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Impact on decision-making of quantified uncertainty for DIC measurements

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European Commission projects

2002-2006



Standardisation
Project for Optical
Techniques for Strain
measurement

2008-2011



Advanced Dynamic
Validation by
Integrating Simulation
& Experimentation

2013-2014



VALidation of Numerical
Engineering Simulations:
Standardisation Actions

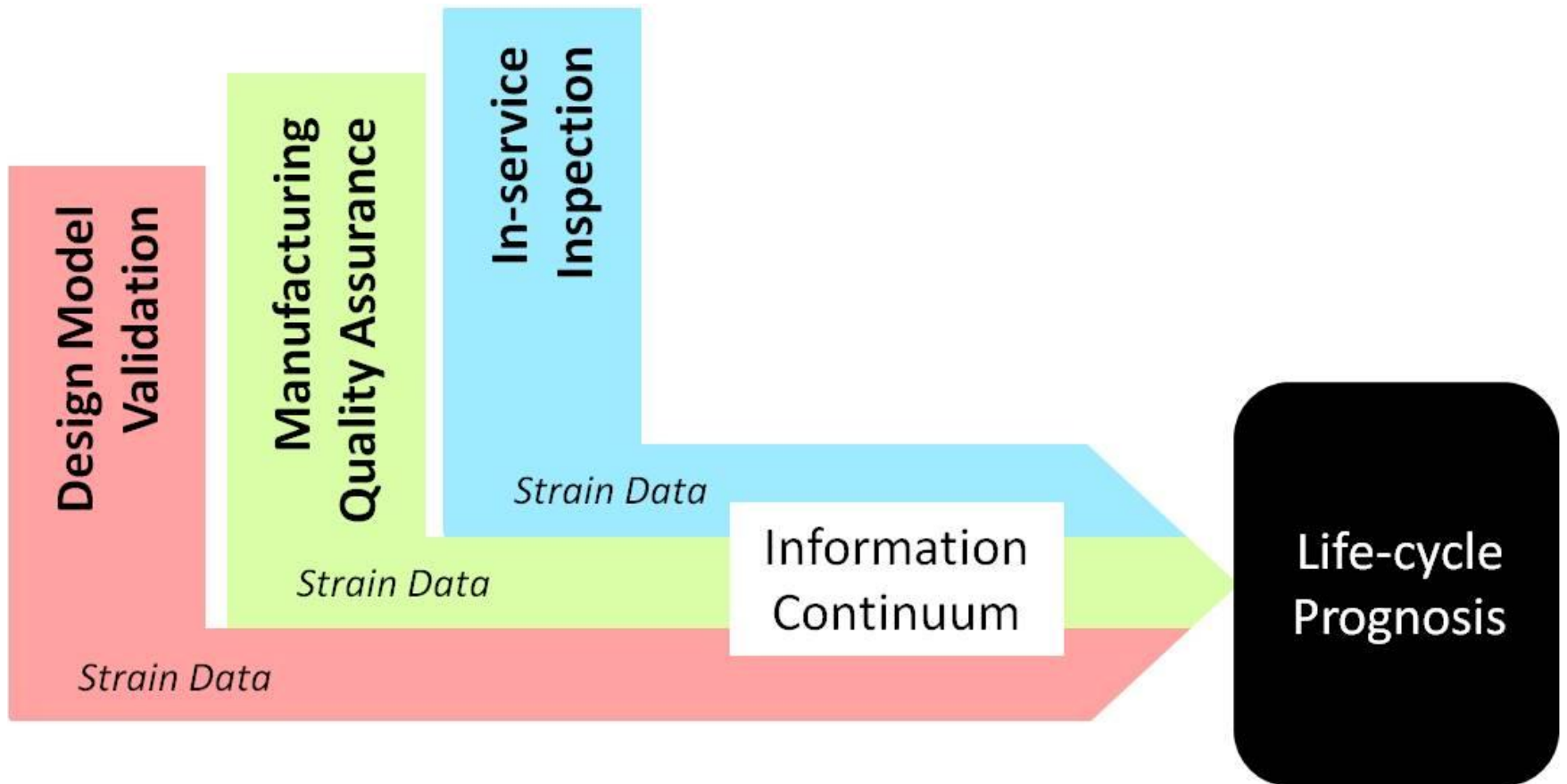
2017-2020



Matrix Optimisation
for Testing by
Interaction of Virtual
And Test Environments

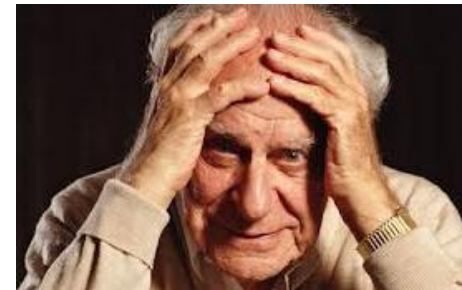


Information continuum



What is an acceptable model?

- Hume [1748] suggested that observational evidence will never support any hypothesis about the unobserved.
- More pragmatic approach required...
- Popper [1959] proposed that observational evidence cannot prove a hypothesis correct, but it can demonstrate its inappropriateness or falsity
- Implies that there is always a possibility of making a mistake when accepting [or rejecting] a hypothesis



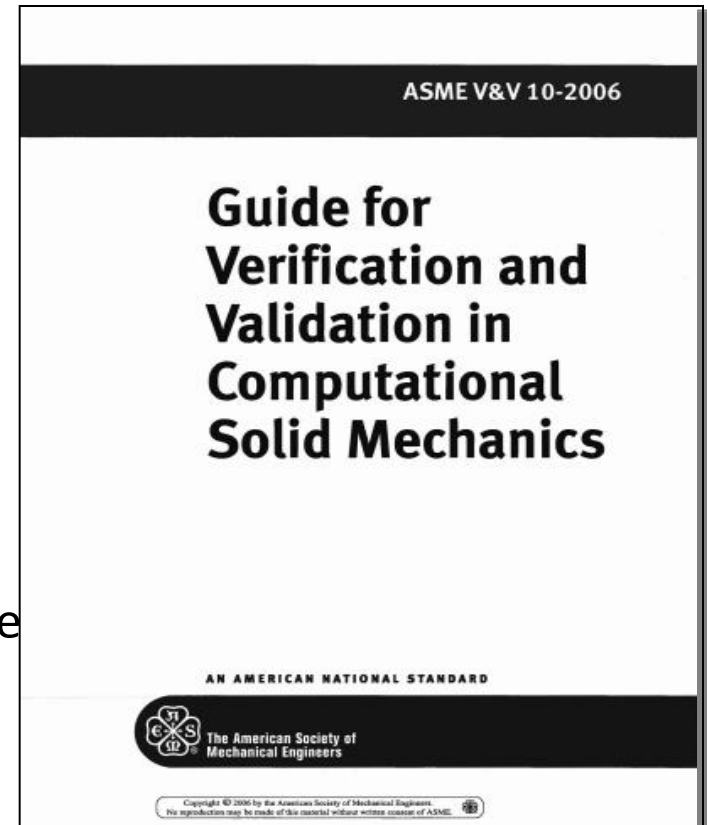
Hume, D., 1748 [1999], *An enquiry concerning human understanding*. Oxford Philosophical Texts, Oxford University Press, Oxford, edited by T.L. Beauchamp.

