Numerical study of Lamb wave mixing for micro-crack detection in plate

Hongtao Lyu[1a], Jingpin Jiao[1], Cunfu He[1] and Bin Wu[1]

[1]Department of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology, Beijing 100124, China

[a]lvhongtao2015@emails.bjut.edu.cn

Abstract. In this paper, the nonlinear interaction of two A0 Lamb waves with microcracks was investigated using finite element method. The simulation results show that resonant conditions based on classical nonlinear theory are valid for such nonlinear interactions, which lead to the generation of transmitted and reflected sum-frequency S0 waves (SFLW). Moreover, the propagation directions of these two sum-frequency S0 waves exhibit different trends with respect to the orientations of microcracks. The transmitted sum-frequency S0 wave can be used to detect microcracks, whereas the reflected one can be used to measure their orientations.

Introduction

Fatigue crack is one of common causes of plate-like structures failure. To avoid the accident, it is important to develop an effective approach to non-destructive testing and evaluation of fatigue-induced damage. Wave mixing technique is a promising method for detecting micrometer-sized defects [1]. However, little research was done to reveal the complex interactions between microcracks and Lamb waves, especially the influence of microcrack orientation on propagation direction of the generated sum-frequency Lamb wave. In this paper, A0 wave mixing was investigated numerically to demonstrate its potential for microcracks characterization.

Theory of Lamb wave mixing

Eq.1 and Eq.2 give the resonant conditions of nonlinear interaction of two plane elastic wave to generate a third wave [2,3].

$$f_g = f_1 \pm f_2 \tag{1}$$
$$\boldsymbol{K}_g = \boldsymbol{K}_1 \pm \boldsymbol{K}_2 \tag{2}$$

where $\boldsymbol{K}_1$, $\boldsymbol{K}_2$ and f_1, f_2 are the wave vectors and frequencies, respectively, of the two incident waves, and $\boldsymbol{K}_g$ and f_g are wave vector and frequency, respectively, of the generated third wave. Previous research showed that those resonant conditions, obtained for the case of bulk waves in an elastic isotropic solid, are also valid for the interaction of Lamb waves [4]. Here, Fig.1 shows the resonant conditions for the interaction of two A0 waves leading to the generation of a SFLW when one incident excitation frequency is fixed to 200 kHz.

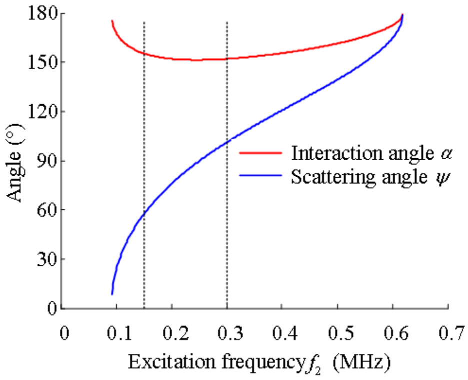

Fig.1. Resonant conditions of A0 wave mixing.

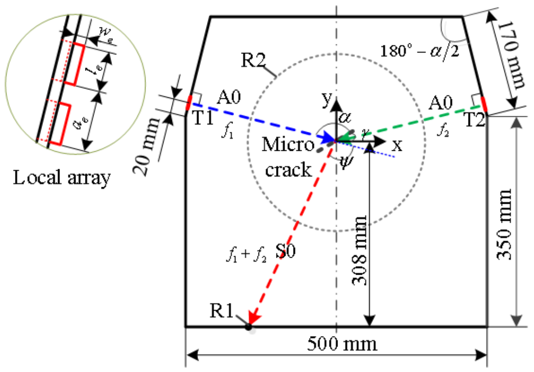

Fig.2. Simulation model of aluminum plate.

Finite-element model and simulation results

A three-dimensional finite element model is represented in Fig.2. The thickness of model is 1mm, and its mechanical properties are those of aluminum. The two A0 waves with different frequencies are stimulated by two linear arrays T1 and T2. An appropriate time-delay law is applied to the two arrays to ensure the beams of the two A0 waves meet at the center of the microcrack. The numerical outputs of velocities are recorded at two sets of positions. One set of positions (R1) at the bottom of model is located in theoretical propagation direction of the generated longitudinal wave; the other set of positions (R2) is a circle of radius 30 mm to analyze the directivity patterns of the generated SFLW.
Fig.3 shows time-frequency distributes of typical signals obtained from model without and with microcrack. It should be noted that for the signal obtained from the model with a microcrack, besides the two peaks at the fundamental frequencies, a new peak appears at the sum frequency (500 kHz), which indicates sum-frequency

generation. According to their group velocities, the interaction of two incident A0 waves with a microcrack can lead to the generation of a sum-frequency S0 wave or a sum-frequency A0 wave.

