Practical assessment of the accuracy of volumetric digital image correlation measurements for the analysis of geomaterials

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1) Introduction: microCT in-situ tests on geomaterials
2) Short review of DIC and DIC error sources
3) Quantification of discrete-DIC errors
4) Quantification of systematic errors
Macro

$$\Sigma(t)$$

Micro

$$\sigma(x)$$

Scale transition

2D and 3D in situ tests + multiscale full field measurement

Microstructures

Physical micromechanisms

Interactions

Heterogeneous field

$$E(t)$$

$$\varepsilon(x)$$
In situ tests in microCT

(cy E. Maire)
Laboratory microCT setup at Navier

2 imagers
2 sources
Air bearings axes
100kg rotation stage
7 in situ testing devices

Manufacturer: RX Solutions, 2010-2012
Example: hyromechanical couplings in granular materials

PhD J.F. Bruchon
(with M. Vandamme, J.M. Pereira P. Delage)

Fontainebleau Hostun

Sand

Glass beads

Load 1kPa

\( \text{Œdometric cell} \)

8cm diam.

Injection of water 1ml/min

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Preliminary test:

Radiographs movie + 2D-DIC

Cross-sections through 3D volumes before/after
Standard volumetric-DIC:
preliminary oedometric test on dry sand

Cross-sections through 3D strain field (von Mises)
**Discrete volumetric-DIC:** ongoing…

Older test

Hall et al, *Géotechnique*, 2010

~65000 grains
Overall displacement field

Step 1 to 7

Standard DIC OK, with regular subsets and (locally) continuous shapes functions

0 (voxels)  230
Grain scale displacement field

Displacement fluctuations at grains scale can be strongly discontinuous

Standard DIC fails…

(separation)  (gliding)  (rolling)  or….
Discrete DIC, example of results: rotation angles

Incremental rotation angle
Questions:

Accuracy of these fields?

Accuracy dependences?

Control of image acquisition and processing procedures to improve accuracy?

...some indications on these complex questions
General DIC framework

$$\Phi_D \approx \text{Argmin } C(D, \Phi_0, f, g)$$

- Estimated local transformation (output)
- Set of possible local transformation (shape function)
- Correlation window
- Grey levels of reference and current images (input)
- Correlation coefficient (measure of similarity)

+ Repeat over all D’s…

Standard DIC:
- D regularly shaped and spaced
- V = \((0, 1, 2\ldots)^{th}\) order polynomial

Discrete DIC:
- D = grains
- V = rigid body motion

Fundamental assumption: convection of grey levels
Classification of DIC errors

(an attempt)

Images → DIC → Displacements

1. Noise

2a. Shape functions

2b. Interpolations

3. Geometric errors
   (3D real space → 2 or 3D image space)

4. Other errors: e.g. bad convection of grey levels...

Here: focus on 1 and 2b
3
Theoretical modelling and experimental validation of angular error in discrete DIC

…related to image noise

(Bornert et al. ICEM14, Poitiers, 2010)
Theoretical analysis:
Perturbation of DIC minimum due to image noise?

Correlation coefficient (SSD):
\[ C(T, R) = \int_D \left[ f(x) - g(X_i + T + R \cdot (x - X_i)) \right]^2 dx \]

Optimality condition:
\[ dC = 0 = \int_D \left[ f(x) - g(\phi(x)) \right] \nabla g(\phi(x)) \left[ dT + dR \cdot (x - X_i) \right] dx \quad \forall dT, \forall dR \]

Perturbation of optimum due to noise: (assuming \( f(\overline{x}) \approx g(\phi(\overline{x})) \))
(assuming \( f(\overline{x}) \approx g(\phi(\overline{x})) \))

\[ \int_D \left[ \nabla f(x) - \nabla g(\phi(x)) \right] \nabla g(\phi(x)) \left[ dT + dR \cdot (x - X_i) \right] dx \quad \forall dT, \forall dR \]

\[ = \int_D \nabla g(\phi(x)) \left[ dT + dR \cdot (x - X_i) \right] \nabla g(\phi(x)) \left[ dT + dR \cdot (x - X_i) \right] dx \]