Popper, K., 1959, *The logic of scientific discovery*, Hutchinson, London.

- **Rudner [1953] argued that the decision to accept or reject a hypothesis depends on the strength of the evidence**
 - And, our judgement on the strength of the evidence depends on the importance of making a mistake in accepting or rejecting a hypothesis
- **For engineering models**
 - Importance of making a mistake will be measured in economic cost and loss of life or injury arising from an engineering failure
- **Strength of evidence required to accept a computational solid mechanics model could be very high in some cases**
 - e.g. design of an aircraft or nuclear power plant
 - So, need comprehensive validation of model



- **Verification**
 - ‘The process of determining that a computational model accurately represents the underlying mathematical model and its solution.’
- **Validation**
 - ‘The process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model.’
- **No methodology provided**



- **Validity of computational models is analogous to scientific hypotheses**
 - Recognise that observational [experimental] data cannot prove its validity
 - Increasing body of evidence can increase degree of belief in the model¹
- **obvious that current practices, based on the strain value at a small number of locations, are inadequate**
- **until now, obviousness over-powered by cost of experimental data**
 - alleviated by new technologies e.g DIC, DVC, ESPI & TSA
- **and, lack of methods for quantitative comparisons of full-field data**
 - Different orientation, coordinate system, scale, pitch of data
 - Resolved by use of image decomposition²
 - Reduces dimensionality of data & is invariant to rotation, scale & translation



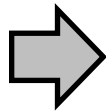
1. Audi, R., 2011, *Epistemology: a contemporary introduction to theory of knowledge*, 3rd ed., Routledge, New York.

2. Wang, W., Mottershead, J.E., Sebastian, C.M., Patterson, E.A., 2011, Shape features and finite element model updating from full-field strain data, *Int. J. Solids Struct.* 48(11-12), 2011, 1644-1657.

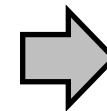
Idealised image decomposition



Image of a three-dimensional shape

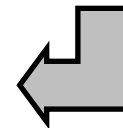


Matrix of grey-scale values, $I(i, j)$



$$S = \begin{bmatrix} s_1 \\ s_2 \\ \dots \\ \dots \\ s_l \end{bmatrix}$$

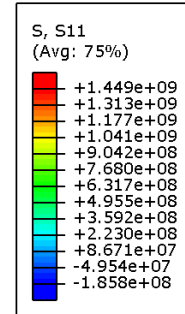
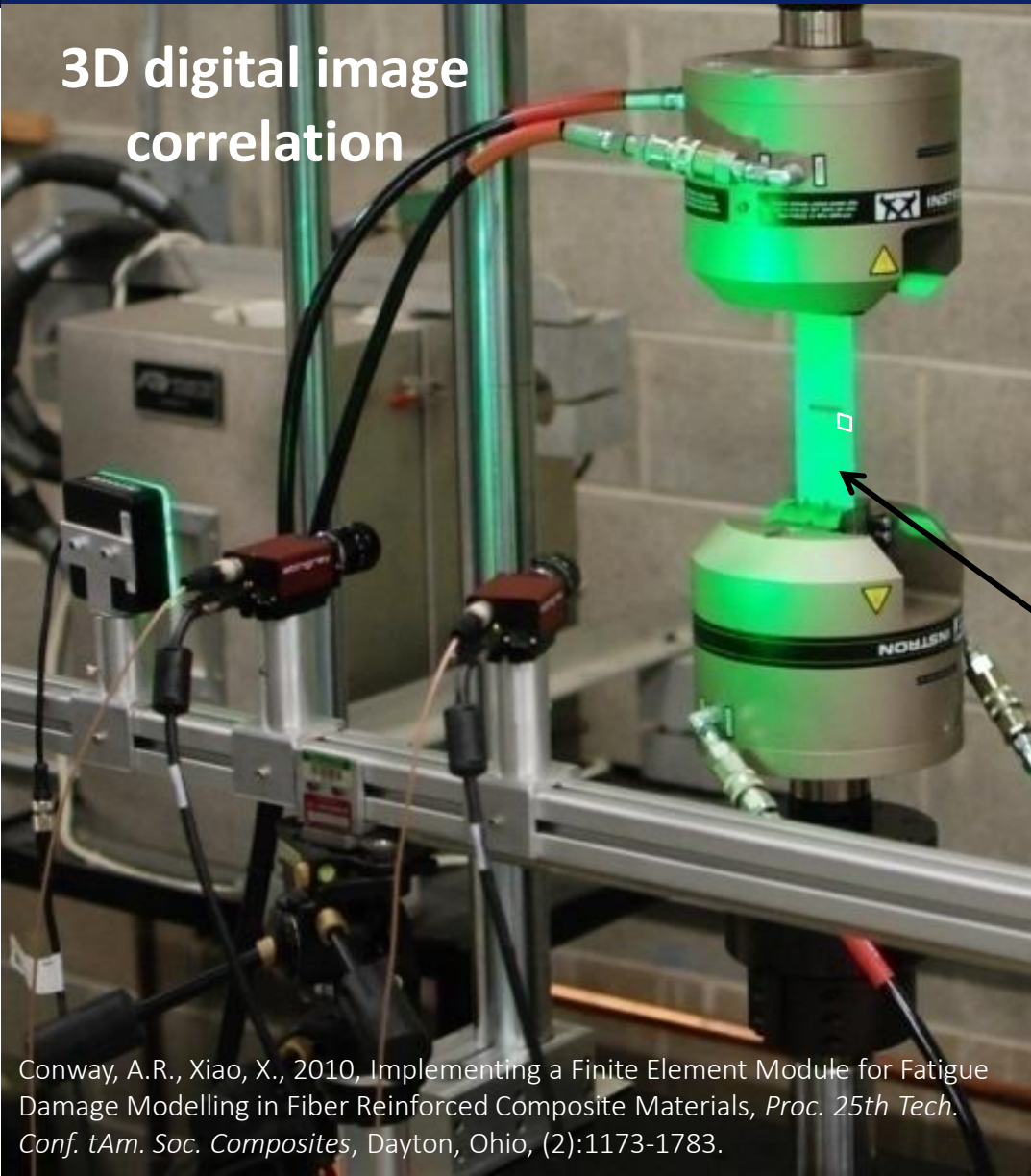
Feature vector consisting of shape descriptors