Fig.4 shows the directivity patterns of the generated SFSWs for different incident frequency combinations. It can be seen that the nonlinear interaction between the two A0 waves and the microcrack generates two types of sum-frequency wave, $S0_R$ and $S0_T$, with different propagation directions, which can be predicted by theoretical resonant conditions. It is demonstrated that the resonant conditions based on classical nonlinear theory are still valid for the nonlinear interactions between Lamb waves and microcracks.

Fig.5 shows the dependencies of the propagation directions of the generated $S0_R$ and $S0_T$ waves on the orientation of the microcrack. With increasing oblique angle of the microcrack, the propagation-direction angle of the $S0_T$ wave generated because of the transmitted A0 wave remains almost constant, whereas the increment of the propagation-direction angle of the $S0_R$ wave generated because of the reflected A0 wave is about twice that of the oblique angle.

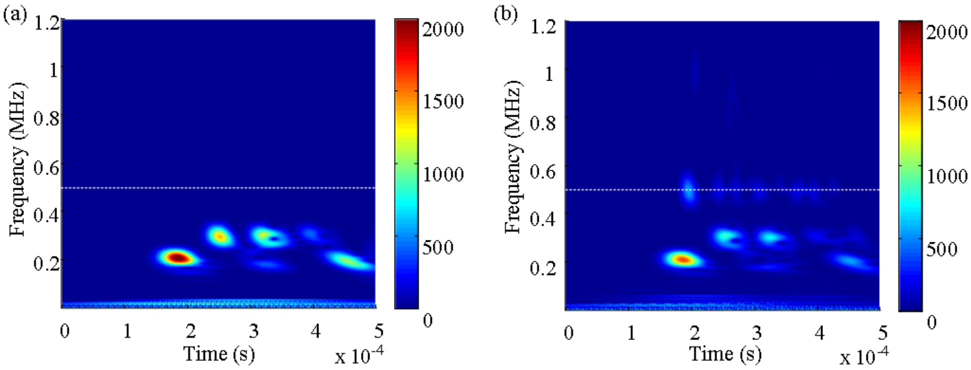

Fig.3. Time–frequency distributions of different signals obtained from the models: (a) without microcrack; (b) with microcrack.

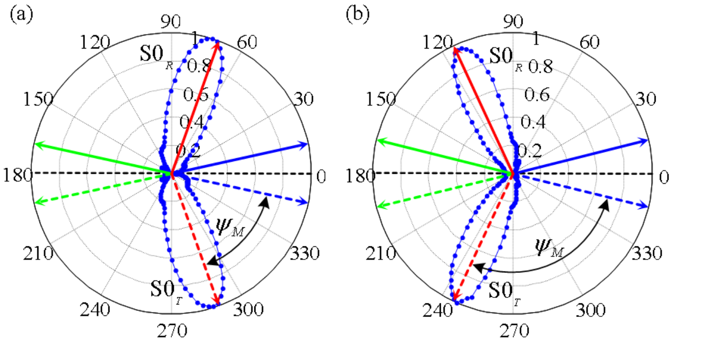

Fig.4. Directivity patterns of generated sum-frequency S0 wave for excitation frequencies:(a) 200kHz, 150kHz; (b) 200kHz, 300kHz;

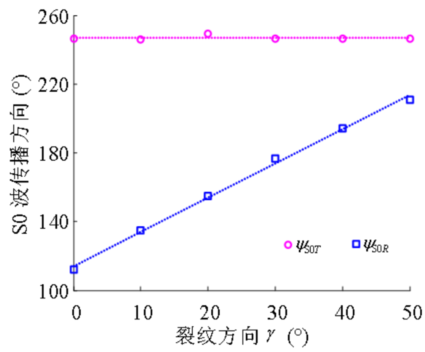

Fig.5. Propagation direction angles of generated $S0_R$ and $S0_T$ waves for cracks with different orientations

Conclusion

In this study, the technique of Lamb wave mixing was investigated numerically to demonstrate its potential for characterizing microcracks. The simulation results show that nonlinear interaction between two A0 waves and a microcrack lead to the generation of reflected and transmitted SFSWs. However, the propagation directions of these two SFSWs exhibited different trends with respect to microcrack orientation. The transmitted SFSW can be used to detect microcracks, and the reflected SFSW can be used to measure their orientations.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (grant nos. 11572010, 11572011) and the National Key Research and Development Program of China (grant no. 2016YFF0203002). The authors also acknowledge the financial support from China Scholarships Council.

References

[1] Croxford A J, Wilcox P D, Drinkwater B W, Nagy P B. *The Journal of the Acoustical Society of America*, Vol. 126 (2009), p. EL117-EL122.
[2] Jones G L, Kobett D R. *The Journal of the Acoustical society of America*, Vol. 35 (1963), p. 5-10.
[3] Taylor L H, Rollins Jr F R. *Physical review,* Vol. 136 (1964), p. A591.
[4] Furgason E S, Newhouse V L. *Journal of Applied Physics,* Vol. 45 (1974), p. 1934-1936.