. Anisotropic hardening – Differential hardening

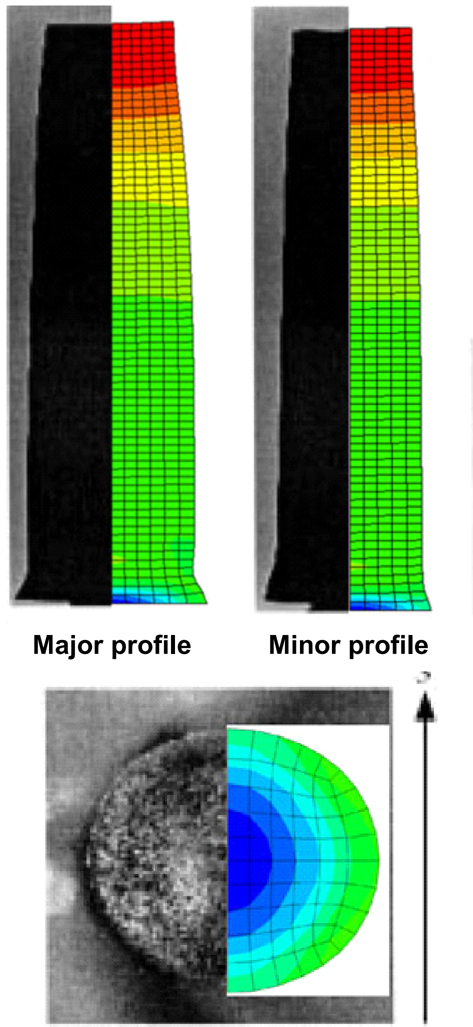
Example based on crystal plasticity of Zr

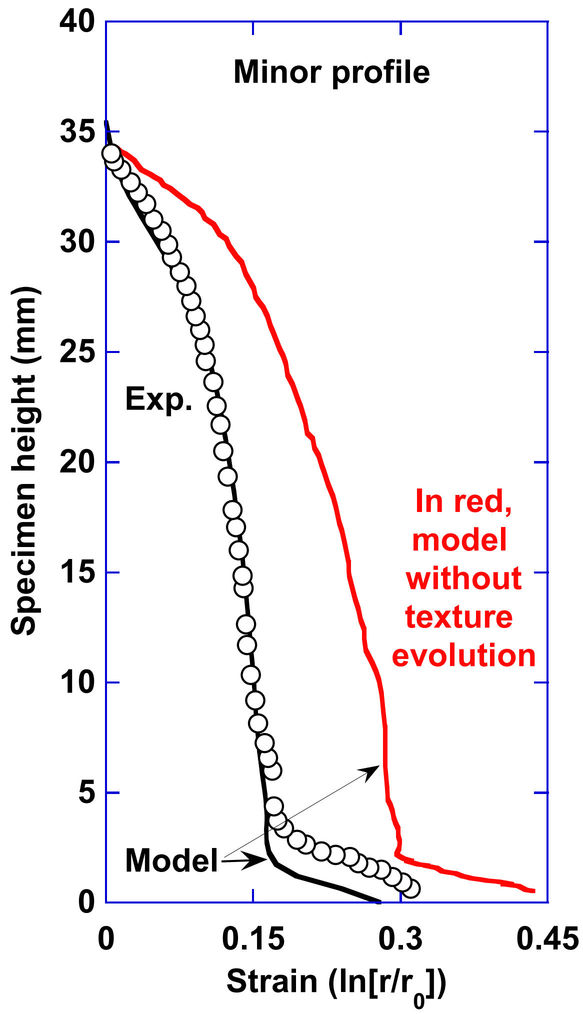


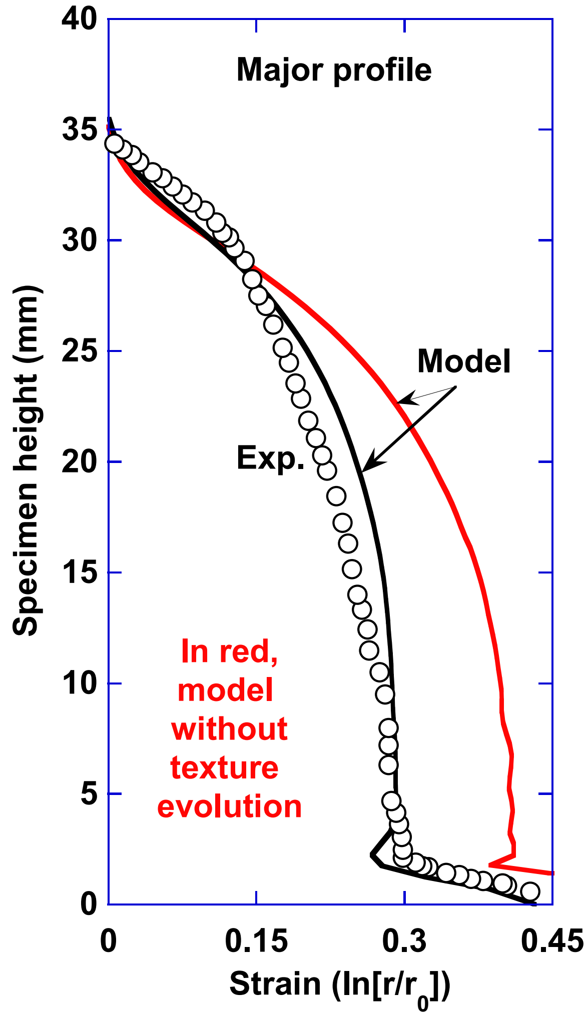
Plunkett et al., 2006

4. Anisotropic hardening – Differential hardening

Example based on crystal plasticity of Zr (Taylor impact test application)