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\[
\begin{bmatrix}
R^T \cdot dT \\
R^T \cdot dR
\end{bmatrix}
\begin{bmatrix}
\int_D \left[ \frac{\partial f}{\partial x} - \frac{\partial g}{\partial \phi(x)} \right] \nabla f(x) dx \\
\int_D \left[ \frac{\partial f}{\partial x} - \frac{\partial g}{\partial \phi(x)} \right] \nabla f(x) \otimes (x - X_i) dx
\end{bmatrix} = \forall dT, \forall dR
\]

\[
\begin{bmatrix}
R^T \cdot dT \\
R^T \cdot dR
\end{bmatrix}
\begin{bmatrix}
\int_D \nabla f(x) \otimes \nabla f(x) dx \\
\int_D \nabla f(x) \otimes (x - X_i) \otimes \nabla f(x) dx
\end{bmatrix}
\begin{bmatrix}
\int_D \nabla f(x) \otimes (x - X_i) \otimes (x - X_i) dx \\
\int_D \nabla f(x) \otimes (x - X_i) \otimes (x - X_i) dx
\end{bmatrix}
\begin{bmatrix}
R^T \cdot dT \\
R^T \cdot dR
\end{bmatrix}
\]

\[
\begin{bmatrix}
R^T \cdot dR
\end{bmatrix}
\[
\text{is a skew-symmetric tensor such that } (R^T \cdot dR) X = dw \wedge X
\]

\[
\frac{d}{dw} = \text{infinitesimal rotation vector (in reference configuration)}
\]

\[
\begin{bmatrix}
\int_D \left[ \frac{\partial f}{\partial x} - \frac{\partial g}{\partial \phi(x)} \right] \nabla f(x) dx \\
\int_D \left[ \frac{\partial f}{\partial x} - \frac{\partial g}{\partial \phi(x)} \right] \nabla f(x) \wedge (x - X_i) dx
\end{bmatrix} = \begin{bmatrix}
\int_D \nabla f(x) \otimes \nabla f(x) dx \\
\int_D \nabla f(x) \wedge (x - X_i) \otimes \nabla f(x) dx
\end{bmatrix}
\begin{bmatrix}
\int_D \nabla f(x) \wedge (x - X_i) \otimes \nabla f(x) \wedge (x - X_i) dx \\
\int_D \nabla f(x) \wedge (x - X_i) \otimes \nabla f(x) \wedge (x - X_i) dx
\end{bmatrix}
\begin{bmatrix}
\frac{\partial t}{\partial w}
\end{bmatrix}
\]

\[
6x6 \text{ matrix } \frac{\partial t}{\partial w} = R^T \cdot dT
\]

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Computation of covariance tensor of errors
(assuming white noise on pixels)

\[
\left\langle \left[ \frac{\partial t}{\partial w} \right] \otimes \left[ \frac{\partial t}{\partial w} \right] \right\rangle \cdot 2 \cdot \left[ M \otimes M \right] = 2 p^3 \sigma_f^2 M
\]

Correlation length of noise (∼voxel size)  
Standard deviation of image noise

General procedure: diagonalize \( M \) …

If \( M \) diagonal:

\[
\left\langle \left[ \frac{\partial t}{\partial w} \right] \otimes \left[ \frac{\partial t}{\partial w} \right] \right\rangle = 2 p^3 \sigma_f^2 \text{Diag}\left( \frac{1}{\mu_1}, \cdots, \frac{1}{\mu_6} \right)
\]

(See Bornert et al. ICEM14, Poitiers, 2010)
Example 1: spherical grain

\[ M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \]

\[ \mu' = \frac{4\pi a^2 (\Delta f)^2}{3b} \]

\[ \sigma_t = p \cdot \frac{\sqrt{6pb}}{2\sqrt{\pi a}} \cdot \frac{\sigma_f}{\Delta f} \]

\[ a < b \]

Linear variation of grey levels

Example 2: cubic grain

\[ M = \begin{pmatrix} \mu' & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu' & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu' & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu^w & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu^w & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu^w \end{pmatrix} \]

\[ \mu' = \frac{8a^2 (\Delta f)^2}{b} \]

\[ \mu' = \frac{(2a)^4 (\Delta f)^2}{3b} \]

\[ \sigma_t = p \cdot \frac{\sqrt{pb}}{2a} \cdot \frac{\sigma_f}{\Delta f} \]

\[ \sigma_w = \frac{p}{a} \frac{\sqrt{6pb}}{a} \cdot \frac{\sigma_f}{\Delta f} \]

(linear variation of grey levels)

(NB: isotropic sensitivity to rotation)

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Experimental validation:

“Zero deformation” experiment

Sample of Hostun sand with large grains
(D50 = 900µm)

