

# Tomographic imaging of displacement and strain fields: Current techniques and applications

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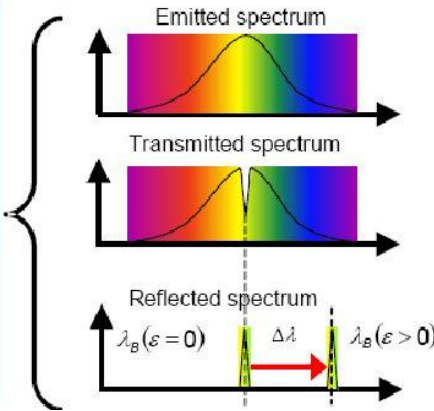
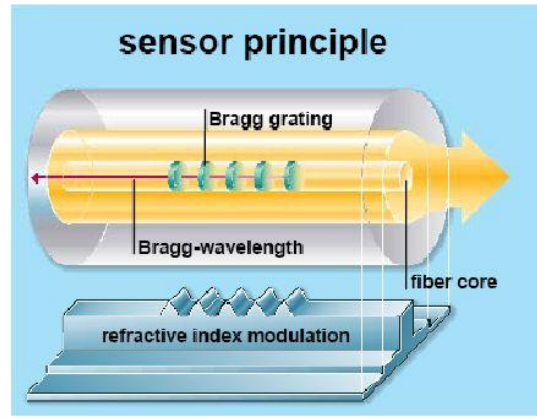
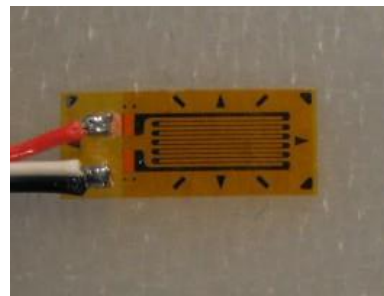


# Outline

- 1-D to 3-D displacement/strain measurement
- Why 3-D strain?
- Techniques
- Comparative table
  - Spatial resolution, strain range, materials, penetration depth

# 1-D strain measurement

- Extensometer
- Resistive strain gauge
- Fibre Bragg sensors

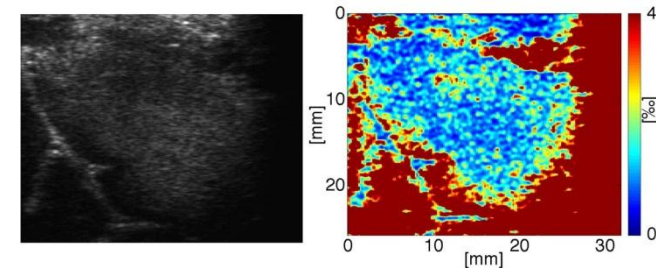
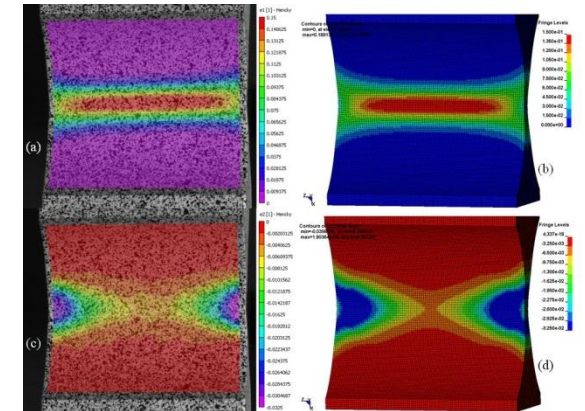
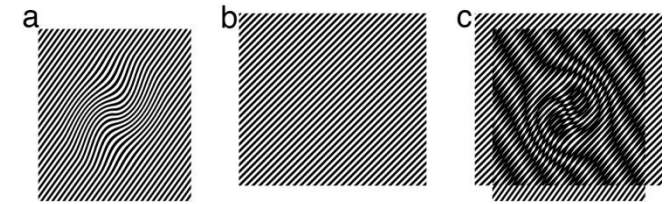


# 2-D strain measurement

- Arrays of 1-D sensors
- Grid method, Moiré
- Triangulation, photogrammetry
- Digital Image correlation
- Neutron diffraction
- X-ray diffraction
- Ultrasound
- Thermal stress analysis
- Photoelasticity
- Moiré Interferometry
- Speckle Interferometry

*In-plane, out-of-plane and slope sensitivity*

## • Static and dynamic applications



# 3-D displacement measurement

- 1-D embedded detector arrays (SGs and FBGs)
- 3-D strain gauge rosette
- Ultrasound
- X-Ray micro CT + DVC
- Magnetic Resonance Elastography
- OCT + DVC
- Wavelength Scanning Interferometry
- Tilt Scanning Interferometry
- 3-D Photoelasticity
- Other varieties of tomographic methods with DVC (PET, acousto-optic, ND, XD, FBG arrays, etc.)

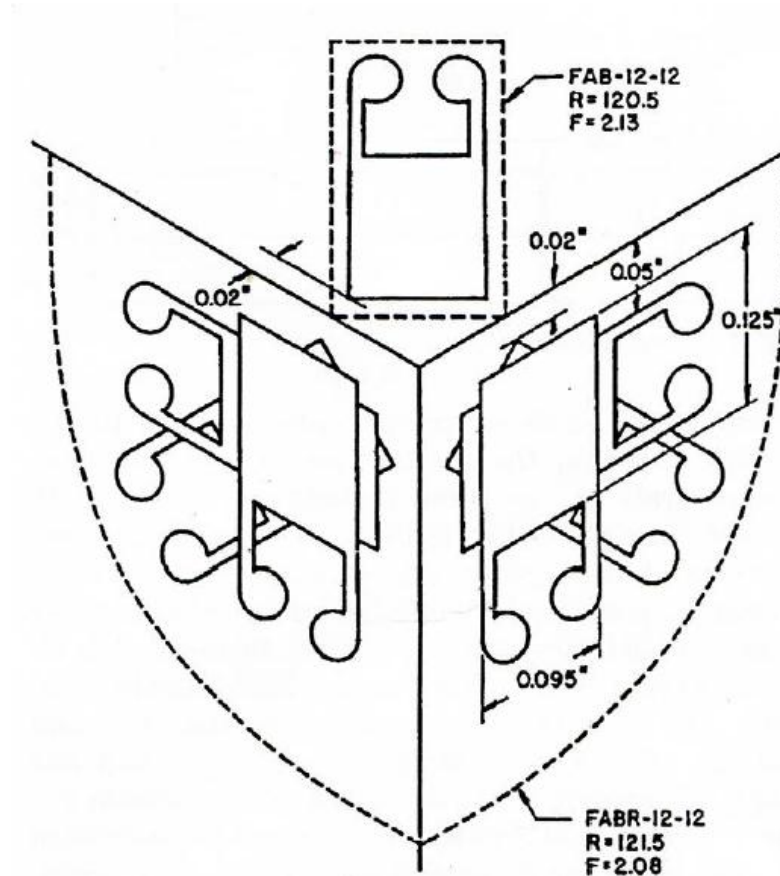
# Methods for 3-D strain imaging

- **Algorithms**
  - Correlation
  - Phase detection
  
- **Mechanical Stimuli used for perturbation**
  - Quasi Static
  - Low frequency vibration (compressive/shear)