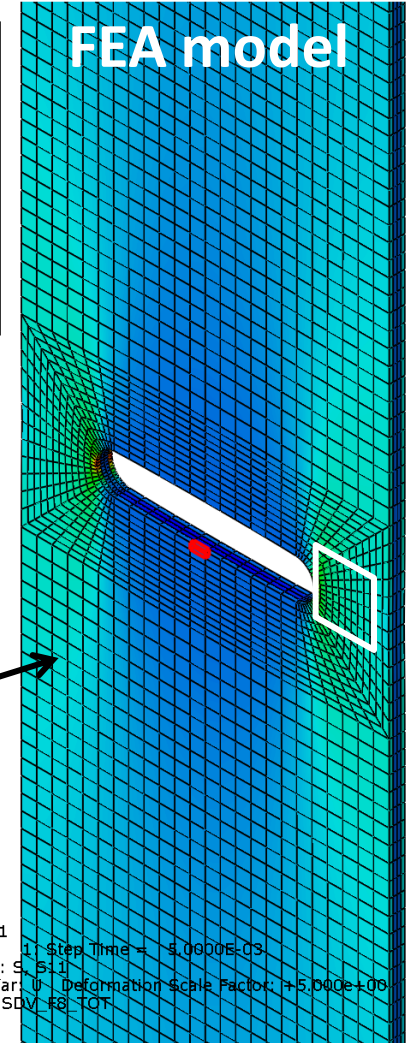
Reconstruction

Massive datasets: $>10^5$ values

3D digital image
correlation



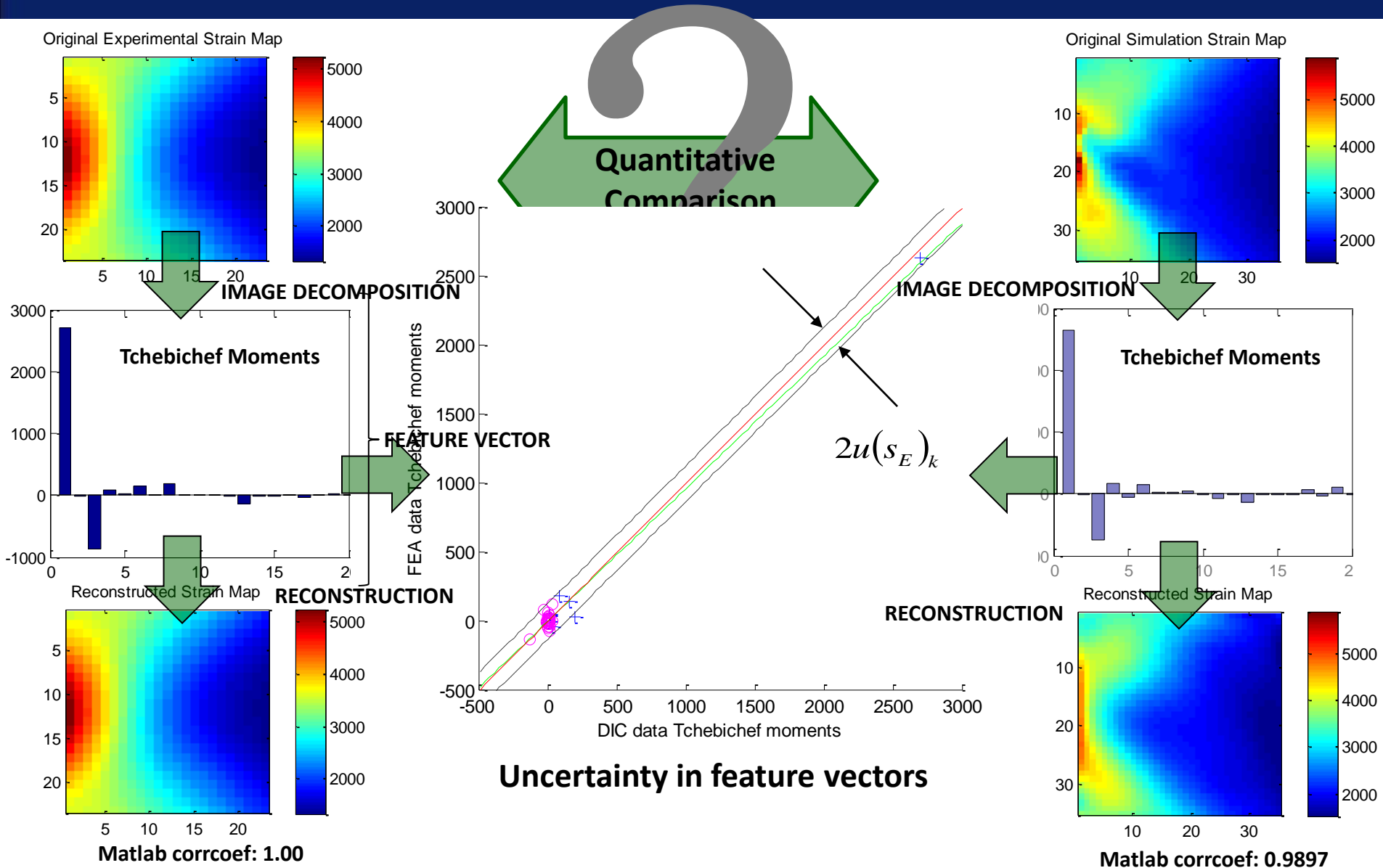
FEA model



Composite
tie bar

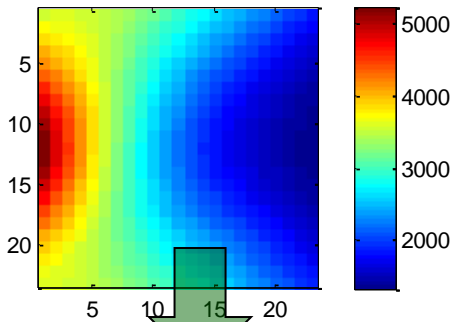
Step: Step-1
Increment: 1
Primary Var: S, S11
Deformed Var: U
Status Var: SDV_F8_TOT
Step Time = 5.0000E-03
Deformation Scale Factor: +5.000e+09

Validation procedure



Validation procedure

Original Experimental Strain Map



Original Simulation Strain Map

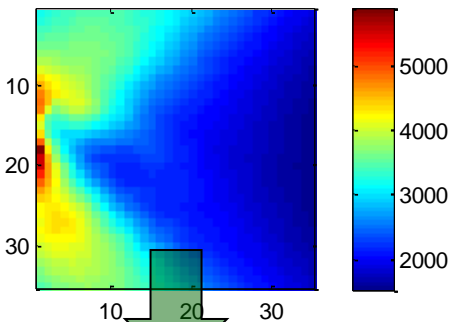
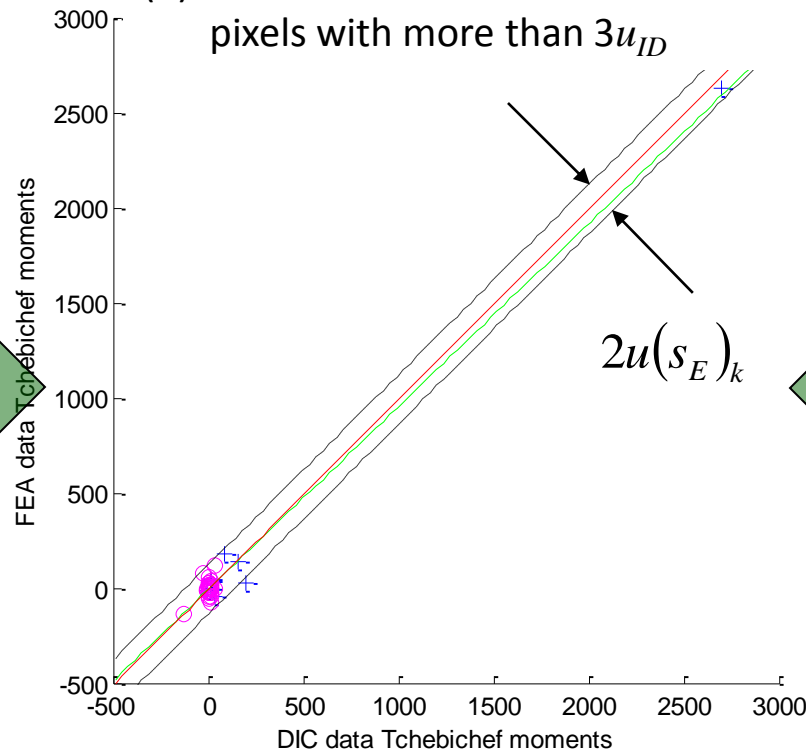


