Thermomechanical analysis of material behaviour - strong coupling effects to quantify stresses

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Summary

- Background to IRT/TSA
- Application to actual structure
- Damage studies
- Field work – strain based NDE
Imaging techniques

• Infra-red thermography (pulse phase, TSA)
• White light -Digital image correlation (DIC), Microscopy
• Coherent light -Electronic speckle pattern shearing interferometry (ESPSI)
• X-ray computed tomography
Thermography overview

- Infrared Thermography (IRT)
- Pulsed and Pulsed Phase Thermography (PPT)
- Thermoelastic Stress Analysis (TSA)

\[ \Delta T = -\frac{T\alpha}{\rho C_p} (\Delta \sigma_1 + \Delta \sigma_2) \]
Thermomechanical coupling

- William Thomson – Lord Kelvin 26 June 1824 – 17 December 1907
- Mathematician, Physicist and Engineer
- Thermodynamics – ca 1850
Thermoelastic effect

\[ \dot{T} = \frac{T_0}{\rho C_\varepsilon} \frac{\partial \sigma_{ij}}{\partial T} \dot{\varepsilon}_{ij} - \frac{\dot{Q}}{\rho C_\varepsilon} \]

for \( i, j = 1, 2, 3 \)

where:

- \( T \) is the temperature
- \( T_0 \) is the absolute (reference) temperature
- \( C_\varepsilon \) is the specific heat at constant strain
- \( \dot{Q} \) is the rate of heat production per unit volume
- \( \rho \) is the mass density
- \( \sigma_{ij} \) is the stress tensor
- \( \dot{\varepsilon}_{ij} \) is the rate of change of the strain tensor

The symbols used in the equation:

- \( T \): Temperature
- \( T_0 \): Absolute (reference) temperature
- \( C_\varepsilon \): Specific heat at constant strain
- \( \dot{Q} \): Rate of heat production per unit volume
- \( \rho \): Mass density
- \( \sigma_{ij} \): Stress tensor
- \( \dot{\varepsilon}_{ij} \): Rate of change of the strain tensor
Thermoelastic stress analysis

\[
\Delta T = -\frac{\alpha T}{\rho C_p} \left( \Delta \sigma_1 + \Delta \sigma_2 \right)
\]

\[
\Delta T = -\frac{T}{\rho C_p} \left( \alpha_1 \Delta \sigma_1 + \alpha_2 \Delta \sigma_2 \right)
\]
Stress-strain temperature equations

\[
\sigma_{ij} = 2\mu\varepsilon_{ij} + (\lambda\varepsilon_{kk} - \beta\delta T)\delta_{ij} \quad \delta_{ij} = \begin{cases} 
1 & \text{for } i = j \\
0 & \text{for } i \neq j
\end{cases}
\]

\[
\beta = (3\lambda + 2\mu)\alpha \\
\mu = \frac{E}{2(1+\nu)} \\
\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}
\]

\[
\frac{\partial \sigma_{ij}}{\partial T} = 2 \frac{\partial \mu}{\partial T} \varepsilon_{ij} + \left( \frac{\partial \lambda}{\partial T} \varepsilon_{kk} - \frac{\partial \beta}{\partial T} \delta T - \beta \right) \delta_{ij}
\]

Neglecting temperature derivatives of the material elastic properties

\[
\frac{\partial \sigma_{ij}}{\partial T} = -\beta \delta_{ij}
\]
Thermoelastic equation

\[ \dot{T} = -\frac{T_0 \beta}{\rho C_\varepsilon} \dot{\varepsilon}_{kk} \quad \Rightarrow \quad \dot{T} = -\alpha \left[ \frac{T_0}{\rho C_\varepsilon} + \frac{1-2\nu}{3\alpha^2 E} \right] \dot{\sigma}_{kk} \]

\( \dot{\sigma}_{kk} \) is the rate of change of the sum of the first stress invariant \( \sigma_{11} + \sigma_{22} + \sigma_{33} \)

\[ C_\varepsilon = C_p - \frac{3E\alpha^2 T_0}{\rho(1-2\nu)} \quad \Rightarrow \quad \dot{T} = -\frac{\alpha T_0}{\rho C_p} \dot{\sigma}_{kk} \]

\[ \Delta T = -\frac{\alpha T_0}{\rho C_p} \Delta(\sigma_1 + \sigma_2) \]

\[ K = \frac{\alpha}{\rho C_p} \]
Detector specification

- **Model:**
  - Cedip Silver 480 M
    (now the FLIR SC5000)

- **Array:**
  - 320 x 256 elements
  - 30 μm pitch
  - ½” chip (12.3 mm diagonal)
  - Indium Antimonide (InSb)
  - 3-5 μm wavelength band

- **Standard operational range:**
  - 278 to 583 K
  - maximum frame rate: 383 Hz (at full frame)
  - sensitivity 4.12 mK / DL (at 298 to 299 K)
    (electronic noise ~17 mK)
Lock-in processing

- Example of a typical noisy measurement signal in TSA.
  - Noise and signal are of similar amplitude.

- A reference signal is used that contains only the frequency of interest.
  - The reference signal is split into a sine and cosine part.
Radiometric calibration

![Radiometric calibration graph](image-url)
Derivation of the thermoelastic constant

- Calibration against known stress

\[ \Delta T = KT \Delta (\sigma_1 + \sigma_2) \]
Why TSA?