# Why 3-D?

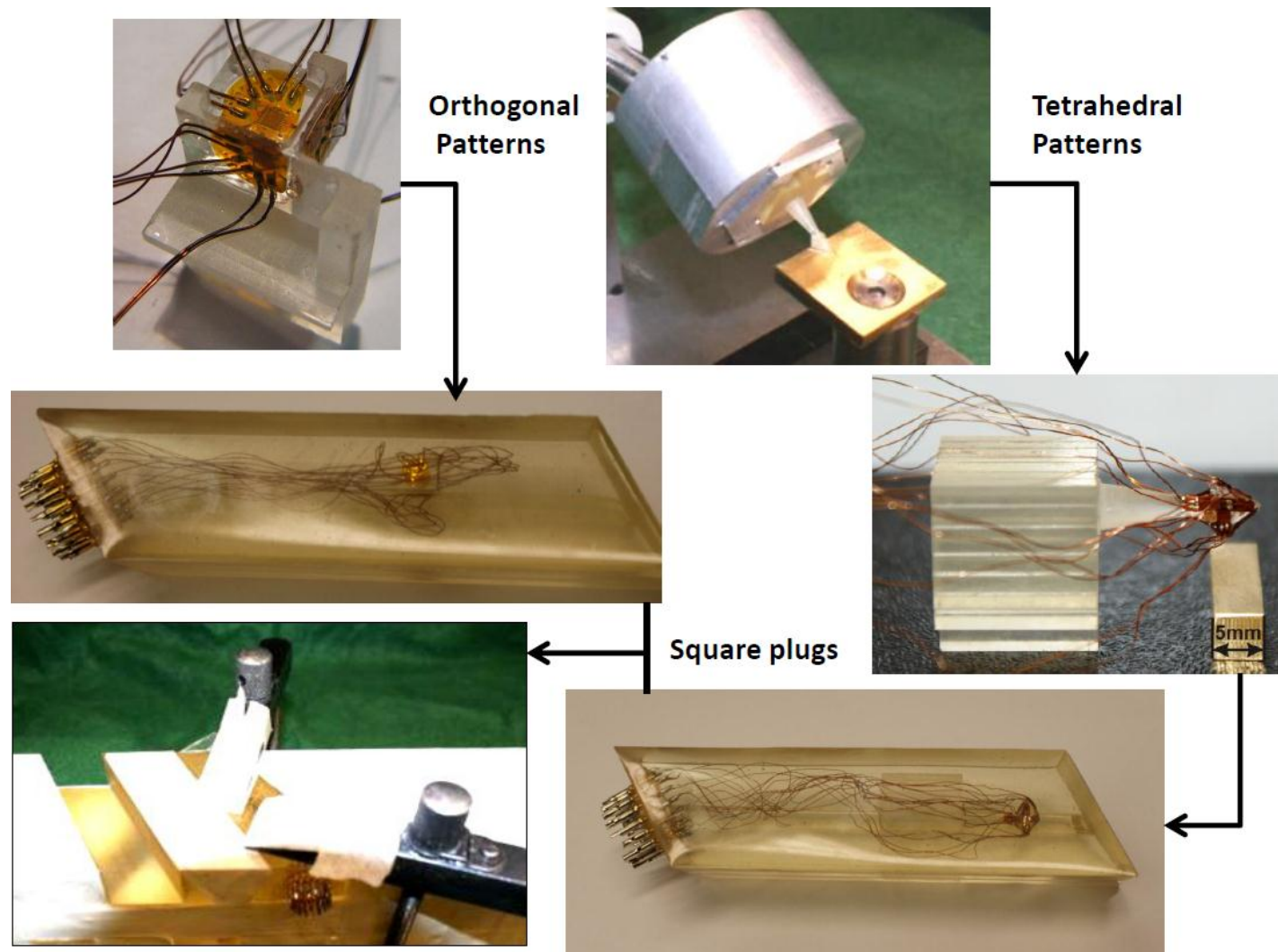
- **Experimental data** is needed to define and validate computer models
- **Material characterization / identification** of constitutive parameters. (Uniqueness issues)
- **3-D elastography** (virtual palpation)
- **Relevant in medicine and biology** (shear modulus  $G^*$ )
  - In soft tissues,  $G^*$  changes due to: Aging, Alzheimer's disease, Normal pressure hydrocephalus, Tumours, Multiple sclerosis, Scarring
- **Engineering** (Composites, functional materials, damage characterization, anisotropy)