Maudlin et al., 1999

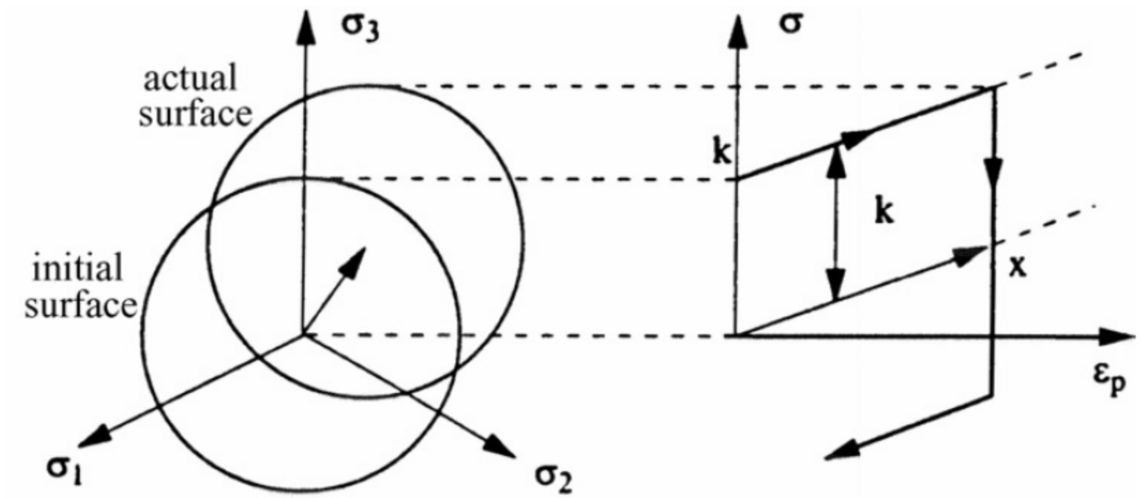
Plunkett et al., 2006

4. Anisotropic hardening – Kinematic hardening

Linear kinematic hardening

$$\Phi(\boldsymbol{\sigma}, x) = \bar{\sigma}(\boldsymbol{\sigma} - \mathbf{X}) - \sigma_y = 0$$ (Prager, 1949)

- One state variable: Back-stress $\mathbf{X} = \mathbf{x}_1$ $\dot{\mathbf{X}} = C\mathbf{D}$



Yield surface translate

4. Anisotropic hardening – Kinematic hardening

Non-linear kinematic hardening

- Chaboche et al. (1979)

$$\bar{\sigma}(\mathbf{s},\mathbf{X}) = \mathrm{Y} + \mathrm{R}(\bar{\varepsilon})$$

Back-stress
Yield stress
Hardening

Evolution equations

$$\dot{\mathbf{X}} = C\mathbf{D} - \gamma\mathbf{X}\dot{\bar{\varepsilon}}$$

$$\dot{R} = \frac{dR}{d\bar{\varepsilon}}\dot{\bar{\varepsilon}}$$

- Hu and Teodosiu (1995)

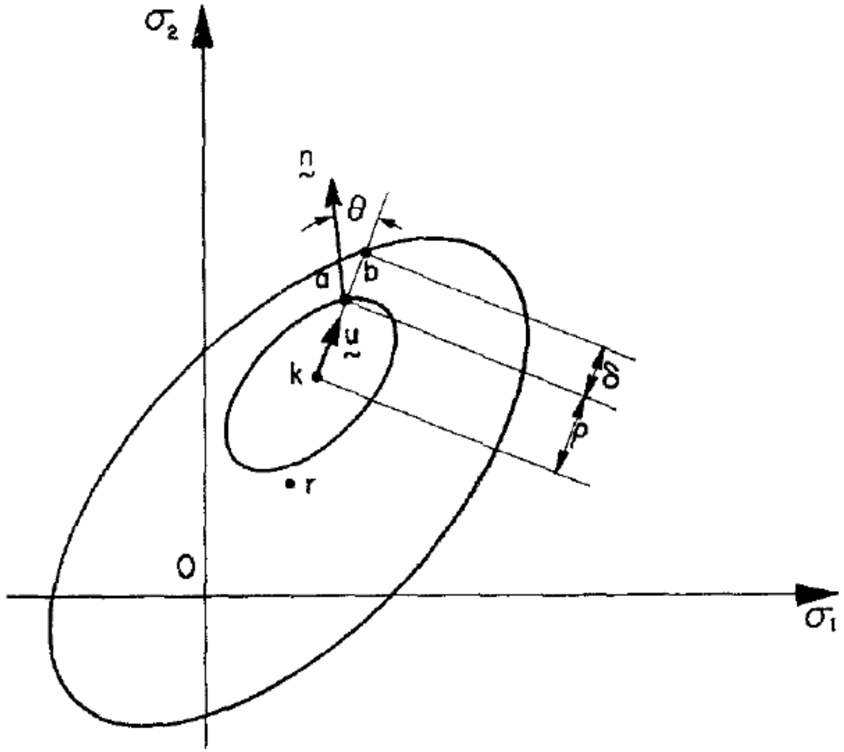
Back-stress
Polarization of dislocation structure

$$\bar{\sigma}(\mathbf{s},\mathbf{X},\mathbf{M}) = \mathrm{Y} + \mathrm{R}(\mathbf{S},\mathbf{P})$$

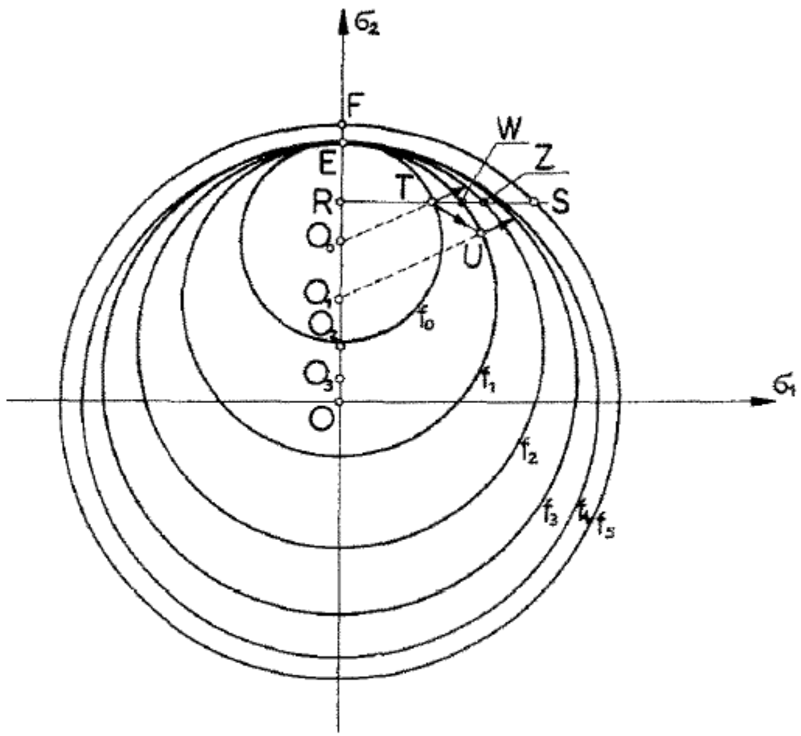
Texture anisotropy (constant)
Strength of dislocation structure