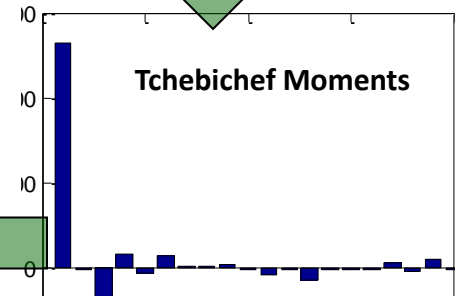
Image decomposition criteria

(a) $u_{ID}^2 < u_{cal}^2(\varepsilon)$

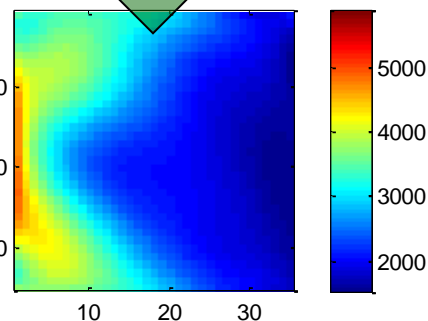
(b) No cluster of more than 0.3% total pixels with more than $3u_{ID}$



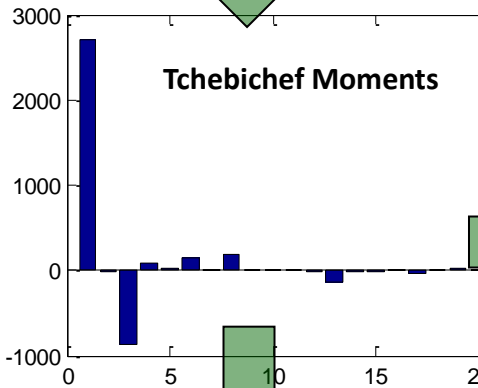
Tchebichef Moments



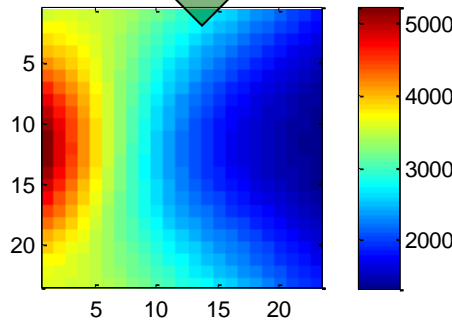
Reconstructed Strain Map



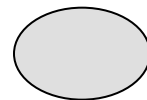
Matlab corrcoeff: 0.9897



Reconstructed Strain Map

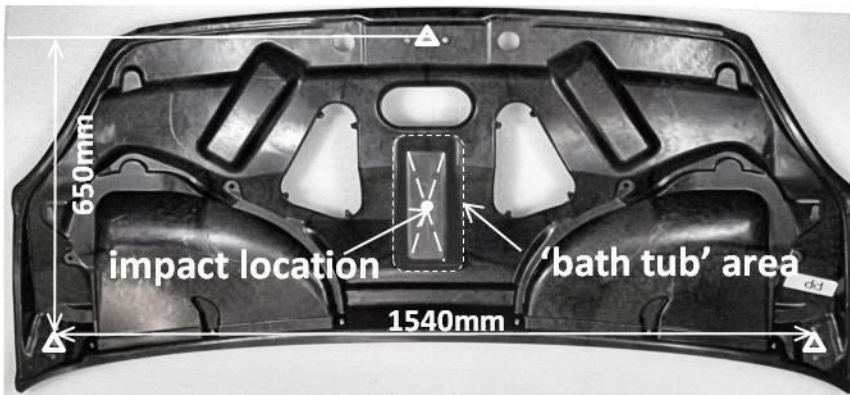


Matlab corrcoeff: 1.00



Uncertainty in feature vectors

Impact on composite bonnet liner