Reference
Translation
Translation + Rotation

Laboratory CT scanner at L3SR

1150x1150 x351 voxels
1 voxel = 15µm

RX-Solutions

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Profile in 16bis CT section

Spheres: $\sigma_t \approx 0.003$ vox  
Cubes: $\sigma_t \approx 0.002$ vox

Theoretical error:

$\sigma_f \approx 300$

$\Delta f \approx 7000$

$b \approx 6$

$a \approx 30$

$\sigma_w \approx +\infty$

$\sigma_w \approx 1/3000$ rad

$\approx 0.02$ degrees
Macroscopic DIC:

\[ \alpha \approx 2.53(\pm 0.01) \, \text{deg} \]

\[ n \approx 0.726 \varepsilon_x + 0.687 \varepsilon_y - 0.02288 \varepsilon_z \]

\[ \Rightarrow \alpha_x = 1.840, \alpha_y = 1.740, \alpha_z = -0.058 \, \text{(deg)} \]

Individual discrete-DIC grain analysis (on ~700 grains, >95% success)

<table>
<thead>
<tr>
<th></th>
<th>( t_x ) (voxels)</th>
<th>( t_y ) (voxels)</th>
<th>( t_z ) (voxels)</th>
<th>( \alpha_x ) (degrees)</th>
<th>( \alpha_y ) (degrees)</th>
<th>( \alpha_z ) (degrees)</th>
<th>( \alpha ) (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Translation</strong></td>
<td>Av. - - -</td>
<td>0.023 -0.023 -0.104 0.109</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \sigma ) 0.148 0.177 0.111</td>
<td>0.133 0.118 0.138 0.126</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Rotation</strong></td>
<td>Av. - - -</td>
<td>1.855 1.759 -0.044 2.565</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \sigma ) 0.094 0.129 0.0651</td>
<td>0.148 0.111 0.151 0.124</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Statistical distribution functions

Motion = Rotation + translation

Motion = Translation

Errors on rotations

Errors on translation
Comments

Consistency exp/theory on $\sigma_w$ $\sigma_w^{th} \in [0.02 ; +\infty]$ (deg)

While apparently $\sigma_i^{exp} \gg \sigma_i^{th}$

But: $X_i$ is not the exact center of grains

$$\sigma_i(X_i) \approx \sqrt{\sigma_t(X_{center})^2 + \sigma_w \left\| X_i - X_{center} \right\|^2}$$

Application to other images (D50 = 280μm)

$\sigma_f \approx 22$ $\sigma_f \approx \frac{1}{4}!$

$\Delta f \approx 90$ $\Delta f \approx \frac{1}{4}!$

$a \approx 10$ $b \approx 1$

$\sigma_w^{th} \in [0.35 ; +\infty]$ (deg)

$\sigma_w^{exp} \approx 1 \text{ deg}$
4) Quantification of full systematic error curve with just two images

Images are discrete data

\[ \int_{u \in D} \alpha(u) \, du \Rightarrow \sum_{(ij) \in D} \alpha_{ij} \]

Interpolations

Fractional part of displacement (pixels)

Displacement

<table>
<thead>
<tr>
<th>n+1</th>
<th>Under-estimated ( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>Evaluated</td>
</tr>
<tr>
<td>n-1</td>
<td>Over-estimated ( \epsilon )</td>
</tr>
</tbody>
</table>

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Lenoir et al, Strain 2007
Experimental evaluation of S-shaped systematic error curve

- Standard procedure:
  Prescribe several real subpixel translations of sample and compare with DIC measurements

- More efficient procedure:
  Prescribe motions to sample or imaging system that generate locally in image an apparent translation with known characteristics

  Rigid rotation or magnification variation
  \[ u = \frac{n}{L} (x - x_0) \]

  If \( \frac{n}{L} \lesssim \frac{0.2}{D} \) displacement is sufficiently uniform in correlation window

  \( \frac{n}{L} \) and \( x_0 \) evaluated (accurately) from overall (apparent) strain

  Typically: \( 1 < n < 6 \)

  Yang et al, 2010, ICEM14
Virtual homogeneous isotropic straining of cylindrical halite sample with Cu markers

GE X-ray 160kV nanofocus tube
   @ 67kV / 100 µA / 6.5 W (mode 1)
Flat Panel Varian 2520,
   @1920x1536, 1s/image, average 30
1440 projections (13h scan)
Images 1840x1840x992 voxels

With M. Bourcier, A. Dimanov, LMS ANR Project « MicroNaSel »

Sample: 10mm Diameter x 20mm Height
   (imaged zone 6.5mm in height)
**Virtual straining:**