# The 3-D strain rosette, 1963

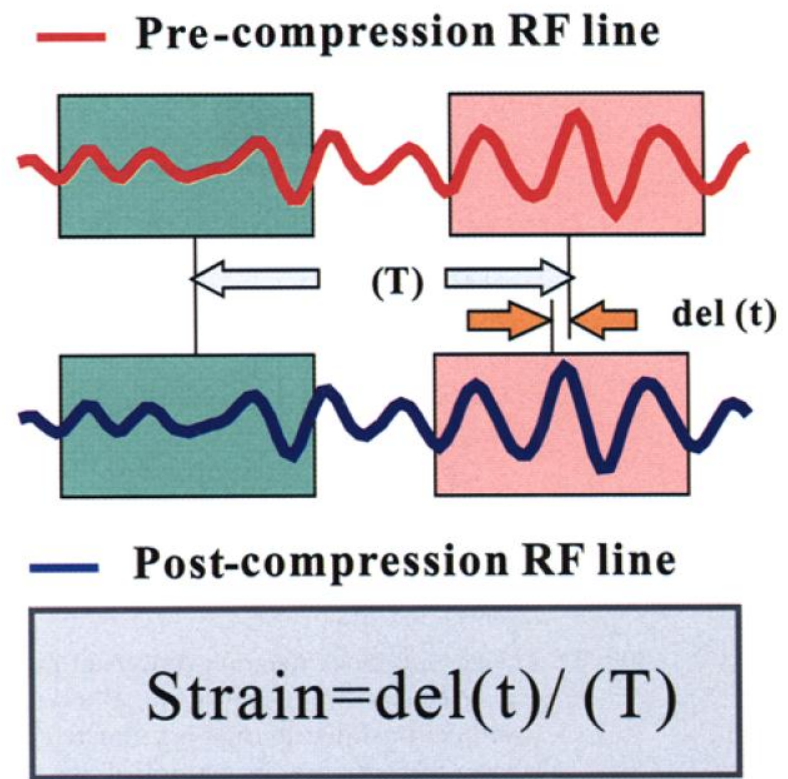
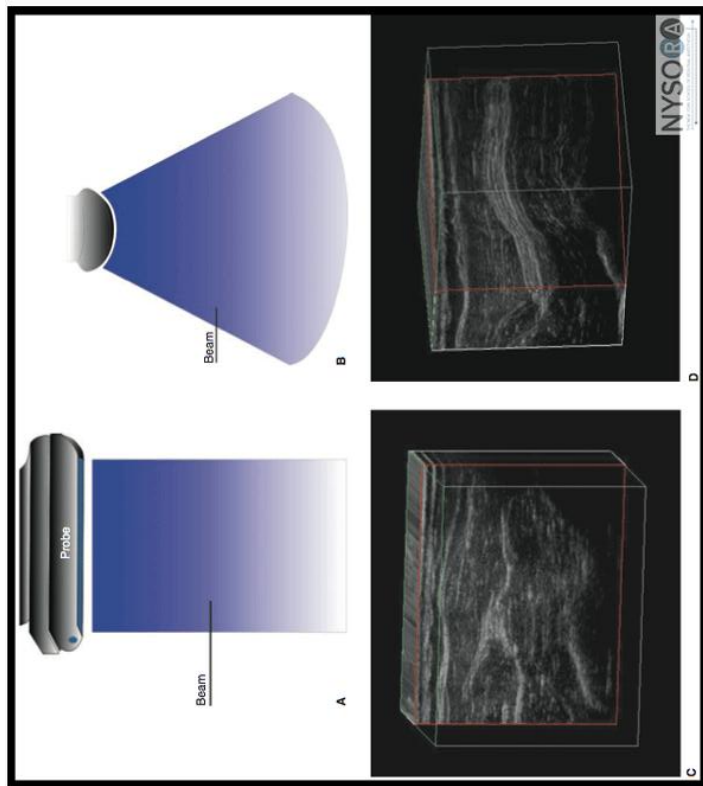




# The 3-D strain rosette, 2011



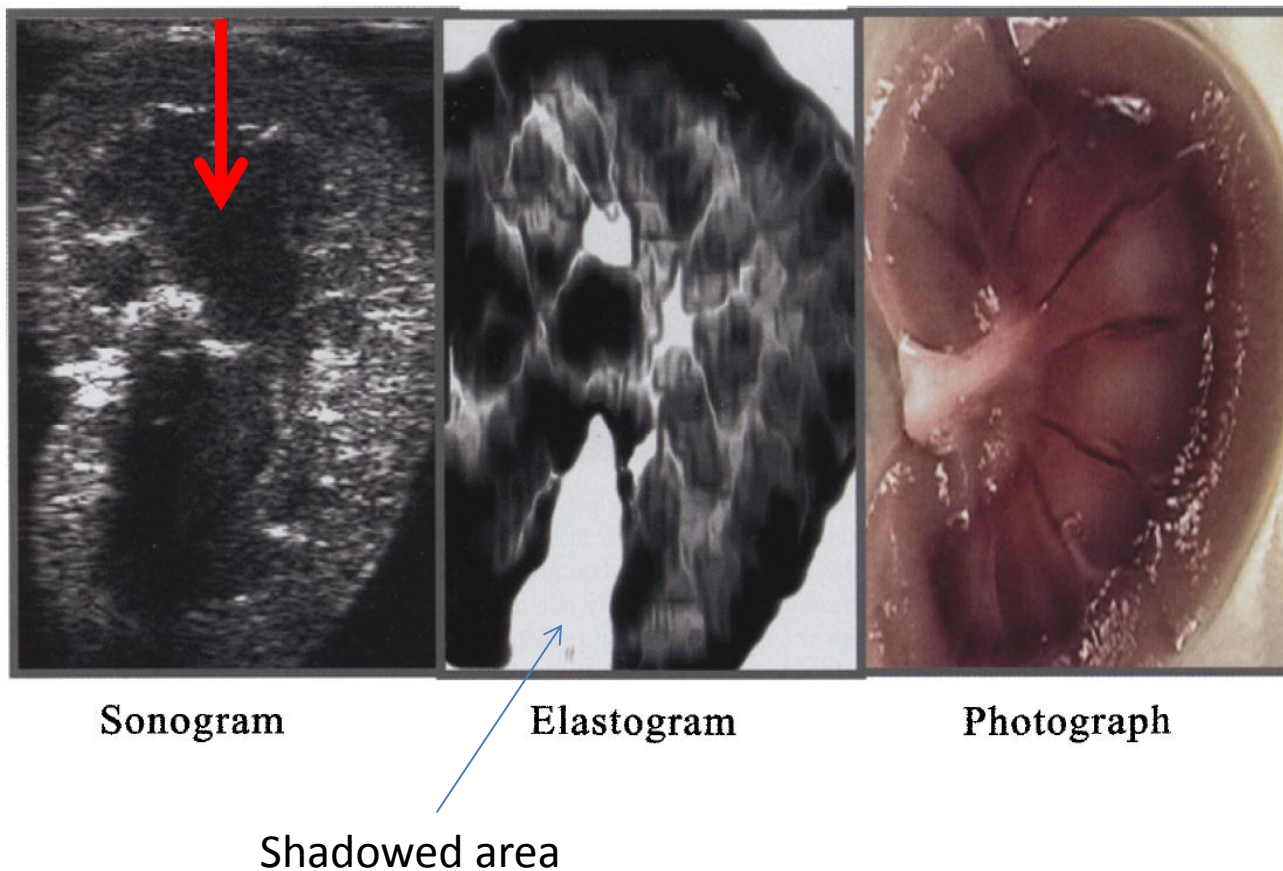
# Ultrasound



- Send acoustic pulse (few MHz) and measure echo delay
- Compare 2 states to measure displacement
- Compare displacement at 2 points to measure strain

