Thermomechanical analysis of material behaviour

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Part 2: Dissipation to characterise irreversible deformation mechanisms

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Goals

Observing kinematic and energetic effects accompanying the material transformations

Constructing energy balances – consistency of behavioral models

Quantitative imaging techniques – full-field measurements


**DIC - Mechanics**
- displacement fields
- strain and strain rate
- stress
- deformation energy rate

**IRT - Thermodynamics**
- absolute temperature
- heat source (via the heat equation)
- intrinsic dissipation
- coupling sources
Outline

**Theoretical background**
- dissipation, energy storage
- thermomechanical (thm) coupling effects

**Experimental tools**
- temperature fields
- displacement fields

**Focusing on cyclic loading**

**Case study #1 : HCF of steel**
- material vs. structure effects
- properties of intrinsic dissipation fields

**Case study #2 : LCF of rubber**
- intrinsic dissipation vs. strong coupling + thermal dissipation

**Concluding comments**
Theoretical background

**Thm constitutive equations**

Generalised standard material formalism

[Halphen & Nguyen, 75]

<table>
<thead>
<tr>
<th>state variables</th>
<th>( {T, \epsilon, \alpha} )</th>
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</thead>
<tbody>
<tr>
<td>internal/free energy potential state equations</td>
<td>( e(s, \epsilon, \alpha) ) ( \psi(T, \epsilon, \alpha) )</td>
</tr>
<tr>
<td>(-s = \psi, T)</td>
<td>(\sigma^f = \rho \psi, \epsilon)</td>
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<tr>
<td>(A_\alpha = \rho \psi, \alpha)</td>
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<tr>
<td>dissipation potential evolution equations</td>
<td>( \varphi(q, \dot{\epsilon}, \dot{\alpha}; T,...) )</td>
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<tr>
<td>(- \frac{\nabla T}{T} = \varphi, q)</td>
<td>(\sigma^{ir} = \varphi, \dot{\epsilon})</td>
</tr>
<tr>
<td>(X_{\dot{\alpha}} = \varphi, \dot{\alpha})</td>
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**Irreversibility**

Material degradation

\[ d_1 = \sigma^{ir} : \dot{\epsilon} + X_{\alpha} \cdot \dot{\alpha} \]

Heat diffusion

\[ d_2 = \varphi, q \cdot q = - \frac{\nabla T}{T} \cdot q \]
Energy balance (I)

\[ w_{\text{def}} = \sigma : \dot{\varepsilon} = \sigma^r : \dot{\varepsilon} + \sigma^i : \dot{\varepsilon} \]
\[ = \sigma^r : \dot{\varepsilon} + A_\alpha \dot{\alpha} + d_1 \]
\[ = \dot{W}_e + \dot{W}_s \]

\( W_e \): rate of elastic energy

\( W_s \): rate of stored energy

\( d_1 \): intrinsic dissipation

... incomplete balance !!
Energy balance (II)

- rate of internal energy

\[ \rho \dot{e} = \rho C \dot{T} + (\sigma^r : \dot{\varepsilon} + A.\dot{\alpha}) - (T \sigma^r_{,T} : \dot{\varepsilon} + TA_{,T}.\dot{\alpha}) \]

\[ = \rho C \dot{T} + w_e^* + w_s^* - w_{\text{thc}}^* \]

« thc » = thermomechanical couplings

- heat equation

\[ \rho C \dot{T} + \text{div} q = \sigma^r_{,r} : \dot{\varepsilon} - A.\dot{\alpha} + T \sigma^r_{,T} : \dot{\varepsilon} + TA_{,T}.\dot{\alpha} + r_e \]

- comments

C.1: C specific heat  C.2: \( q = -k \cdot \text{grad}T \)  C.3: \( \dot{T} = \frac{\partial T}{\partial t} + \nabla \cdot \dot{T} \)
Experimental tools

Quantitative imaging – full field measurement system

- **CCD camera**
  - max frame rate: 20 Hz
  - 1280×1024 pixels
  - 13×13 μm²
  - 14 bits
  - δx ≈ 0.1 mm
    (min 15 μm)

- **IRFPA camera**
  - [3,5] mM
  - max frame rate: 250 Hz
  - 640×320 pixels
  - 18×23 μm²
  - 14 bits
  - δx ≈ 0.1 mm
    (min 25 μm)
  - δT ≈ 0.02 °C

- **Hydraulic testing machine**
  - load cell: ± 25 kN
  - frame: ± 100 kN
  - max(f_L) = 50 Hz en R_s = -1

Refs

[Wattrisse et al., Exp. Mech, 2001]
[Chryso et al., IJES, 2000]
Combining DIC & IRT

Speckle image

Grey scale

$X_{DIC}$ (pixel)

$Y_{DIC}$ (pixel)