4. Anisotropic hardening – Kinematic hardening

Translating surfaces



Two surfaces
(Dafalias and Popov, 1975)



Multiple nested surfaces
(Mroz, 1967)

4. Anisotropic hardening – Kinematic hardening

Two surfaces

- Yoshida and Uemori (2002)

- Yield surface evolution

$$f = \phi(\mathbf{s} - \alpha) - Y = 0$$

Back-stress 1

Yield stress

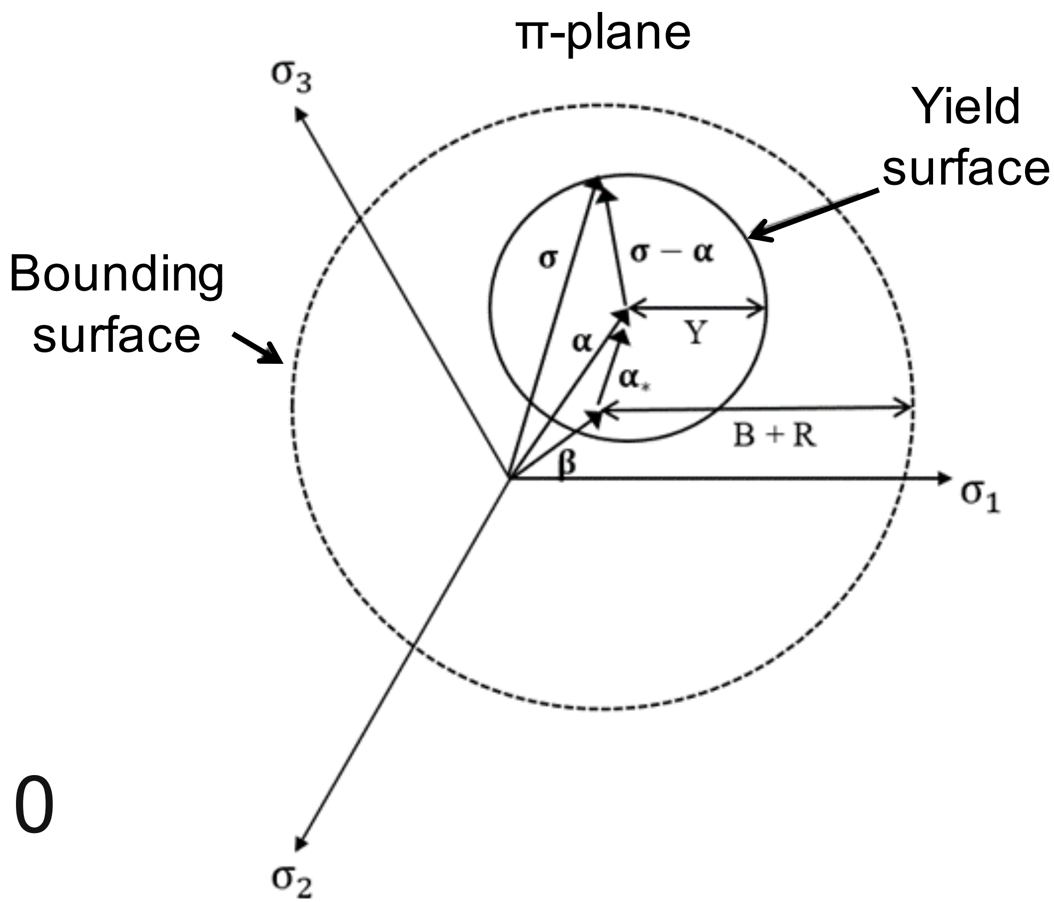
- Bounding surface evolution