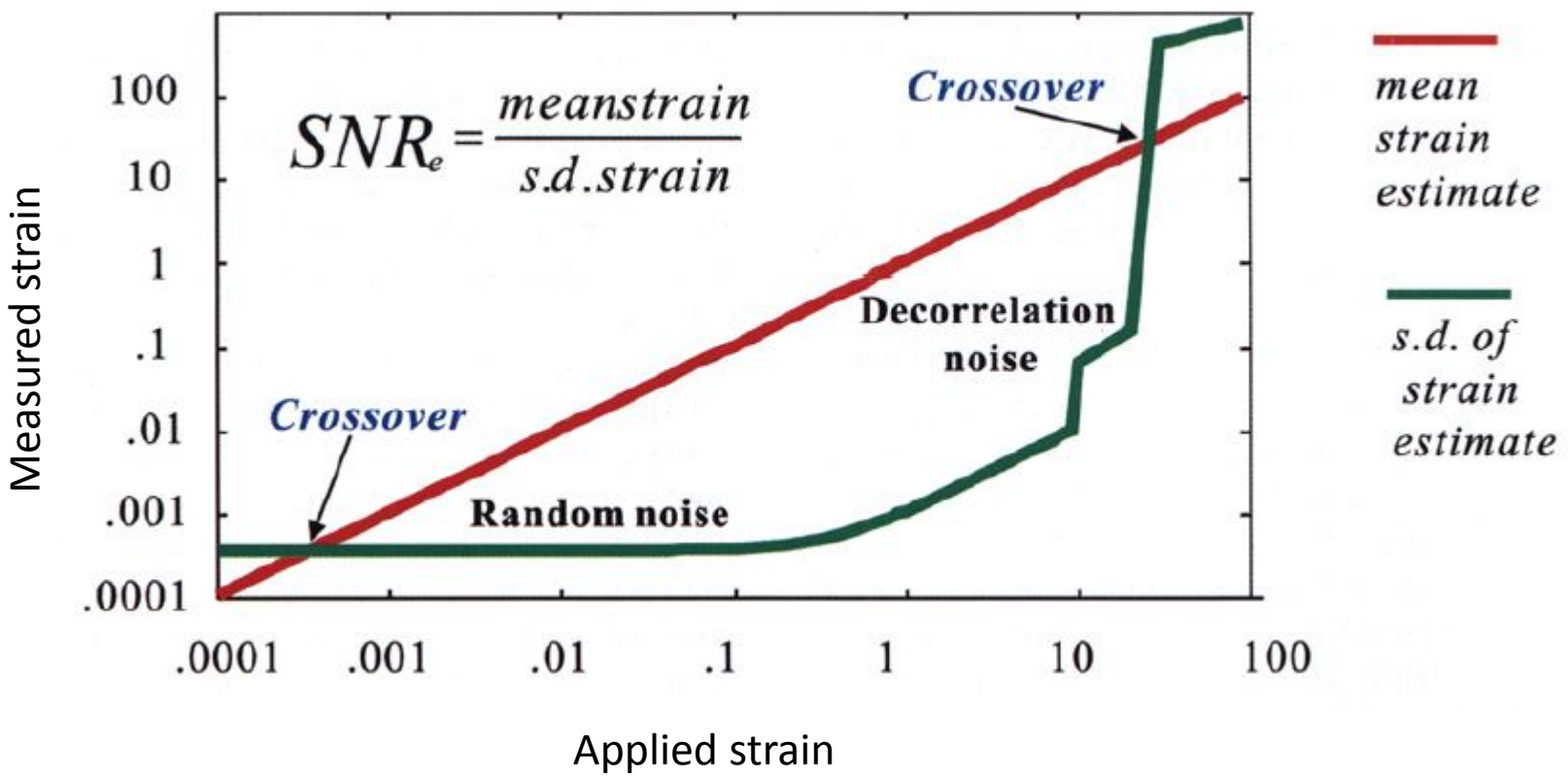
# Ultrasound elastography example

- Ovine kidney in-vitro
- Bright = 'soft'; dark = 'stiff'