IR image

$T (°C)$

$X_{IRT}$ (pixel)

$Y_{IRT}$ (pixel)

Map-to-map correspondence

$U(t_{DIC}, x_{DIC}, y_{DIC})$

$T(t_{IRT}, x_{IRT}, y_{IRT})$

[Chryso et al., JoMMS, 2010]

$t_{DIC}$

synchrocam target

$(x_{DIC}, y_{DIC})$

$(x_{IRT}, y_{IRT})$

$t_{IRT}$

(50 µs)

$(\approx 0.5$ pixel)
Stress derivation

Thin, flat sample

\[ \sigma_{xx}(x,t) = \frac{F(t)}{S_0} \exp(\varepsilon_{xx}(x,t)) \]

\[ \sigma_{xy}(x,y,t) = -\sigma_{xx}(x,t) \frac{\partial \varepsilon_{xx}(x,t)}{\partial x} y \]

\[ \sigma_{yy}(x,y,t) = \frac{\partial}{\partial x} \left( \frac{\sigma_{xx}(x,t) \partial \varepsilon_{xx}(x,t)}{2} \right) \left( \frac{w(x,t)^2}{4} - y^2 \right) \]

1. Stress triaxiality neglected
2. No volume variation
3. Uniform distribution of tensile stress over a cross-section
4. No overall shear loading
5. No lateral stress
Heat rate

Heat equation averaged over the sample thickness

\[
\rho C \left( \frac{\partial \bar{\theta}}{\partial t} + v_x \frac{\partial \bar{\theta}}{\partial x} + v_y \frac{\partial \bar{\theta}}{\partial y} + \frac{\bar{\theta}}{\tau_{th}} \right) - k \left( \frac{\partial^2 \bar{\theta}}{\partial x^2} + \frac{\partial^2 \bar{\theta}}{\partial y^2} \right) = \bar{W}_h
\]

Direct estimate of heat sources using noisy and discrete thermal data

Thermal noise

• uniform power spectrum
• Gaussian probability distribution

Estimate of the partial derivative operators

• thermal data projection onto spectral solutions (1995)
• periodic expansion and convolutive filtering by DFT (2000)
• local approximation of \( \theta \) using l.sq. fitting (2004)
• POD: pre-filtering of thermal fields (2013)

Refs:
IJES : 2000
EXP-MECH : 2007
JoMMS : 2010
EXP-MECH : 2014
Focusing on cyclic tests

Monochromatic uniaxial loading
Focus on a load-unload cycle

(i) $A \neq B$

(ii) $\varepsilon_A = \varepsilon_B$

(iii) $A = B$

(i) $w_{\text{def}} = \int_{t_A}^{t_B} \sigma : \dot{\varepsilon} \, dt = \int_{t_A}^{t_B} d_1 \, dt + \int_{t_A}^{t_B} (\rho \dot{e} - \rho C \dot{T} + w_{\text{thc}}) \, dt$

(ii) Hysteresis loop: $w_{\text{def}} = A_h$ (for uniaxial loading)

(iii) Load-unload cycle = thermodynamic cycle $w_{\text{def}} = \int_{t_A}^{t_B} d_1 \, dt + \int_{t_A}^{t_B} w_{\text{thc}} \, dt$
IR image processing

\[ \rho C \left( \frac{\partial \overline{\theta}}{\partial t} + \frac{\overline{\theta}}{\tau_{th}} - \frac{k}{\rho C} \left( \frac{\partial^2 \overline{\theta}}{\partial x^2} + \frac{\partial^2 \overline{\theta}}{\partial y^2} \right) \right) = \overline{d}_1 + \overline{s}_{\text{the}} \]

Slow evolution of mean dissipation

\[ \overline{d}_1 = \int_{\text{cycle}} f_L \overline{d}_1 \, d\tau \]

In phase with loading

\[ \tilde{w}_{\text{the}} = \int_{\text{cycle}} f_L \, \overline{s}_{\text{the}} \, d\tau \]

Linear PDE + Linear BC

\[ \overline{\theta} = \overline{\theta}_d + \overline{\theta}_{\text{the}} \]

Local approximation function

\[ \theta_{\text{app}}(x,y,t) = p_1(x,y) \cos(2\pi f_L t) + p_2(x,y) \sin(2\pi f_L t) + p_3(x,y) \, t + p_4(x,y) \]

periodic response

\[ p_i(x,y), i=1,\ldots,4, \text{ are 2nd order polynomials of } x \text{ and } y \]
Fatigue loading

$m_i$: series of “mini” cycle blocks (3000 cycles) at different stress ranges: energy balance at “constant fatigue state”