- Heterogeneous materials
- Monitor damage evolution
- Stress metric
- Small parasitic effects - not kinematic
- High resolution
- High speed
Stresses in secondary aircraft structure

• Increased use of composite materials in aircraft structure
  – weight saving
  – improved life time
• Development of new manufacturing techniques and new materials

Generic panel
Face sheet material characterisation

UD prepreg/autoclaved

NCF RFI Oven cure
### Thermoelastic Constants

<table>
<thead>
<tr>
<th>Thermoelastic constant</th>
<th>M1, $K_L$</th>
<th>M1, $K_T$</th>
<th>M2, $K_L$</th>
<th>M2, $K_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPa$^{-1}$ (x 10$^{-6}$)</td>
<td>1.592 ± 0.83</td>
<td>3.112 ± 1.19</td>
<td>1.58 ± 0.81</td>
<td>2.837 ± 1.55</td>
</tr>
</tbody>
</table>
Full-scale tests

- Panels cyclically loaded at 1 Hz on custom design rig to allow a pressure load – imparting 10 ± 5 kPa

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Results

- Standard autoclaved prepreg offered a panel with a maximum deflection of 6.3 mm whilst RFI, NCF panel deformed by 4.6 mm.

- The measured stress response indicated a reduction in stress peak when using RFI and NCF.

Woven material

- Damage in textile composites can occur at very low stress levels, < 20 % of $\sigma_f$.
- Damage can be identified using TSA despite the heterogeneous thermoelastic material response.
- Phase data provides a means for damage identification without \textit{a priori} knowledge of the thermoelastic field.

Is a test machine required?

Advanced MUlti-scale Strain-basED non-destructive evaluation (AMUSED)
Full-scale inspection-Demonstrator

- TSA measurement of full-scale secondary aircraft panel with incremental damage.
Full-scale inspection

Forced loading data

Natural frequency loading data
GFRP Damage Propagation

- Damage created at three points across specimen width using a punch and hammer
- Specimens coated in matt black paint, followed by a light coat of white speckles to enable DIC processing
- Cyclically loaded at 10 Hz, 4.5 kN ± 3 kN
- $\sigma_{\text{mean}} = 141$ MPa
- $\Delta\sigma = 188$ MPa
- $R$ ratio = 0.2

<table>
<thead>
<tr>
<th>FRP material</th>
<th>Stacking Sequence</th>
<th>Quasi-static properties</th>
<th>Young’s Modulus</th>
<th>UTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass–Epoxy</td>
<td>$[90,0,90,0]_s$</td>
<td></td>
<td>21.22 ± 0.72 GPa</td>
<td>513 ± 40 MPa</td>
</tr>
<tr>
<td>ACG MTM 28-1</td>
<td></td>
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</tbody>
</table>
Specimen found to fail at around 4400 cycles

Damage propagation can be clearly seen in the normalised TSA data

Initial regions of damage coalesce forming a central region which shows little thermoelastic output. Central region therefore has very little load bearing properties

\[ \frac{\Delta T}{T_0} = K_1 \Delta \sigma_1 + K_2 \Delta \sigma_2 \]

\[ K_1 = \frac{-\alpha_1}{\rho C_p}, \quad K_2 = \frac{-\alpha_2}{\rho C_p} \]
GFRP Damage Propagation

**Longitudinal strain** $\varepsilon_{yy}$

Image correlations performed between an unloaded reference image and images captured at the point of maximum load.

Cell size = 31 x 31 pixels, stepsize of 15
0.32 mm/strain data point
GFRP Damage Propagation

Central area experiences high strain but a low stress sum
Methodology development: Combining TSA and DIC

Challenges:
1. Triggering the saving of image sets automatically at various points in fatigue cycle without interrupting cyclic load.
2. High resolution white light cameras preferred for enhanced DIC accuracy. Accurate camera triggering and intense light therefore required.

Calibrated 3D DIC setup to be used to account for out of plane movement and camera perspective

IR images taken

White light images taken

IR Camera (Cedip Silver 480M)

2 x white light cameras (Manta G504-B/C, 5Mpixel)
Lock-in Application to DIC

1. Acquire images & reference signal
2. Process reference signal to assess frequency and truncate video accordingly
   - Process images using DIC to obtain strain field
   - Process reference signal to have an amplitude of 1 and a mean of 0.
     - Generate quadrature (i.e. cosine) of reference signal
3. Apply lock-in, point by point to Calculate amplitude and phase
Under sampling?

- Example:
  - Signal frequency: 7.1 Hz
  - Recording frequency: 2.0 Hz
  - Reconstructed signal: 0.9 Hz
Brazilian disc

- Visual comparison between static and dynamic results.

Lock-in DIC and TSA

**TSA**
Cedip 480M infrared photon detector
20 mK threshold, reduced to ~4 mK using lock in
Altair and Altair LI software

**DIC**
3D DIC
2 x LaVision E-lite 5 MP
NILA LED lighting
DaVis software

**Loading**
LDS V201 permanent magnet shaker from Brüel & Kjær
Rigid stinger used to impart load attached via beeswax.
Aluminium plate – mode 2

\( \Delta T/T \) vs. Pixel

Phase

TSA

DIC

z-displacement
Conclusions

• Introduced TSA and described the basic physics

• Shown how full-field experimental mechanics techniques can be used to validate FEA

• Presented convincing case studies that demonstrate the applicability and ease of using TSA in damage studies

• Shown potential as a strain based NDE technique

• Shown how TSA can be combined with DIC during the same test
Learn more about thermography/TSA

• BSSM Experimental Mechanics Workshop, University of Southampton, 23rd -27th 2015.

• See flyer in pack

• Covers also DIC, GM, ESPI, high speed imaging.

• Hands on

• Data processing