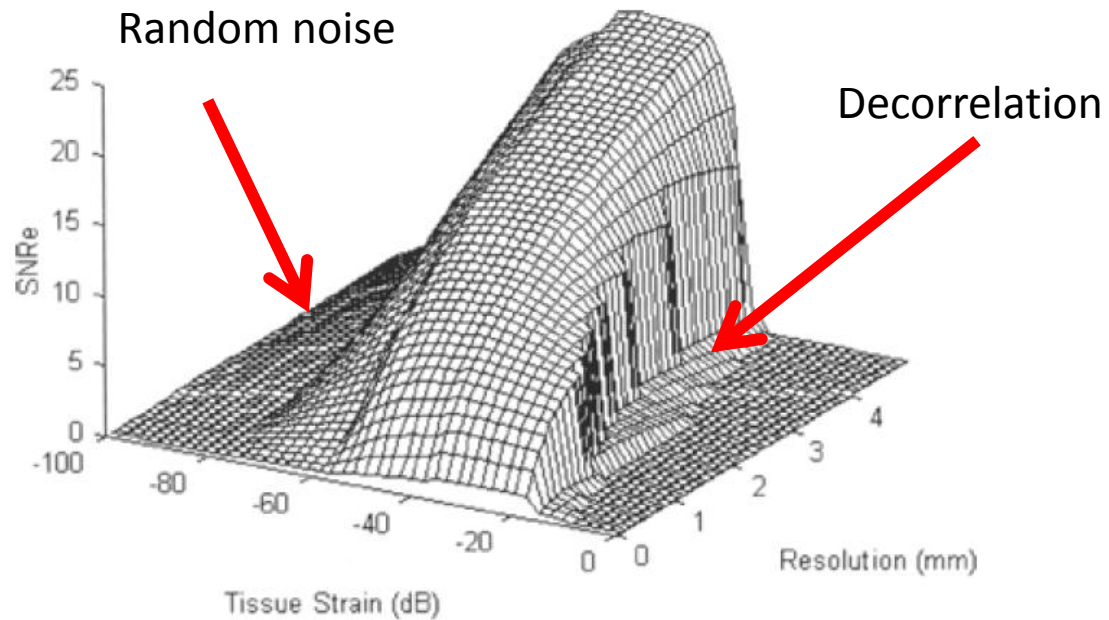


# Ultrasound

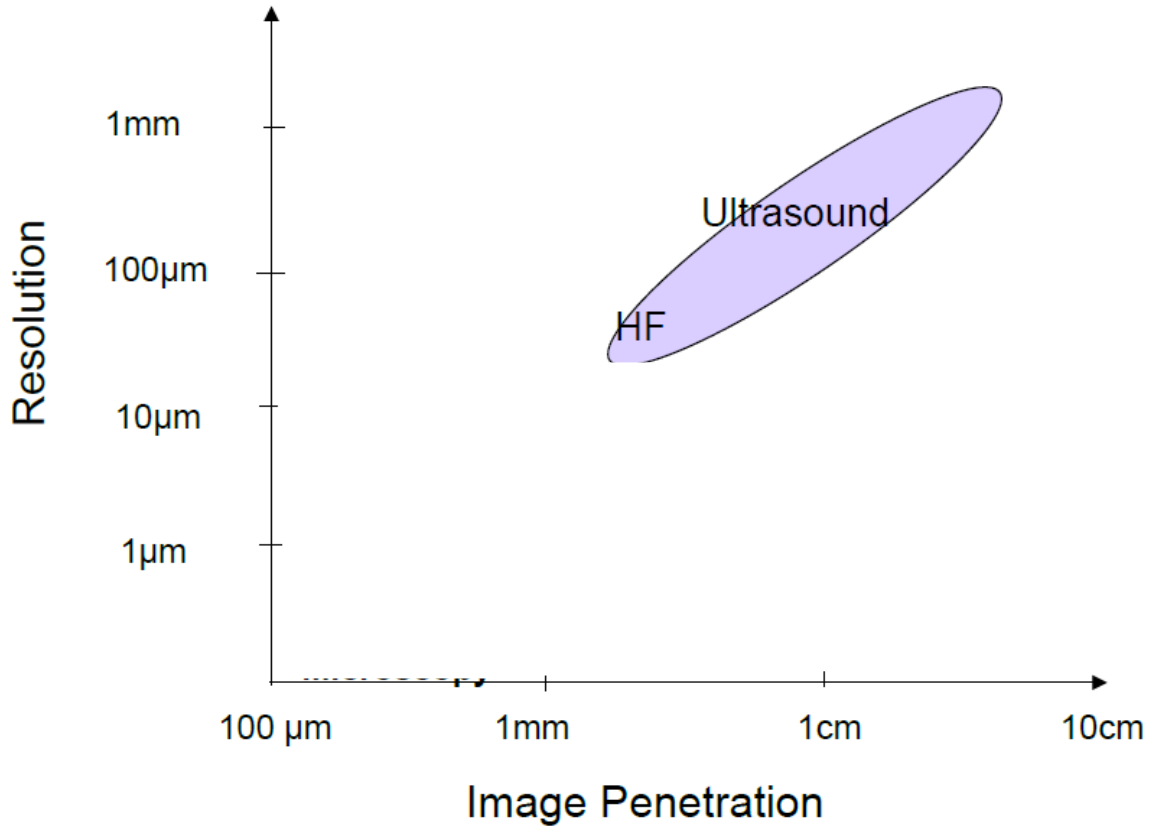
- In soft tissues:  $K \sim 2\rho c$  ( $\rho$ : density,  $c$ : wave velocity  $\sim 1540\text{m/s}$ )
- Most attempts to map  $K$  distributions failed, as  $c$  nearly uniform in tissues (low contrast-to-noise ratio).
- Shear modulus  $G$  is the main parameter identified in US elastography.



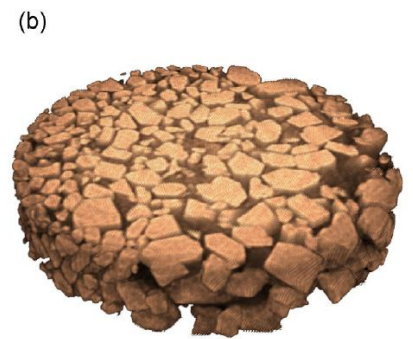
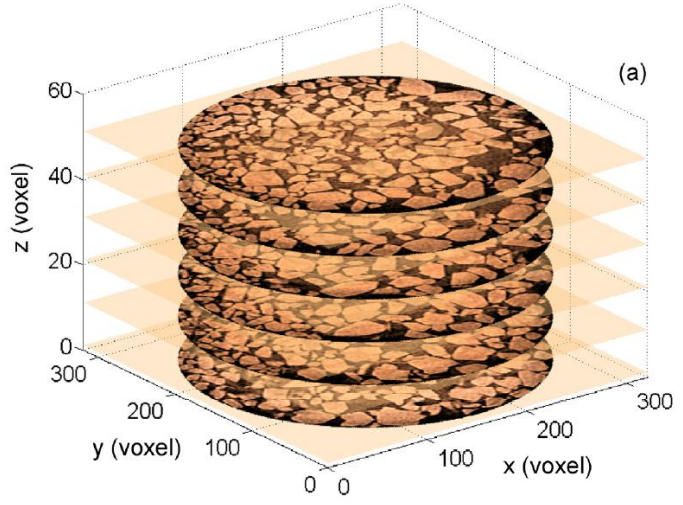
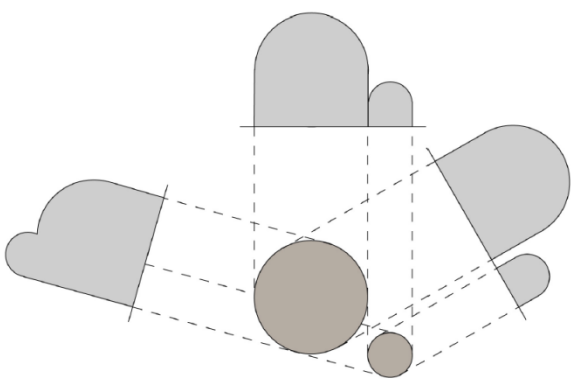
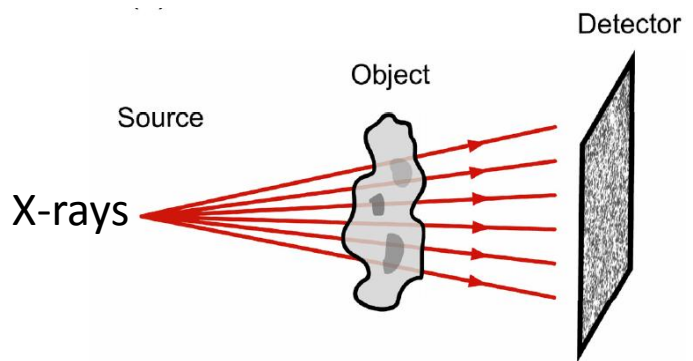
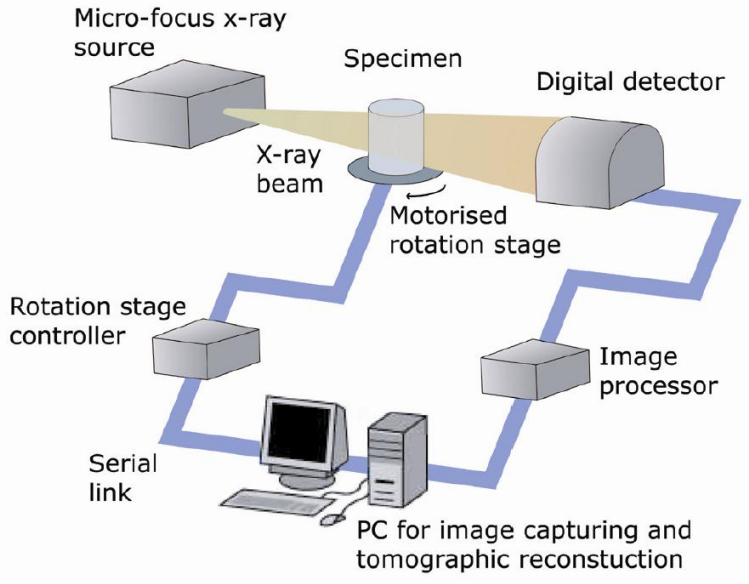
# Strain Filter concept