$$F = \phi(\mathbf{s} - \beta) - (B + R) = 0$$

Back-stress 2

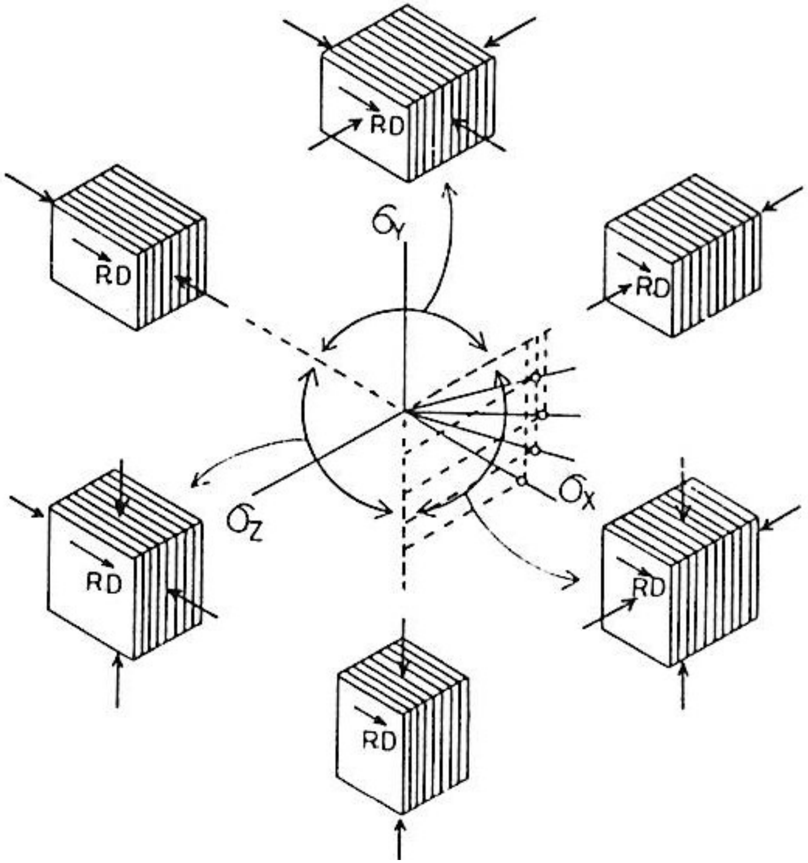
Initial size boundary surface

Hardening

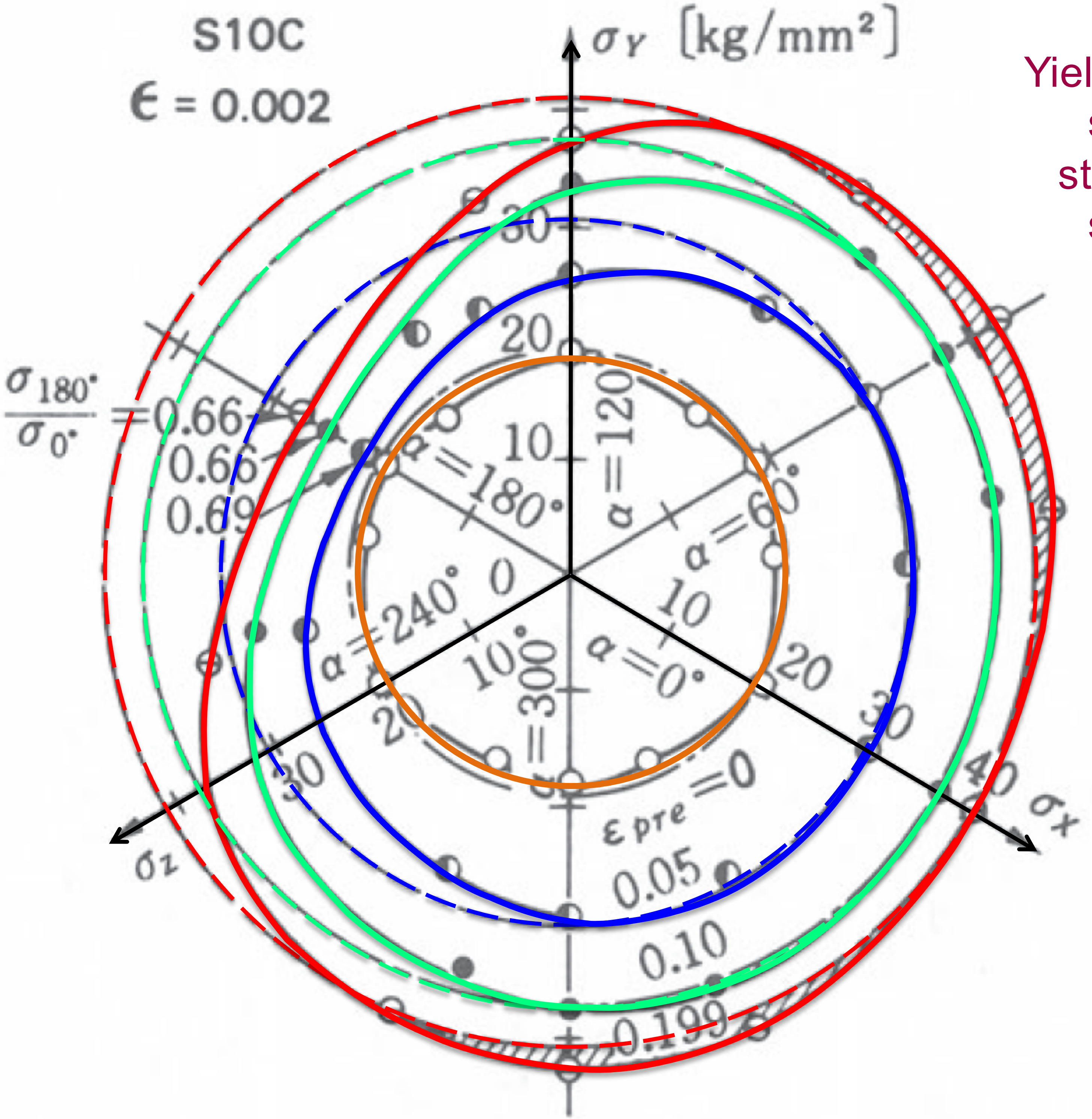


4. Anisotropic hardening – Validation

Biaxial compression testing – Stacked sheet specimens



Tozawa, 1978



Yield loci at ε = 0.2% for steel (S10C) pre-stretched by various strains in tension

