

158 Evaluation of lock-in signal data processing procedures for Thermoelastic Stress Analysis

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Abstract. Thermoelastic Stress Analysis generally requires the evaluation of the harmonic content of thermograms acquired over a time window, while the test piece is subject to cyclic loading. Traditionally, such evaluation has been performed by means of commercial lock-in signal processing procedures/devices. These allow limited customisation and flexibility and are usually integrated into a whole generally expensive package, including both the hardware and software. This work reviews three lock-in algorithms suitable for offline evaluation of the harmonic content. The similitudes of the three approaches and the influence of spectral leakage are highlighted by implementing the data processing via simple Matlab[®] scripts, and by testing a Single Edge Notched Tension (SENT) steel sample with a fatigue crack growth.

Introduction

Cyclic loading allows the modulation of some thermo-mechanical coupling responses of the material, such as the thermoelastic effect. If the loading frequency is sufficient to establish an adiabatic behaviour, then the amplitude of the temperature harmonic component at such load frequency is proportional to the first stress invariant, according with the first order Thermoelastic effect law for isotropic media [1]. Thermoelastic Stress Analysis (TSA) then requires the filtering of specific harmonics from a sequence of thermograms sampled over a time window. Three procedures have been mainly implemented, in different ways, to filter out the temperature harmonic content: Digital Cross-Correlation (DCC), Least Square Fitting (LSF), Discrete Fourier Transform (DFT) [2–4]. All such approaches can be applied numerically on sampled datasets. In particular, after the advent of Focal Plane Array sensors, lock-in is performed numerically and off-line, even if buffering the thermal signal with a suitable reference signal might provide a faster, almost in-line, response. In this work, the three numerical procedures are applied via simple Matlab[®] scripts, allowing more flexibility of operation and eliminating the need to use IR Thermographic systems specifically developed for TSA.

Signal Data Processing

Digital Cross-Correlation (DCC) provides the in-phase, X , and in-quadrature, Y , harmonic components with respect to a reference signal built upon the frequency being filtered:

$$X = \frac{2}{N} \cdot \sum_{i=1}^N T_i \cdot \sin\left(\frac{2\pi}{N} k \cdot i + \phi_r\right); \quad Y = \frac{2}{N} \cdot \sum_{i=1}^N T_i \cdot \cos\left(\frac{2\pi}{N} k \cdot i + \phi_r\right) \quad (1)$$

where N is the number of samples, k is the bin number representing the discretised frequency being filtered, and ϕ_r is the phase shift between the reference signal (loading signal in the case of TSA) and the time when sampling is started, which is generally unknown if no synchronisation is applied.

The DFT formula providing the harmonic content can be written as:

$$H_k = \sum_{j=1}^N T_j \cdot \left[\cos\left(\frac{2\pi}{N} k \cdot i\right) - i \cdot \sin\left(\frac{2\pi}{N} k \cdot i\right) \right] \quad (2)$$

where it is noticed that Eq. (2) is equivalent to Eq. (1) with a reference signal synchronised on the cos wave: $\cos(2\pi k i / N)$. In this work, the DFT is implemented via the Matlab[®] built in function *fft* (*fast Fourier Transform*). Finally, the LSF approach is based on the minimisation of the sum of square difference between the sampled temperature and its analytical model representation T_{mod} :

$$T_{mod}(t) = A + B \times t + C_{TE} \sin(\omega \cdot t + \phi_{TE}) + D_{SH} \sin(2\omega \cdot t + \phi_{SH}) + \sum_{k=others} H_k \sin(\omega_k t + \phi_i) \quad (3)$$

$$\Delta = \sum_{i=1}^N (T_{exp}(i) - T_{mod}(i))^2 \rightarrow \frac{\partial \Delta}{\partial x_j} = 0; \quad x_j = A, B, E, \phi_E, D, \phi_D, \dots, H_j, \phi_j \quad (4)$$

where, in the case of TSA, ω is the pulsation at the loading frequency, E and ϕ_E the amplitude and phase of the Thermoelastic Signal, D and ϕ_D the amplitude and phase of the Second Harmonic signal. It is noticed that

eliminating the linear term B from Eq. (3) and taking a number of total terms $k=1:(N-1)/2$ will end up into a number of harmonics equivalent to those of the DFT [3]. All three approaches have then a common spectral analysis physical basis, which should turn out into exhibiting similar performances. In particular, it is expected that errors will be mainly due to spectral leakage, i.e. related to discrete sampling and finite time windowing.

Experimental case study

A SENT specimen made of stainless steel AISI 304l is cyclically loaded with load ratio $R=0.1$. Temperature is measured with a cooled FLIR X6540sc IR camera, with integration time set at $650 \mu\text{s}$ and sampling frequency at 200 Hz. Figure 1 shows an example of power spectrum obtained with the DFT on a point near the crack tip, while Fig. 2 shows the thermoelastic amplitude and phase obtained from the three algorithms.