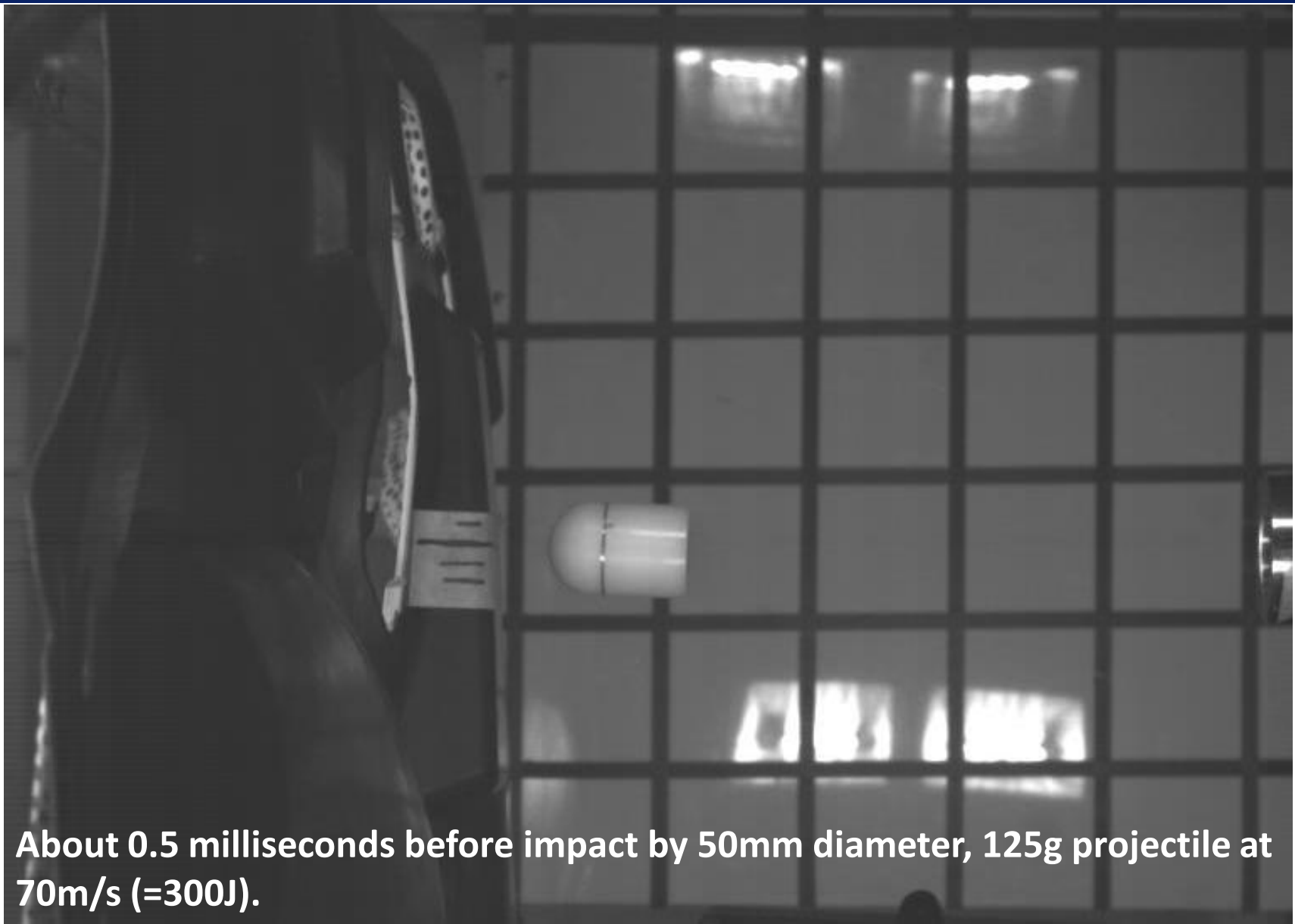


Bonnet liner made with short fibres in polyamide matrix



Finite element model of bonnet liner

High velocity, low energy impact

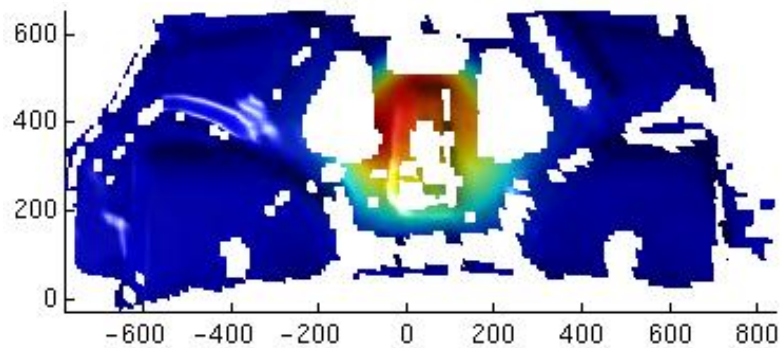


About 0.5 milliseconds before impact by 50mm diameter, 125g projectile at 70m/s (=300J).

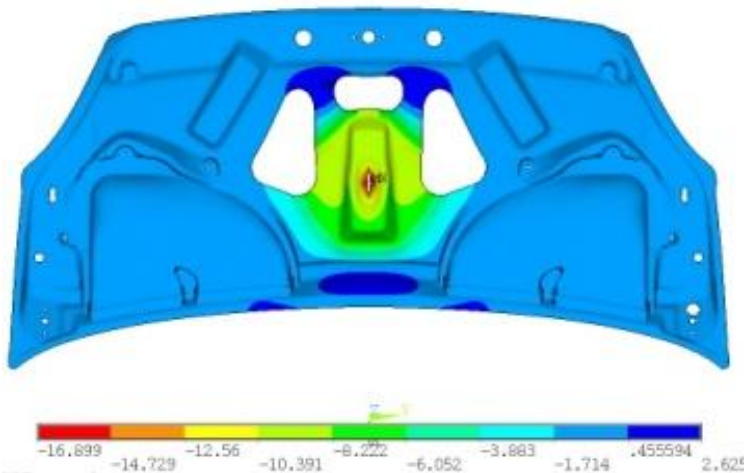
Dynamic analysis of bonnet

Displacements 40 msec after impact

from DIC

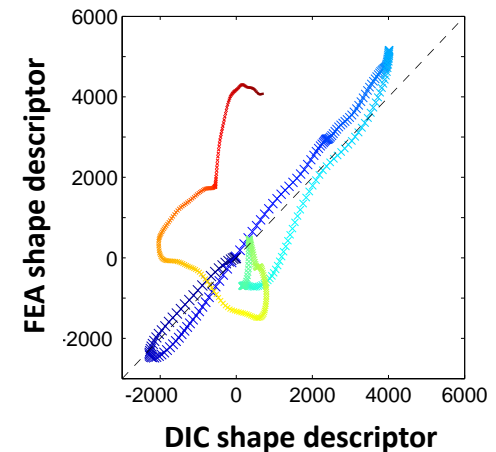
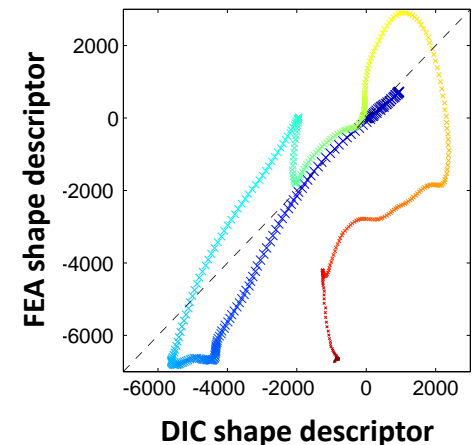
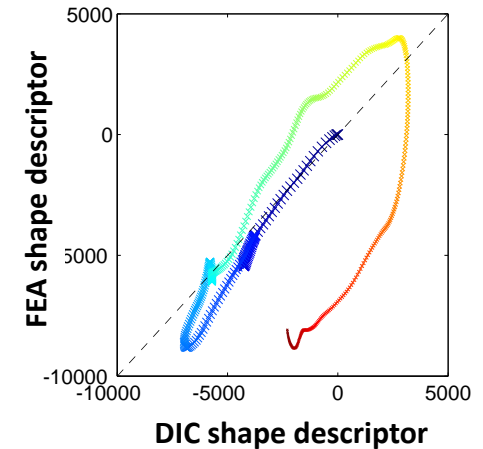
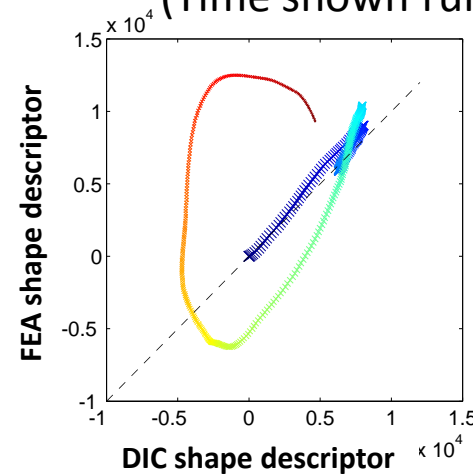