- Virtually deformed image:
  - Same sample, same conditions
  - with imager shifted by 0.9mm
  - + sample shift $\Delta Y = 100\mu m$

  MORE PRECISELY (according to geometry of system):
  - apparent dilatation $= 1.0031962 = \frac{\text{ratio of voxelsizes}}{6.50022 / 6.47951}$

  *This corresponds to ~5 voxels increase in sample diameter*

**Vol-DIC analysis:**

- Grid:
  - 20 voxels steps,
  - 80x80x47 points
  - = 300800 points, 232683 in sample

- Trilinear g.l. interpolation
- Rigid transformation
- In-house code (CMV3D)

- Various window sizes
  - From $20^3$ to $50^3$
  - Fixed or adjustable

Initial voxel size = 6.5 $\mu m$

New voxel size = 6.48$\mu m$
A) Direct processing of original images

Average deformation gradient:  
(example of result)  
0.003196

(very close to prescribed magnification variation)  

<table>
<thead>
<tr>
<th></th>
<th>0.003198</th>
<th>0.000441</th>
<th>-0.000067</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.000087</td>
<td>0.003191</td>
<td>-0.000009</td>
</tr>
<tr>
<td></td>
<td>-0.000118</td>
<td>0.000076</td>
<td>0.003268</td>
</tr>
</tbody>
</table>

(Accuracy better than 0.0001)

Statistical analysis of local evaluations of displacement

Compare DIC measurements with theoretical displacement

1) Global analysis

Standard deviation on 3 displacement components

2) Local analysis as a function of fractional part of theoretical displacement

Standard deviation + bias on 3 displacement components
1) Global analysis

<table>
<thead>
<tr>
<th>Window Size</th>
<th>Std. Dev. X</th>
<th>Std. Dev. Y</th>
<th>Std. Dev. Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>20³ constant</td>
<td>0.158734</td>
<td>0.128389</td>
<td>0.254016</td>
</tr>
<tr>
<td>20³ variable</td>
<td>0.146954</td>
<td>0.121243</td>
<td>0.195838</td>
</tr>
<tr>
<td>30³ variable</td>
<td>0.133726</td>
<td>0.106016</td>
<td>0.185546</td>
</tr>
<tr>
<td>40³ constant</td>
<td>0.128136</td>
<td>0.100171</td>
<td>0.182437</td>
</tr>
<tr>
<td>50³ constant</td>
<td>0.126381</td>
<td>0.098852</td>
<td>0.181389</td>
</tr>
</tbody>
</table>

No significant change

Improvement

~ <
2) Local analysis

Full S-shaped curve obtained with two images

Similar X and Y behaviour (as expected), consistent with 2D observations

Behaviour along Z is again quantitatively different
Window = 20\(^3\)

Window = 30\(^3\)

Window = 40\(^3\)

Von Mises Strain
Window = 40\(^3\)
(Fe derivatives)

Residual shape function mismatch error
(can be shown to be \(\sim 0.05\) voxels)
A) Processing of binned images

- Original (6.5 µm)
- Binning 2x2x2
- Binning 3x3x3
- Binning 4x4x4 (voxel 26 µm)

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Bining 2x2x2
Window = $10^3$ (=20$^3$)

Bining 2x2x2
Window = $15^3$ (=30$^3$)
Von Mises Strain
Bining 2x2x2
Window = $10^3 (=20^3)$

(Computation time = minutes)
( Acquisition time divided by 2)

Von Mises Strain
Window = $40^3$

(Computation time = hours)
Bining 2x2x2
Window = 15^3 (=30^3)

Bining 3x3x3
Window = 10^3 (=30^3)
Reversed curvature of S curve
Increasing amplitude

OPTIMAL COMBINATION OF
(SAMPLE + IMAGING + DIC)
TO REDUCE SYSTEMATIC ERRORS

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Simular results in 2D-DIC (with contrast controlled by lens aperture)

Lens F4.5/90mm, G=1, 1pixel = 7.4μm

Argilite

Aluminium

Aperture-1

Aperture-3

Aperture-5

(Yang et al, ICEM14, Poitiers, 2010)

Airy \approx \frac{0.6}{\alpha}

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Concluding remarks

MicroCT in situ test combined with (Discrete-)DIC provide highly valuable insights for the micromechanics of (geo)materials

Several DIC error sources

We need to understand them, to model them and to quantify them for real experimental conditions

Some simple and accurate procedures are proposed

But still a lot to do….