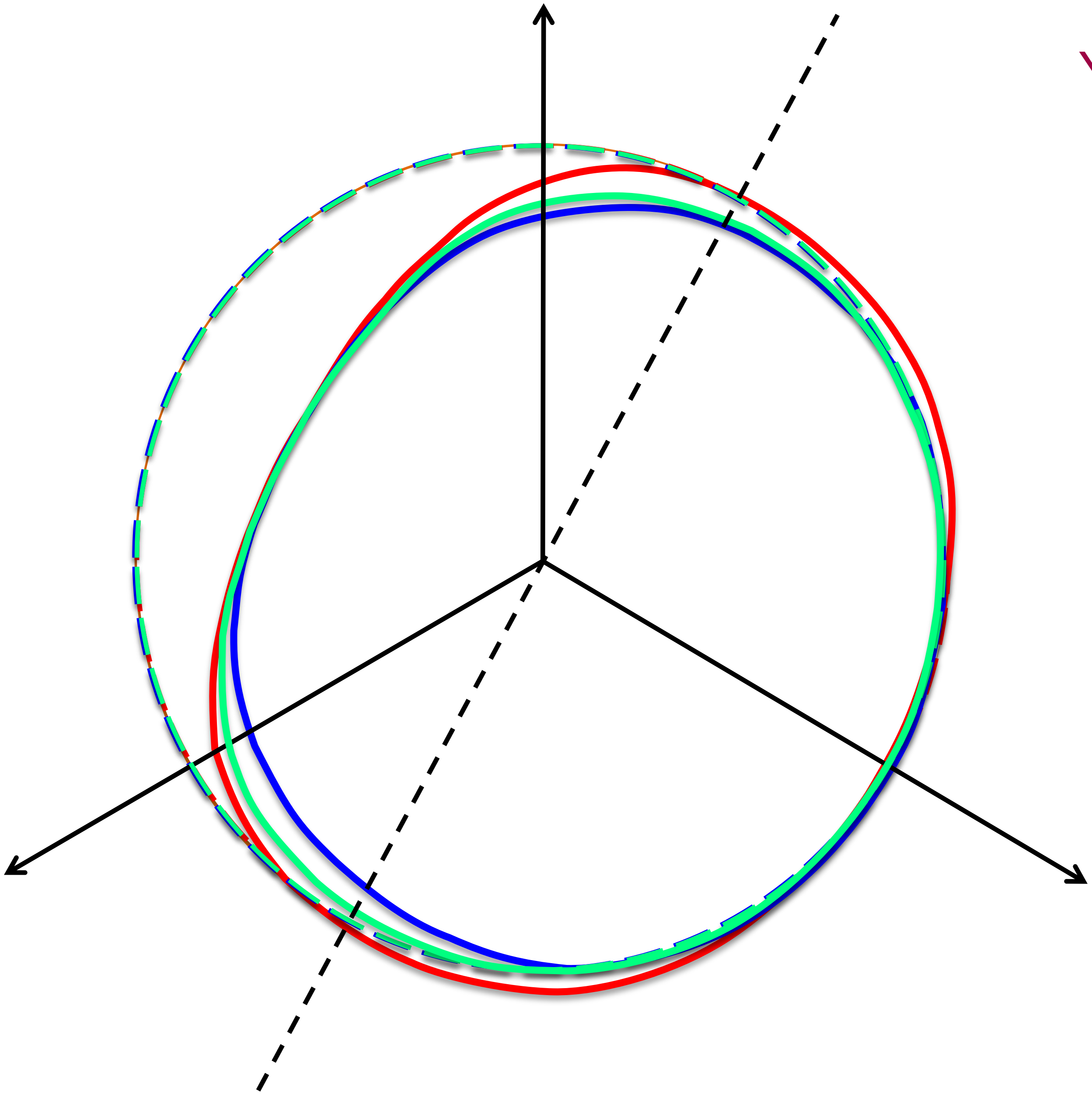
$\varepsilon_{pre} = 0.05$

$\varepsilon_{pre} = 0.10$

$\varepsilon_{pre} = 0.20$

Tozawa, 1978

4. Anisotropic hardening – Validation



Yield loci at ε = 0.2% for steel (S10C) pre-stretched by various strains in tension

$\varepsilon_{pre} = 0.05$

$\varepsilon_{pre} = 0.10$

$\varepsilon_{pre} = 0.20$

4. Anisotropic hardening – Distortional plasticity only

Homogeneous anisotropic hardening (HAH)

Homogenous function

No kinematic hardening

$$\bar{\sigma}(\mathbf{s}) = \left\{ \boxed{} \; \bar{\varpi}(\mathbf{s})^{q} \right\}^{\frac{1}{q}} = \sigma_{R}(\bar{\varepsilon})$$

Effective stress | Fluctuating component (Bauschinger effect) | Stable component | Flow stress

- f_1^q, f_2^q describe the amount of distortion
- $\bar{\varpi}(\mathbf{s})$ replaced by $\bar{\varpi}_{CL}(\mathbf{s})$ for cross-loading & latent effects

Microstructure deviator

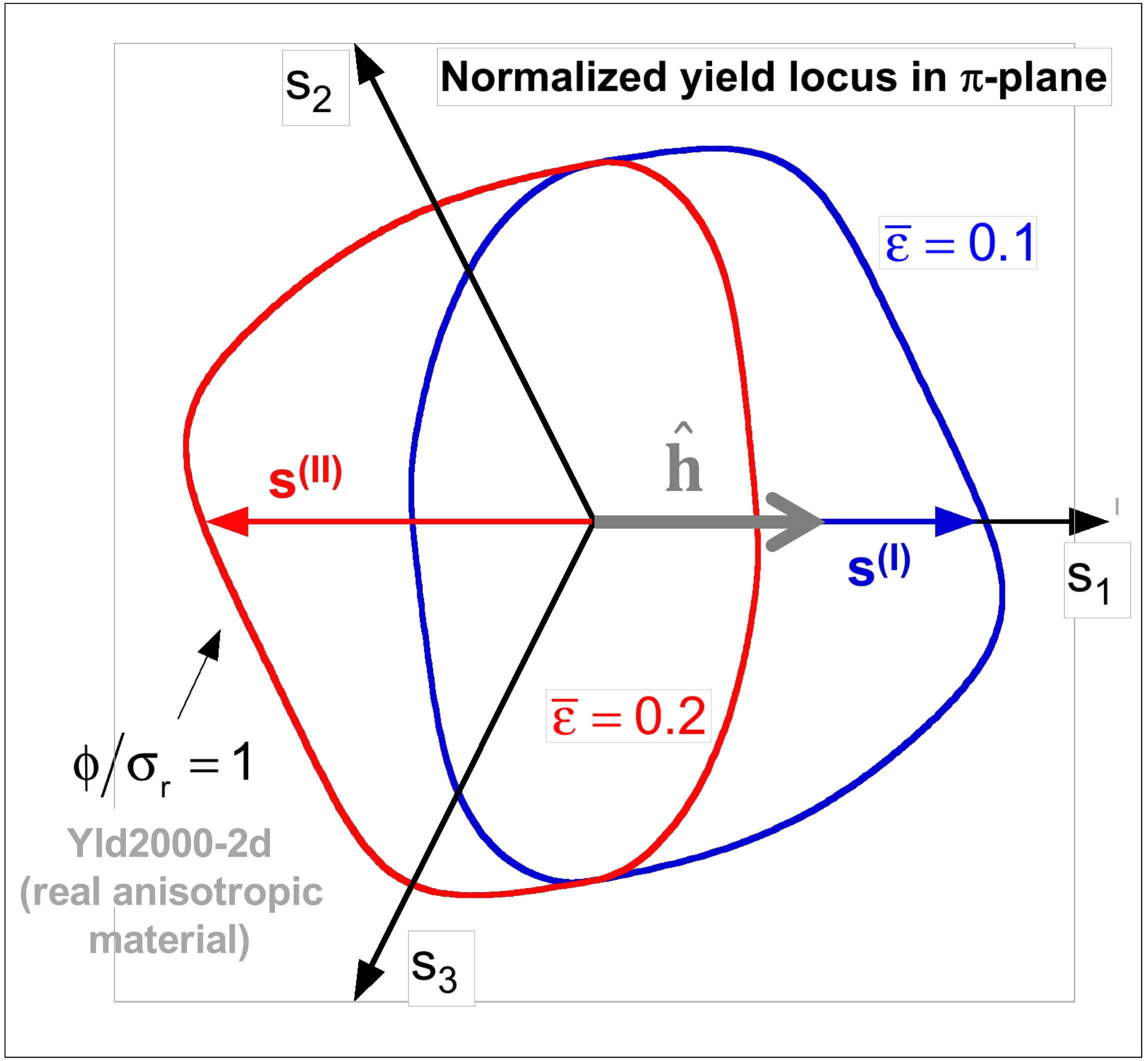
$\hat{\mathbf{h}}$

- Tensorial state variable with evolution rule
- Mimics delays in formation / rearrangement of dislocation structures
- Provides a reference for distortion