# Depth and resolution

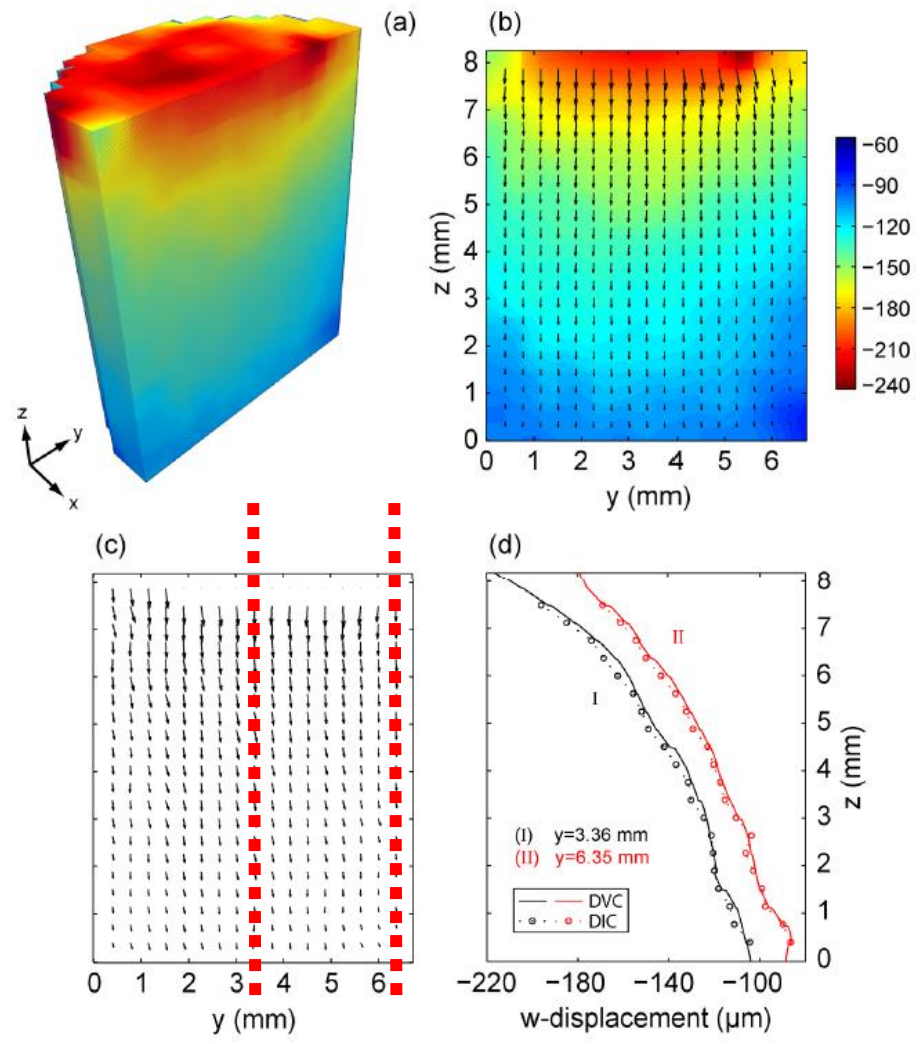
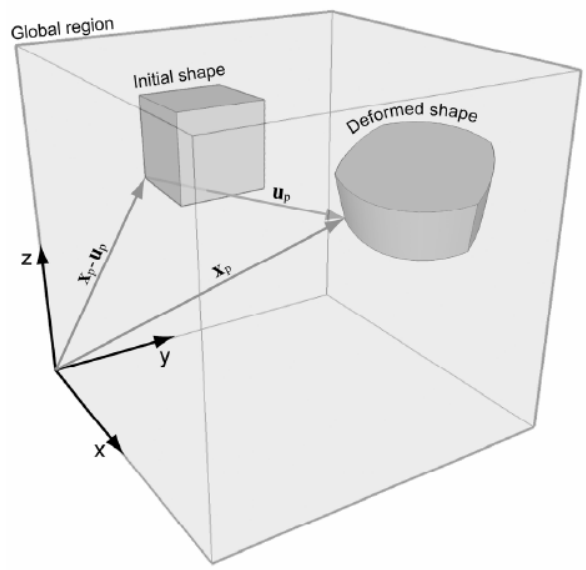