from FEA

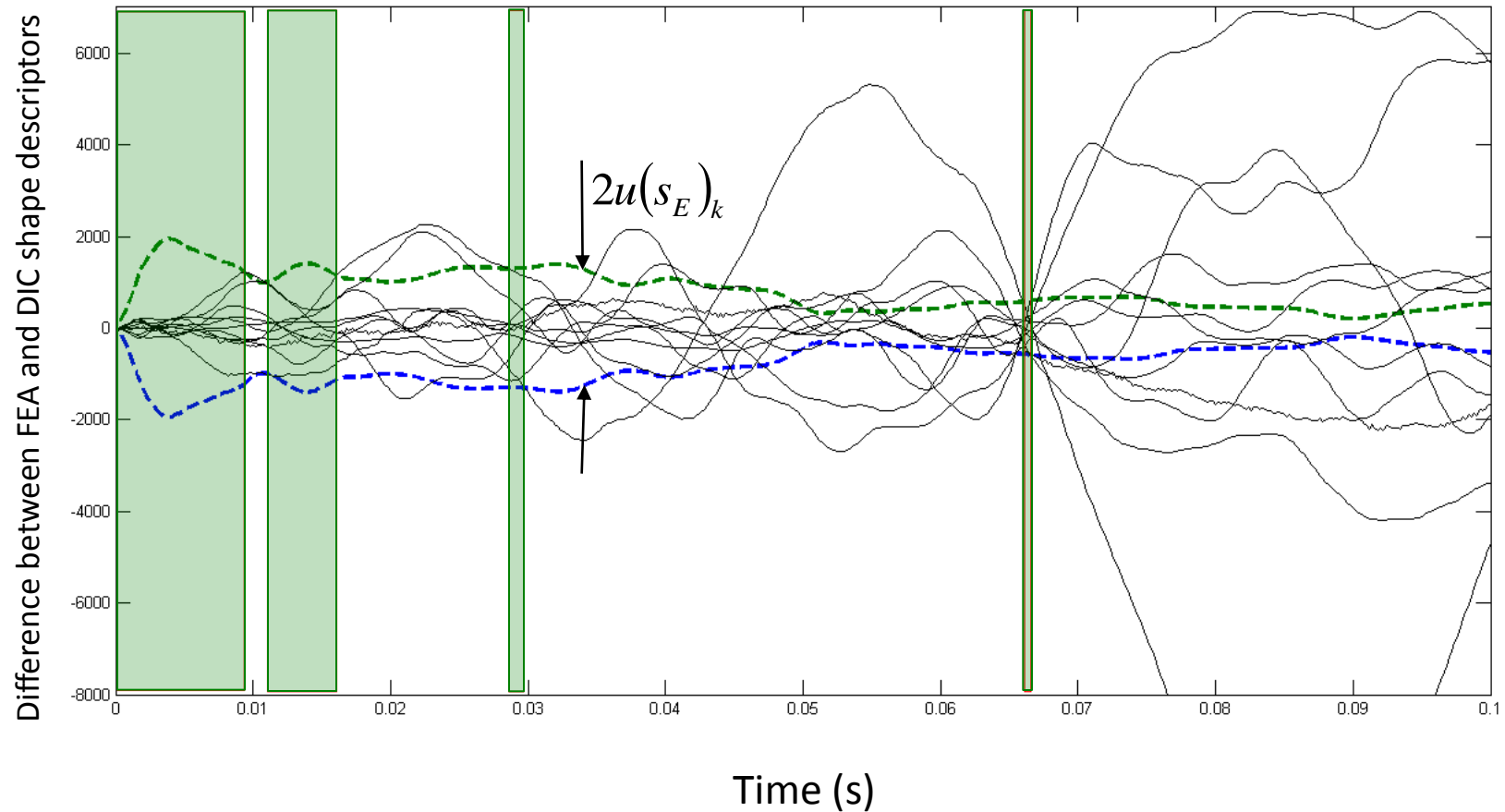


Largest four of 20 shape descriptors

(Time shown running from blue to red)



Comparison for validation of model



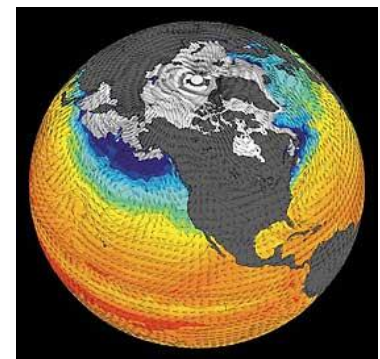
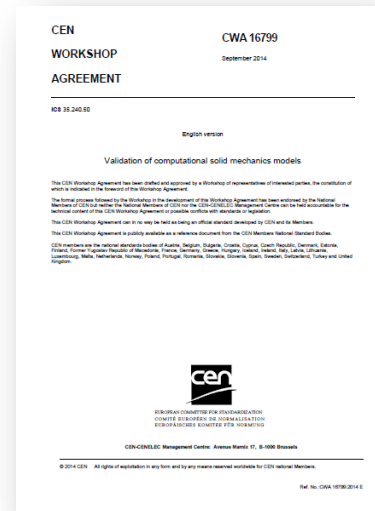
Consequences/Opportunities?

Experimentalists: “How do I implement all this?”

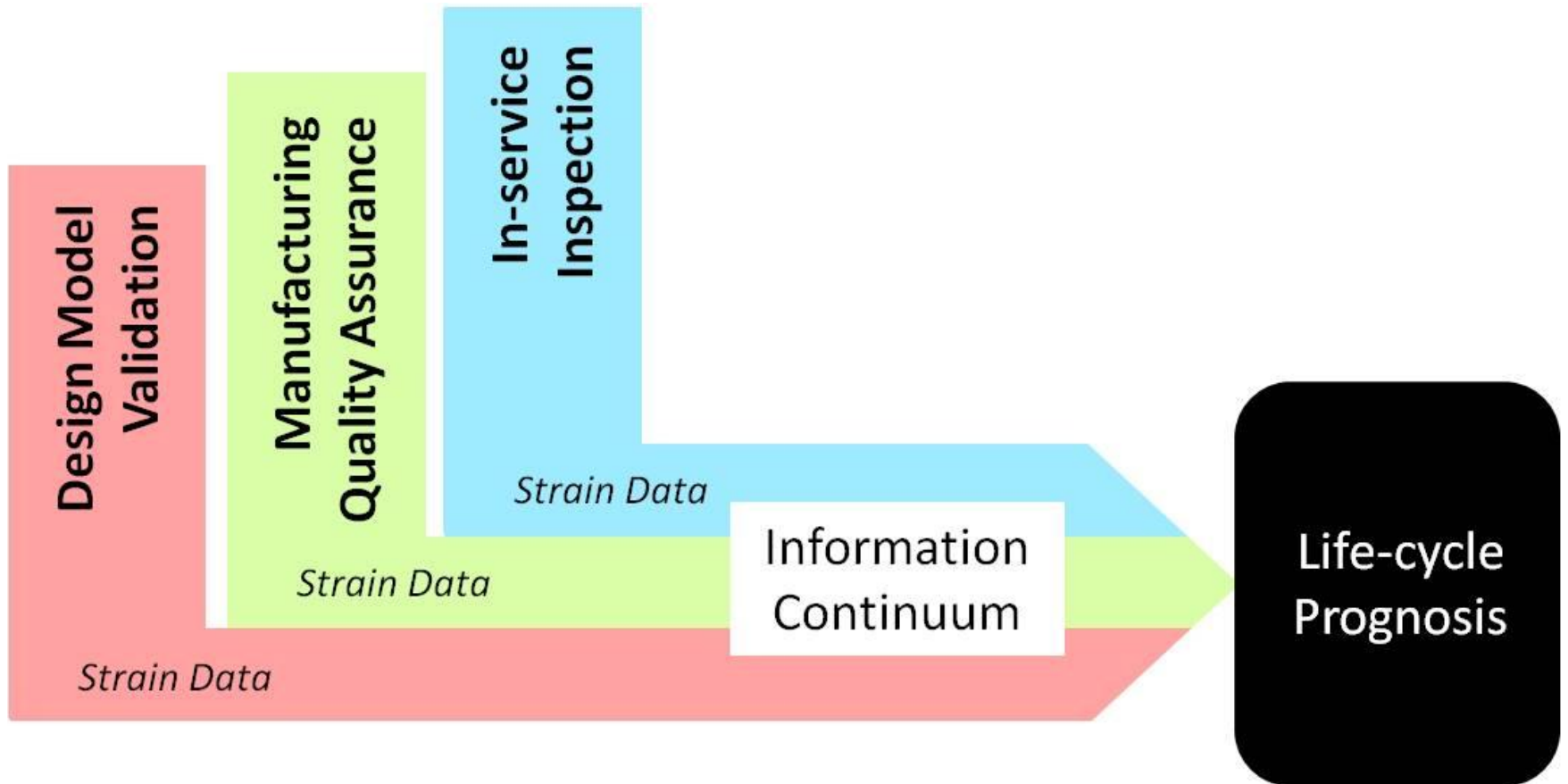
- **CEN Workshop Agreement 16799: 2014**
Validation of computational solid mechanics models