4. Anisotropic hardening – HAH model

Reverse loading

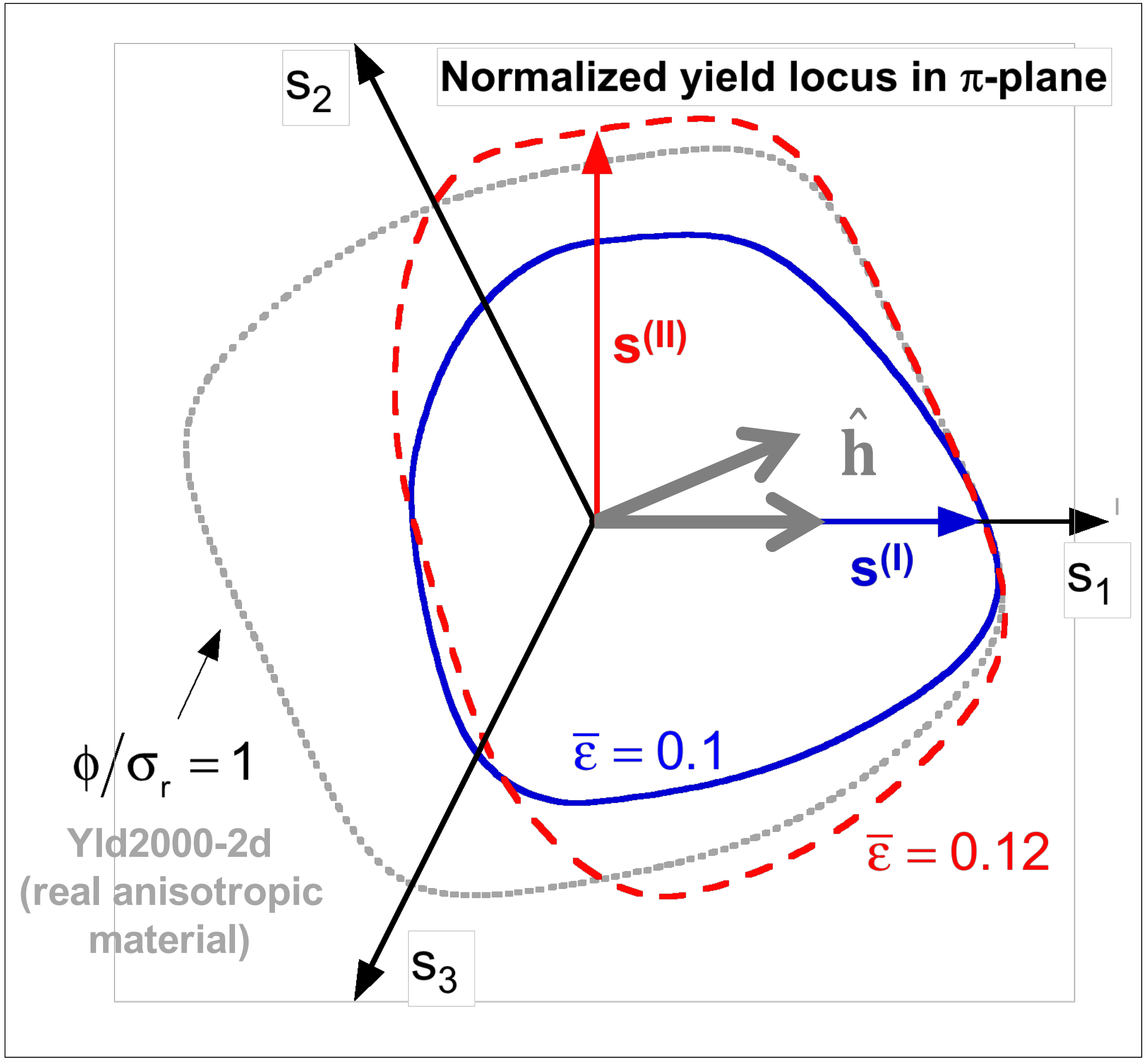
- **Anisotropic material 1**
- **Loading sequences**
  - (I) RD tension
  - (II) RD compression
- **Effects captured**
  - Distortional hardening
  - Bauschinger effect
- **Note**
  - Half and half



4. Anisotropic hardening – HAH model

Cross-loading

- **Anisotropic material 2**
- **Loading sequence**
  - (I) RD tension
  - (II) Near TD plane strain tension
- **Effects captured**
  - Distortional hardening
  - Bauschinger effect
  - Cross-loading contraction
  - Latent hardening
- **Note**
  - Proportional loading (proof)



4. Anisotropic hardening – HAH model

Coefficient identification

- Sequential

Proportional loading

- HAH reduces to isotropic hardening response (anisotropic yield function)
- Same identification procedure as that of isotropic hardening ($\sigma_R\left(\bar{\varepsilon}\right)$, $\bar{\omega}\left(\mathbf{s}\right)$)