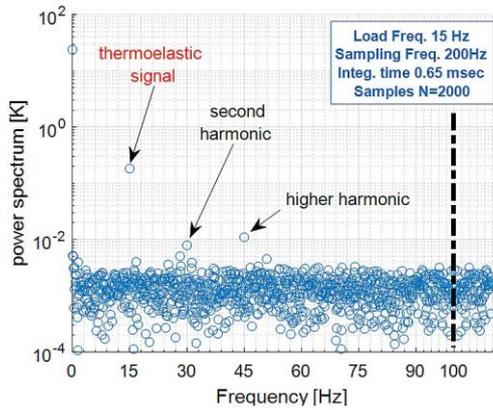


Fig. 1: power spectrum from DFT

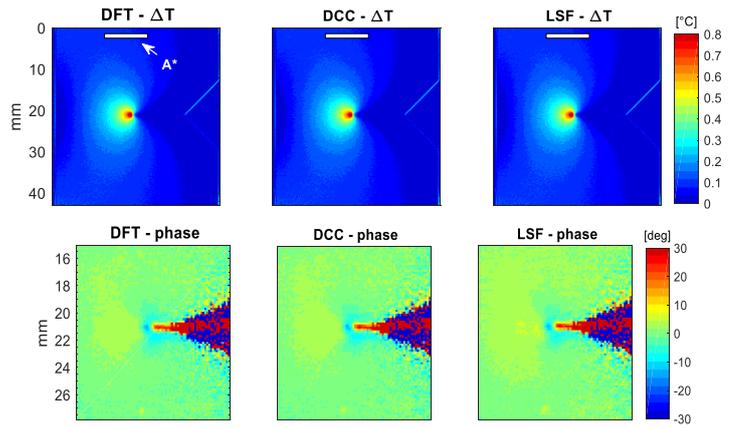


Fig. 2: Thermoelastic and Phase maps

		5 Hz			10 Hz			15 Hz			20 Hz		
		DFT	DCC	LSF									
N=5995 ≈30 sec	ΔT [K]	0.078	0.078	0.078	0.078	0.078	0.078	0.076	0.076	0.076	0.054	0.054	0.054
	St. dev. [K]	0.0097	0.0097	0.0097	0.0097	0.0097	0.0097	0.0092	0.0092	0.0092	0.0068	0.0068	0.0068
	ΔK [MPa×m ^{0.5}]	25.1	25.1	25.1	25.75	25.76	25.76	24.85	24.85	24.84	17.65	17.65	17.65
N=6000 30 sec	ΔT [K]	0.088	0.088	0.088	0.087	0.087	0.087	0.083	0.083	0.083	0.084	0.084	0.084
	St. dev. [K]	0.0109	0.0109	0.0109	0.0107	0.0107	0.0107	0.01	0.01	0.01	0.0102	0.0102	0.0102
	ΔK [MPa×m ^{0.5}]	27.11	27.11	27.1	28.75	28.75	28.75	27.1	27.1	27.1	26.42	26.42	26.43
N=2000 10 sec	ΔT [K]	0.089	0.089	0.089	0.087	0.087	0.087	0.086	0.086	0.085	0.084	0.084	0.084
	St. dev. [K]	0.0117	0.0117	0.0117	0.0112	0.0112	0.0112	0.0109	0.0109	0.0109	0.0104	0.0104	0.0104
	ΔK [MPa×m ^{0.5}]	26.72	26.72	26.73	27.76	27.76	27.75	26.63	26.63	26.62	26.73	26.73	26.74

Table 1: results from different lock-in procedures, at varying load frequency and sampling N.

Table 1 reports values of the Thermoelastic Signal ΔT at various load frequencies, averaged over the white rectangle area A^* shown in Fig. 2 (see DFT- ΔT map). The value of the Stress Intensity Factor range, calculated with the Stanley-Chan method [5], is also reported. It is found that all three lock-in procedures yield pretty similar results under the same operative settings. Data obtained with 6000 and 2000 sampled frames are characterised by having a frequency bin at the same value of the applied frequencies. It is observed that in this case, results are very similar and there is no apparent improvement between 10 and 30 sec sampling. When the number of cycles is slightly changed from 6000 to 5995, there is no more a frequency bin at the loading frequencies and this produces differences in results due to spectral leakage, which can lead to significant errors such as in the case of 20 Hz load frequency.

The present work concludes that there is no significant difference of performances between Digital Cross Correlation, Discrete Fourier Transform and Least Square Fitting, and all of them are affected by spectral leakage errors which require a careful choice of sampling parameters. Moreover, Discrete Fourier Transform provides a full representation of the harmonic content on selected points, and can be a powerful tool for exploring at a glance the spectral content for TSA and other thermo-mechanical analyses.

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