Modellers: “Do we have to do all of this?”

- **Well, yes and no**
- **No, for product designs that evolve relatively slowly**
 - models with epistemic values are more likely reliable
 - epistemic values include: simplicity, explanatory power, internal consistency, external consistency
- **Yes, for radical design innovation**
 - Models unlikely to have epistemic properties
 - Comprehensive validation required to support credibility
- **Yes, for high risk applications**



Information continuum



Strain fields in damaged laminates

Virgin & impact damaged
9-ply GFRP 3.5mm thick
[0/90/0/90/0/90/0/90/0]^o
240x60mm laminates

Time of
flight
ultrasound
with binary
filter

DIC strain fields
in tension

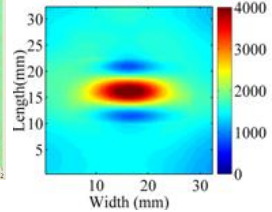
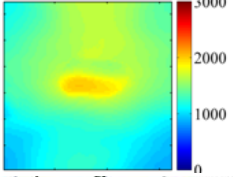
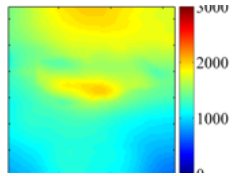
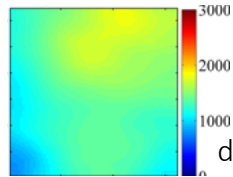
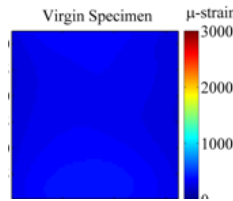

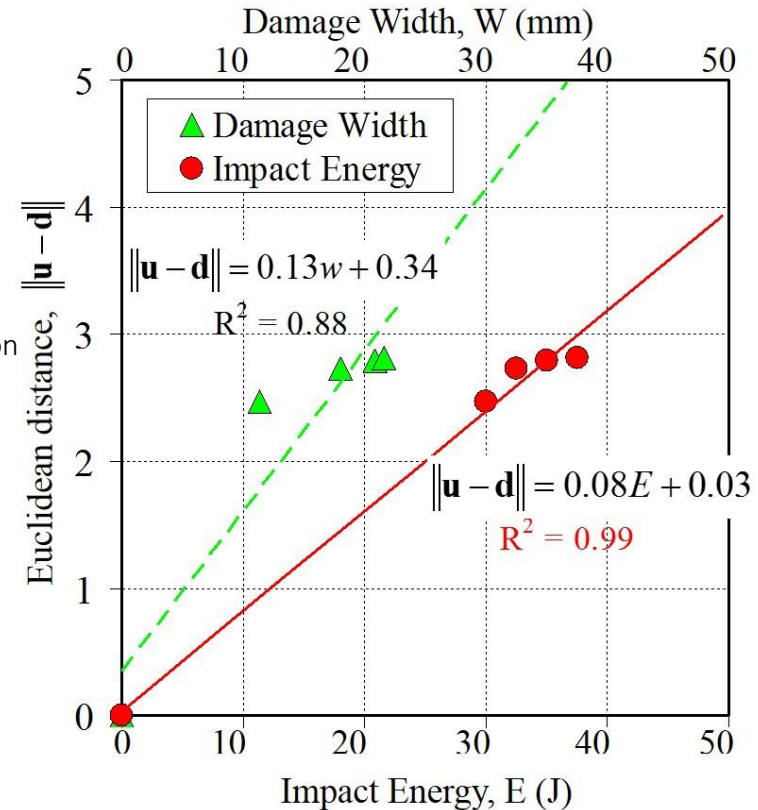


Image
decomposition



Euclidean distance between DIC strain fields in
damaged and virgin specimen is excellent
indicator of impact energy & damage