Reverse loading

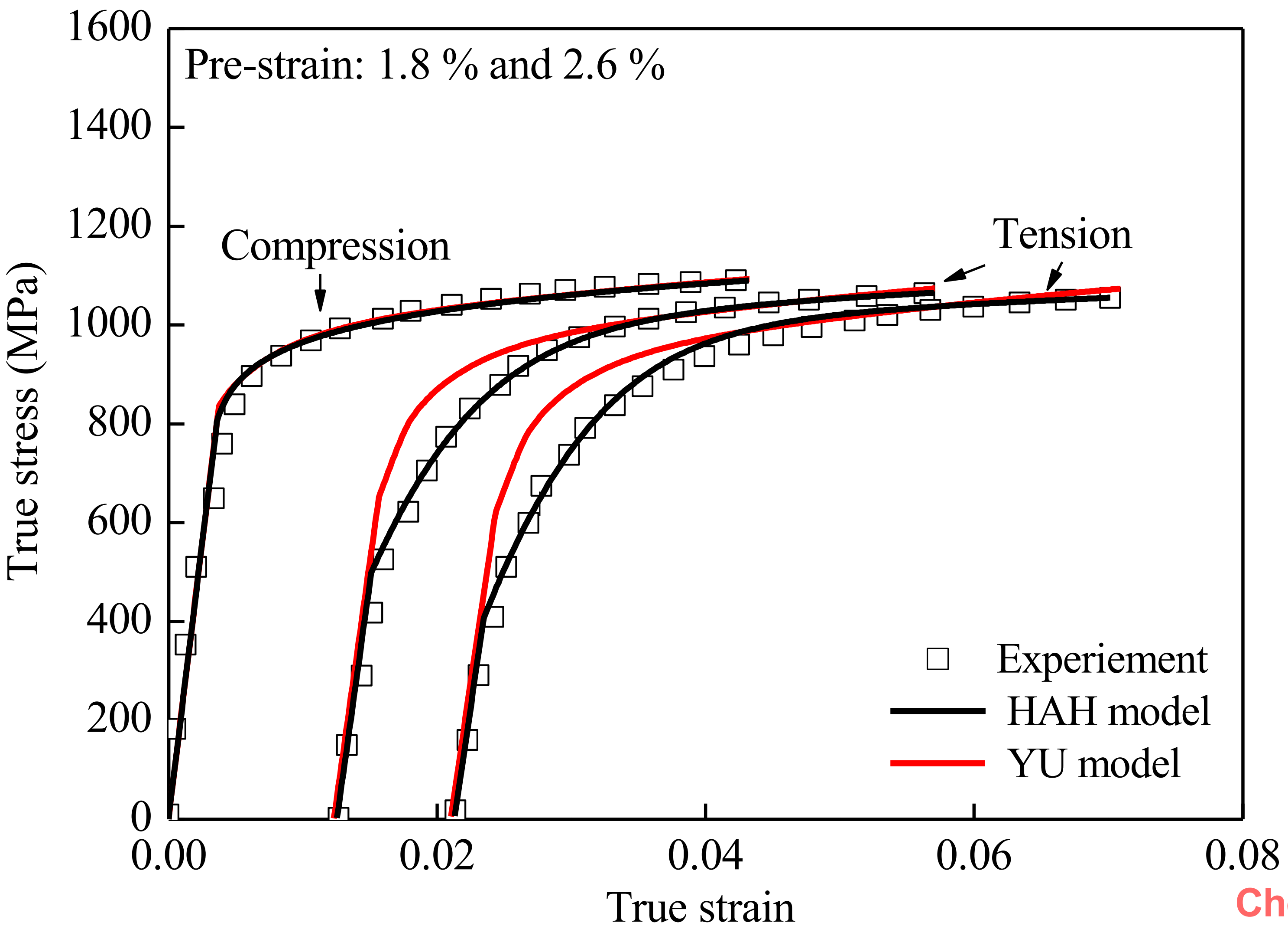
- Independent identification of coefficients (Bauschinger and other effects)

Cross-loading

- Independent identification of coefficients (latent hardening and other effects)