# X-ray micro CT



# X-ray micro CT

- Digital Volume Correlation



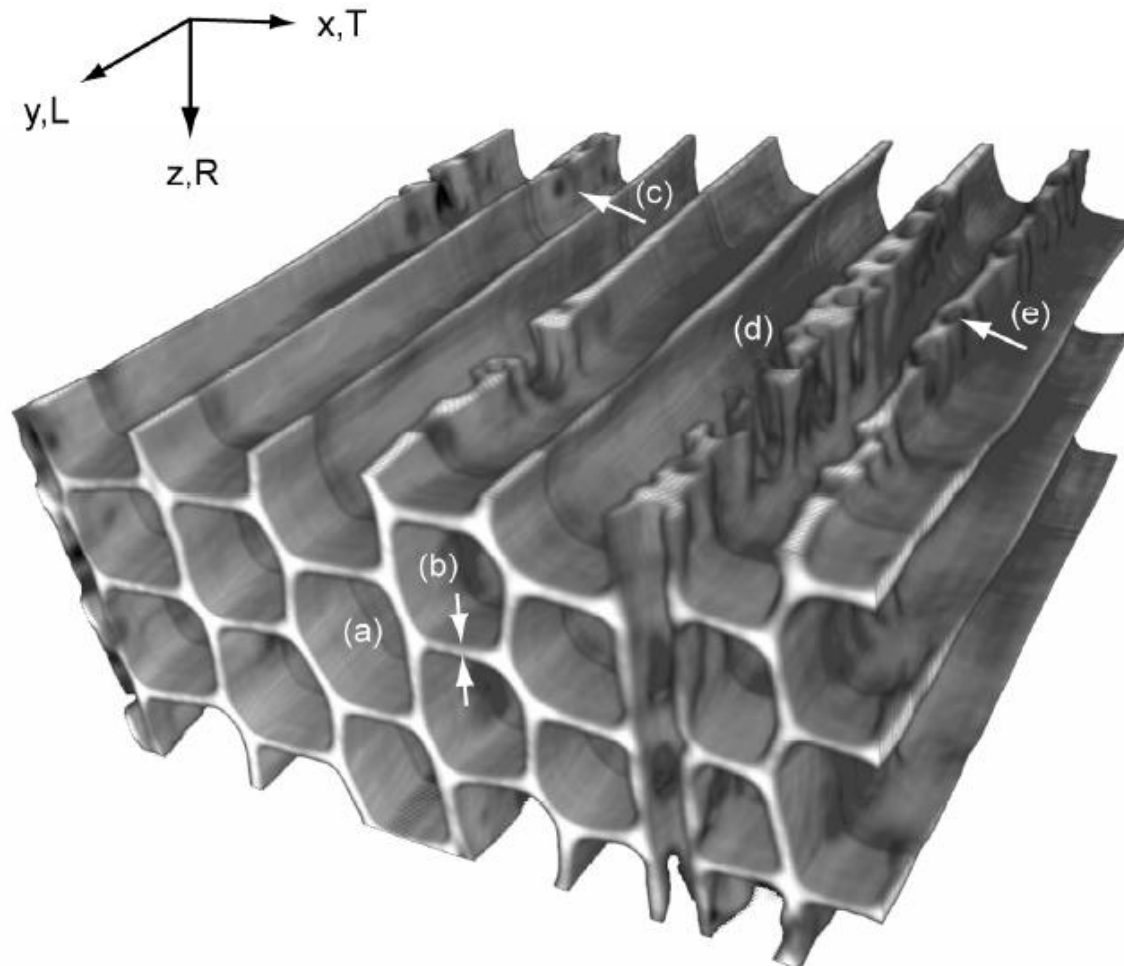


# X-ray intense source

- Synchrotron facility
  - Dynamic applications at high resolution



# X-ray micro CT

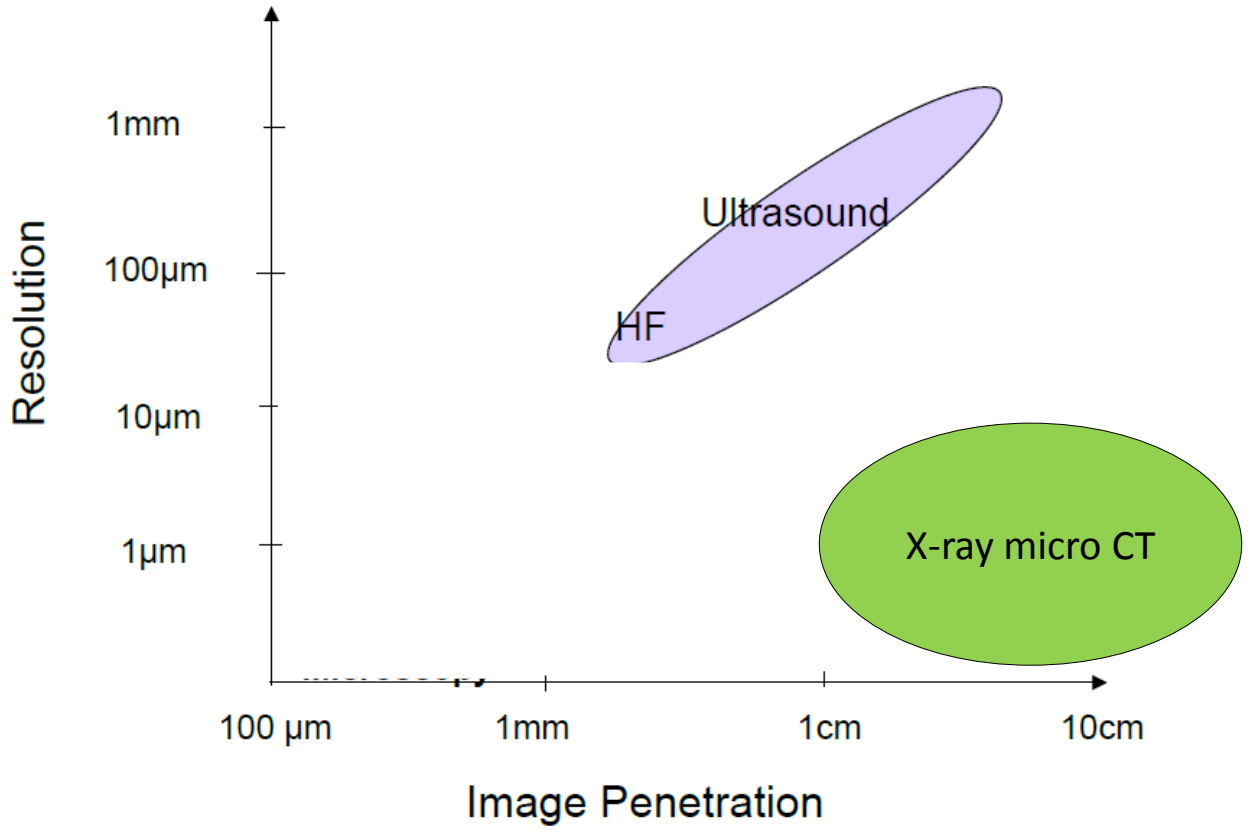


Wood microstructure,  $112 \times 112 \times 56 \mu\text{m}^3$

# X-ray micro CT

	Synchrotron	Desktop systems
Sample average diameter	<50 mm	<200 mm
Spatial resolution max / typical	0.2 $\mu\text{m}$ / 0.2 $\mu\text{m}$	0.5 $\mu\text{m}$ / 5-10 $\mu\text{m}$
Scan time	1 sec - 10 minutes	1-2 hours