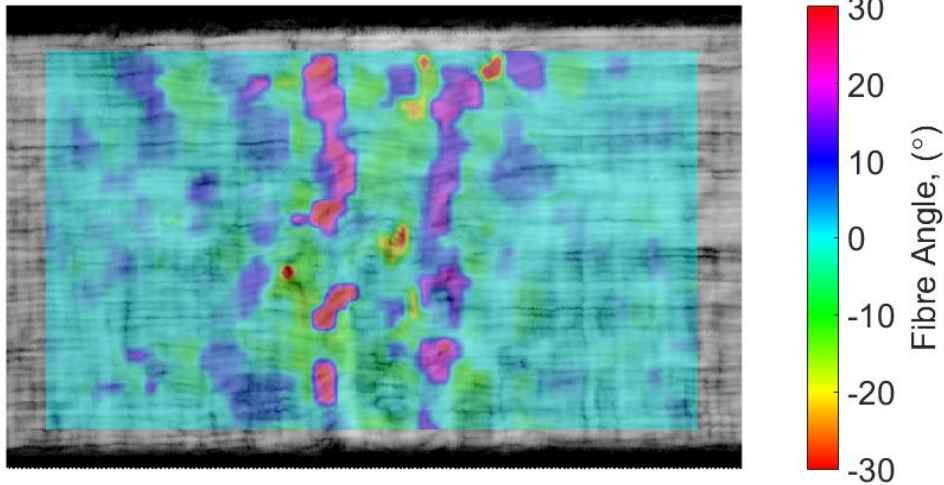


$$\|\mathbf{u} - \mathbf{d}\| = \sqrt{\sum_{i=0}^N (u_i - d_i)^2}$$

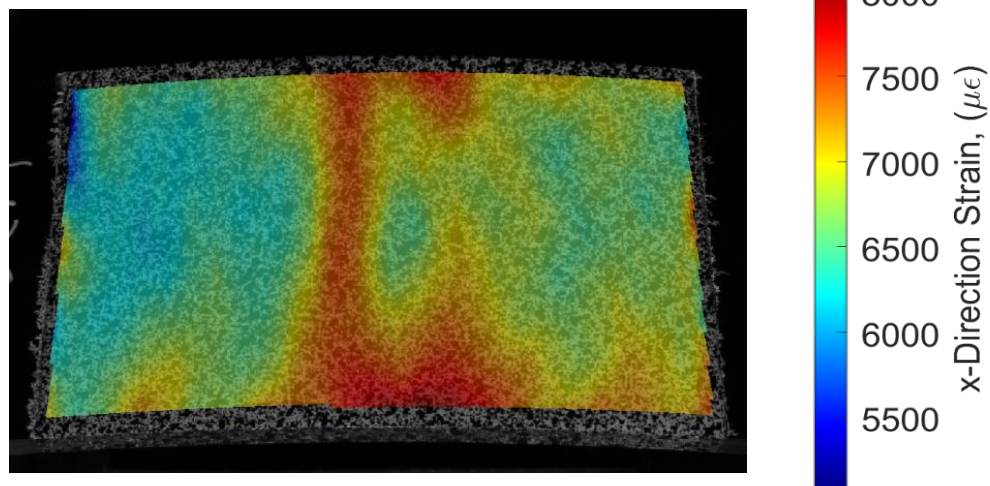
u_i, d_i are feature vector
coefficients representing
undamaged /damaged
strain fields

Manufacturing Quality Assurance

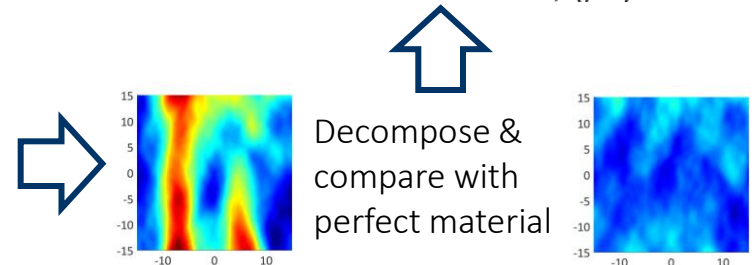
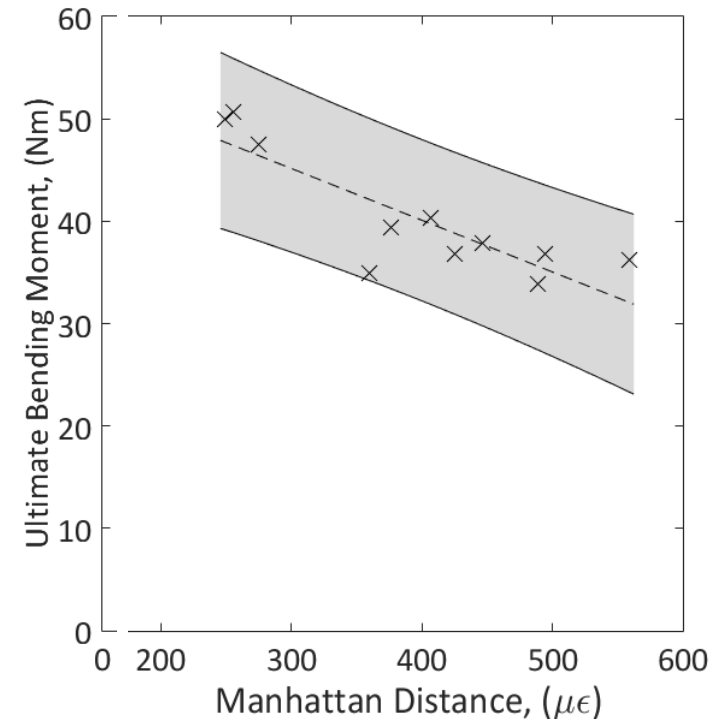
Ultrasound evaluation of fibre waviness in CFRP



Strain[DIC]-based evaluation of fibre waviness

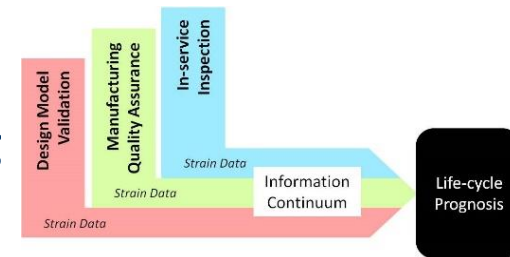


Residual strength prediction using DIC evaluation of fibre waviness



Concluding remarks

- **Opportunity to unite design validation, manufacturing assurance & in-service inspection in digital twins**
- **Model validation is about quantifying the strength of evidence**
 - Uncertainty quantification is an essential step in validation
 - Aim to use ‘observational evidence’ of maximum quantity & quality
- **Manufacturing quality assurance viable in composites**
 - Strain data indicate likely structural performance relative to design model
- **In-service inspection viable for composites laminates**
 - Strain data provide input to more reliable residual life assessments
- **All enabled by proper orthogonal decomposition**
 - low-dimensional descriptions that capture the features of interest in large quantities of high dimensional data

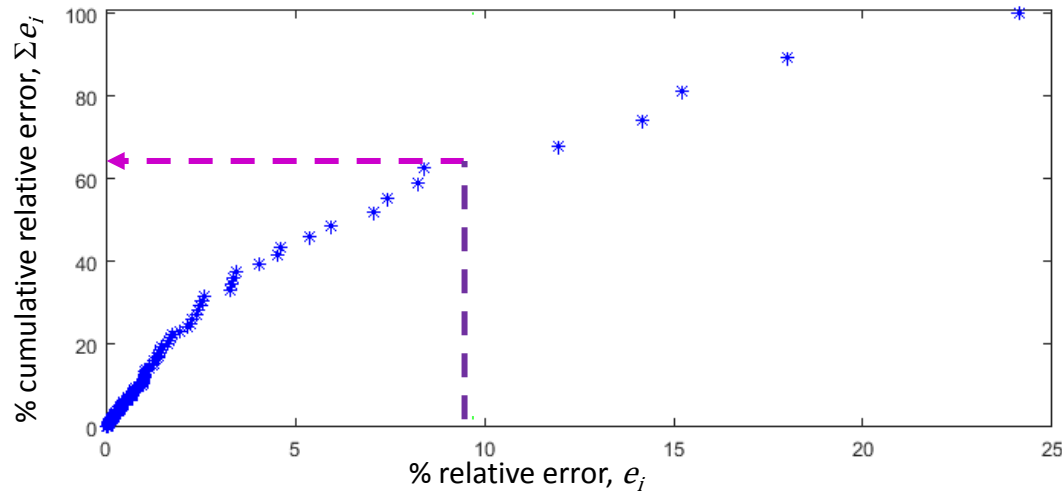




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Current work: frequentist approach

- Relative error for each data pair,
$$e_i = \frac{|S_{M,i} - S_{E,i}|}{\max |S_{E,i}|}$$



64% probability of model being valid



when simulating specific parameter & conditions



given 9% relative uncertainty in experimental data