$p_i$: large blocks (100 000 cycles) at constant stress range: energy balance evolution induced by fatigue mechanisms

highest stress range $\approx$ fatigue limit

[Boulanger, PhD 2004]
[Berthel, PhD 2008]
[Blanche, PhD 2012]
Thermal and calorific effects

HCF test on DP 600 steel

\[ \Delta \sigma = 500 \text{ MPa}, \ R_\sigma = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} = -1 \ 	ext{et} \ f_L = 30 \ \text{Hz} \]

\[ \frac{\Delta s_{\text{the}}}{\rho C} \approx 75 \ ^\circ\text{C}.s^{-1} \]

\[ \frac{d_1}{\rho C} \approx 0.1 ^\circ\text{C}.s^{-1} \]
Dissipation properties (I)

- heterogeneous distribution
- similar distribution, whatever $\Delta \sigma$
- $\tilde{d}_1(\Delta \sigma)$ non linear, $\tilde{d}_1 \propto f_L$
- slow time evolution of dissipation
- plastic shakedown? ($TQ$ ratio $\approx 50 \%$!)

$f_L = 30\text{Hz}$ and $R_\sigma = -1$

$\Delta \sigma = 260\text{MPa}$

$\Delta \sigma = 340\text{MPa}$

$\Delta \sigma = 512\text{MPa}$

$\tilde{d}_1 / \rho C$
Dissipation properties (II)

Interpretation of curves

\( m_i \) = dissipation induced by activated “micro-defects” at constant fatigue state for different stress ranges

\( \rho_i \) = dissipation drift at constant stress range, reflecting a slow evolution of the fatigue state

\( \frac{\langle \tilde{d}_1 \rangle}{\rho C} \) (°C.s\(^{-1}\))

375k cycles

260k cycles

145k cycles

30k cycles

energy safeguard: kinetics of fatigue progress
Case study #2 : back to thm couplings

The most simplistic non-adiabatic thermoelastic (the) model…

\[
\begin{align*}
\varepsilon &= \frac{\sigma}{E} + \lambda_{\text{th}}(T - T_0) \\
\dot{T} + \frac{T - T_0}{\tau_{\text{th}}} &= -\frac{E\lambda_{\text{th}} T \dot{\varepsilon}}{\rho C}
\end{align*}
\]

« 0D » approach
linear heat losses

\[
E = 1000 \text{ MPa} \\
\rho = 1000 \text{ kg.m}^{-3} \\
C = 1000 \text{ J.kg}^{-1}.\text{K}^{-1}
\]

\[
\lambda_{\text{th}} = 50 \times 10^{-5} \text{ K}^{-1} \\
\tau_{\text{th}} = 30 \text{ s} \\
T_0 = 294 \text{ K}
\]

William Thomson
Lord Kelvin (1824-1907)

Thm couplings + thermal dissipation

\[
\tilde{W}_{\text{def}} = A_h = \tilde{W}_{\text{the}}
\]
Thm couplings vs. viscosity

Thermoelastic coupling (i.e. $d_1=0$)

\[ \begin{align*}
\varepsilon &= \frac{\sigma}{E} + \lambda_{th} \theta \\
\dot{\theta} + \frac{\theta}{\tau_{th}} &= -\frac{E\lambda_{th}(T_0 + \theta)}{\rho_0 C_0} \dot{\varepsilon} \\
\sigma + \tau_{th} \dot{\sigma} &\approx E\varepsilon + E\tau_{th}(1+\chi)\dot{\varepsilon}
\end{align*} \]

Viscous dissipation

Isothermal framework

\[ \begin{align*}
\sigma &= E(\varepsilon - \varepsilon_v) \\
\sigma &= h\varepsilon_v + \mu \dot{\varepsilon}_v
\end{align*} \]

Rheological equation

Visco-analysis of polymers - Dynamic Mechanical Analysis (DMA)
Entropic elasticity of natural rubber

Gough (1805) – Joule (1857) – Treloar (1960)

\[ e(s, \varepsilon) = e_{nr}(T) \quad \text{perfect gas analogy} \]

\[ \psi_{nr}(T, \varepsilon) = TK_1(\varepsilon) + K_2(T) \]

\[ w_{\text{def}} = w_h \]

[Saurel, PhD 99]
[Honorat, PhD 06]
[Caborgan, PhD 11]
Concluding comments

- **Full-field measurements**
  Material vs. Structure effects

- **Temperature, the 1st state variable** …
  thermal effects vs. calorimetric effects
  not totally intrinsic (heat diffusion)

- **Heat sources of different nature**
  Thm coupling sources: thermo-sensitivity
  dissipation sources: irreversibility of material deformation

- **Energy balance and constitutive equations**
  stored energy / state laws
  dissipated energy / evolution law

- **Rate-dependent behaviour**
  $d_1$ (viscosity) vs. [thm coupling + $d_2$ (heat diffusion)]