4. Anisotropic hardening – HAH model

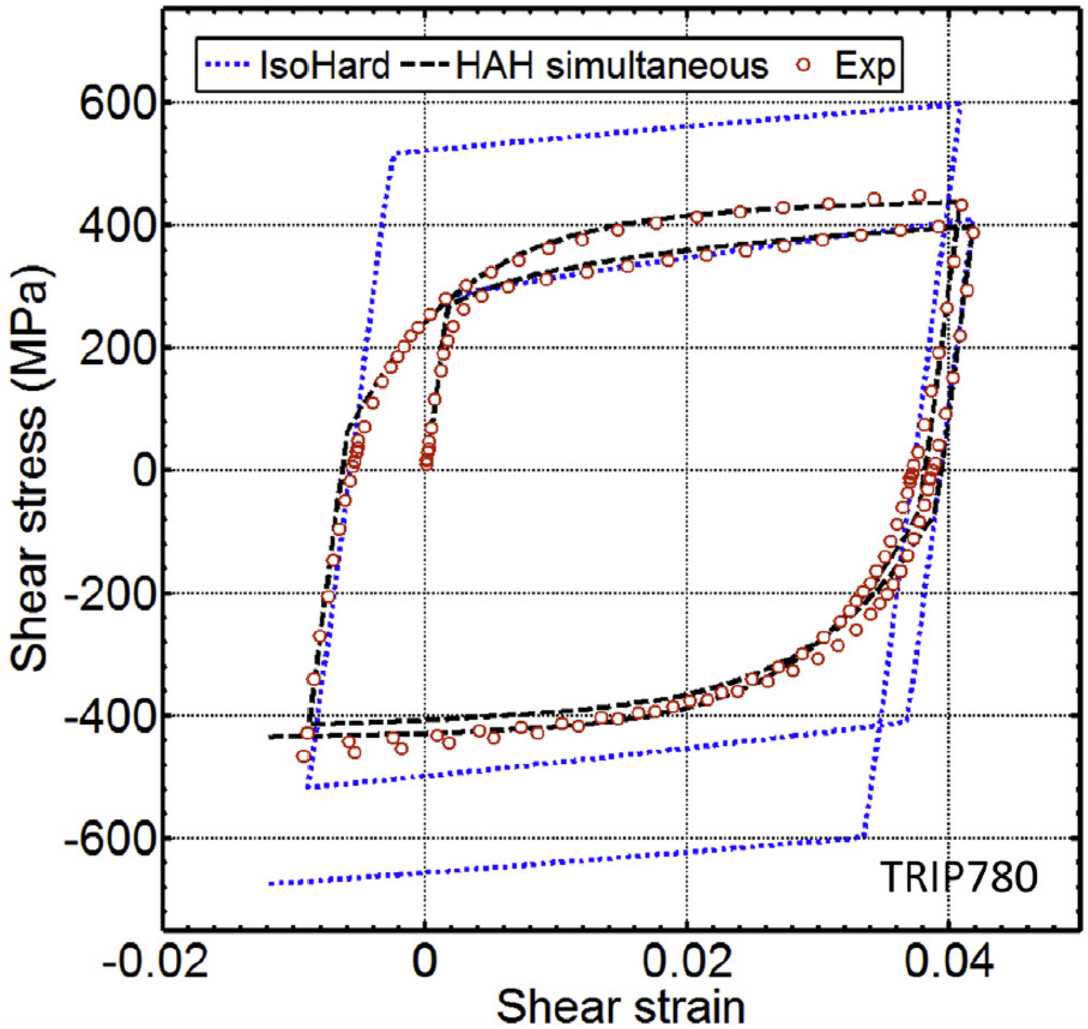
Forward and reverse simple shear



Choi et al., 2017

4. Anisotropic hardening – HAH model

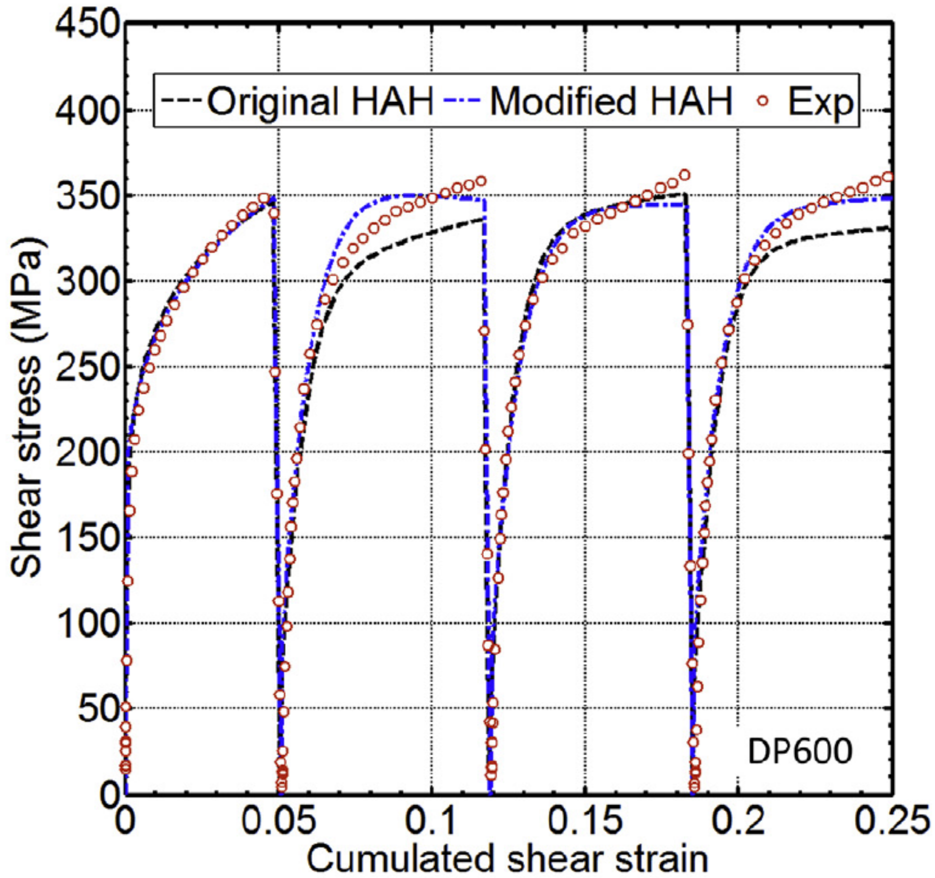
Forward and reverse simple shear (TRIP 780 steel)

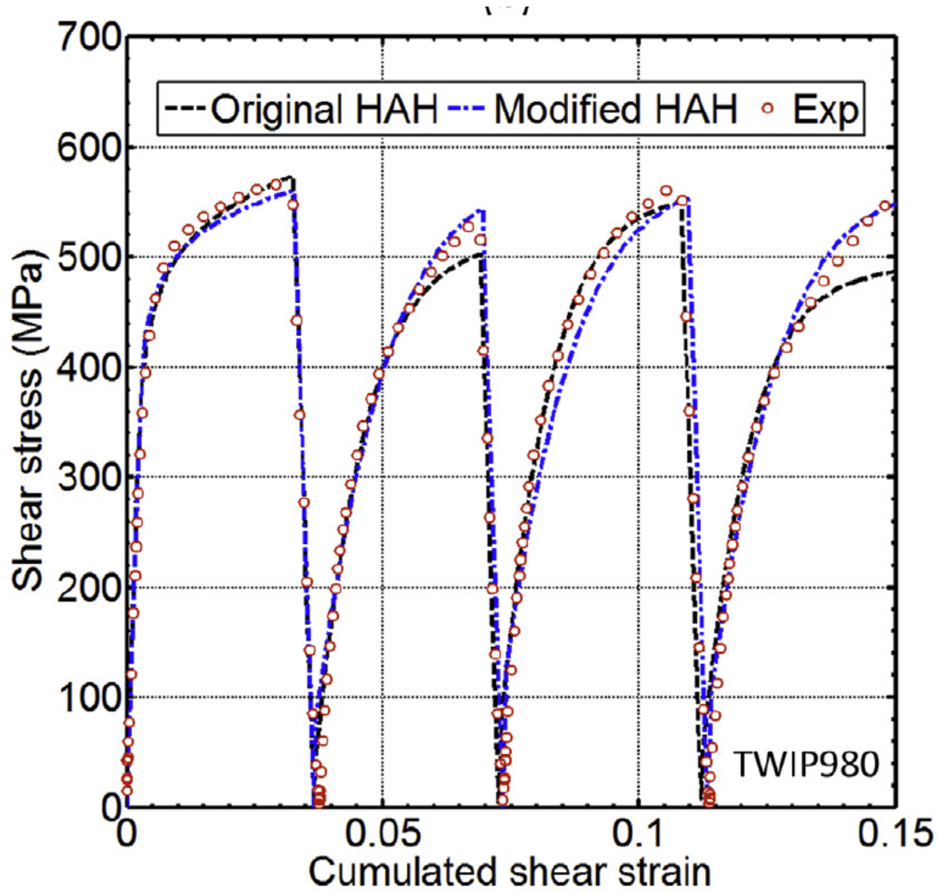


Fu et al. (2017)

4. Anisotropic hardening – HAH model

Forward and reverse simple shear cycles (DP 600 & TWIP 980 steels)





Fu et al. (2017)

5. Final remarks

Plasticity remains a topic with many challenges

- Anisotropic material under isotropic and anisotropic hardening
- Numerical implementation
- Identification with complex evolution equations
- Applicability for industrial problems