# Depth and resolution



# Magnetic Resonance Elastography

## MRI

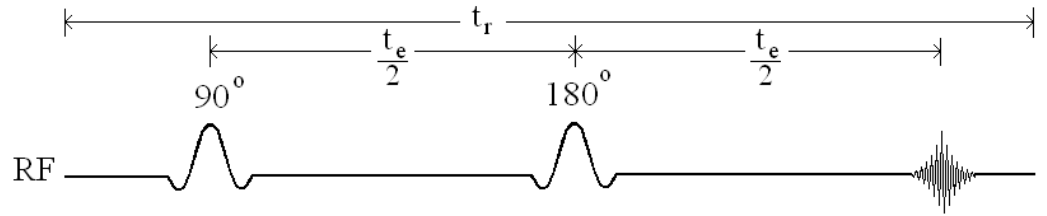
- Measure nuclear spin precession
- Encode position using magnetic field gradients (frequency proportional to magnetic field)
- Non-magnetic samples

## MRE

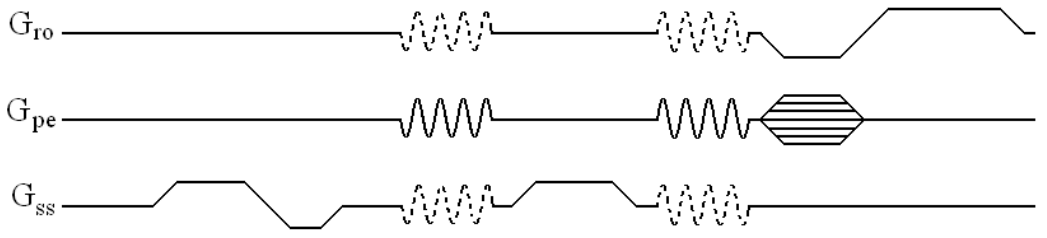
- Visualize **mechanical waves** in tissue
- Wave velocity and wavelength depend on **elastic modulus**

# Phase contrast measurement of shear waves

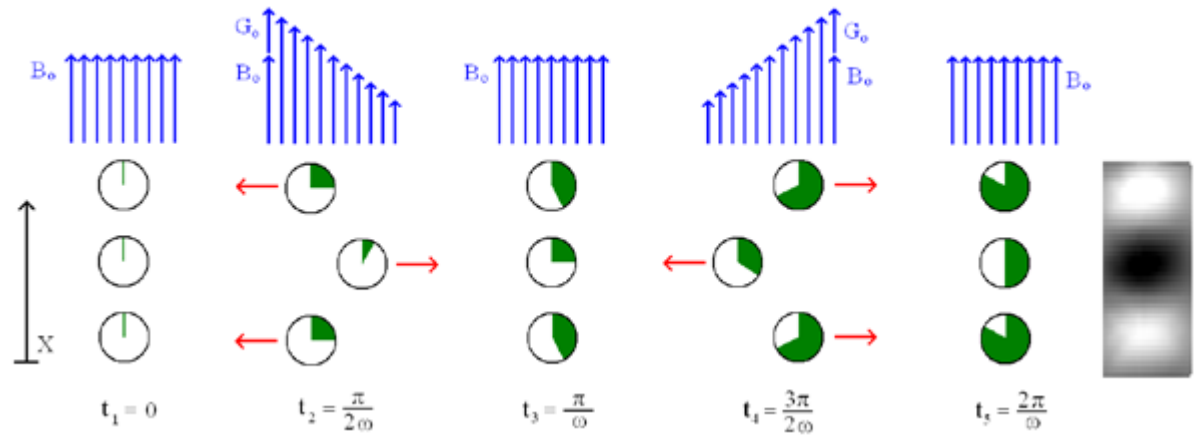
Pulse sequence



Oscillating gradients

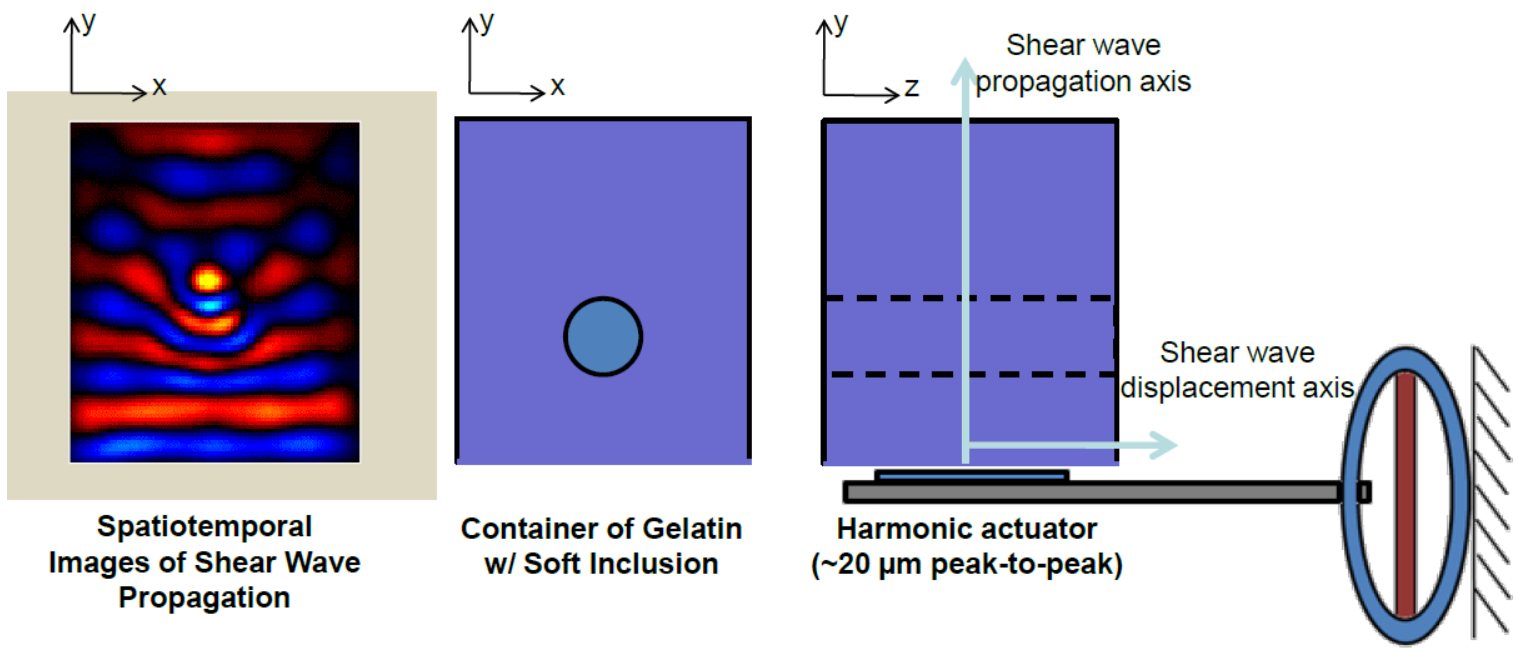


Phase accumulation  
Proportional to  
displacement

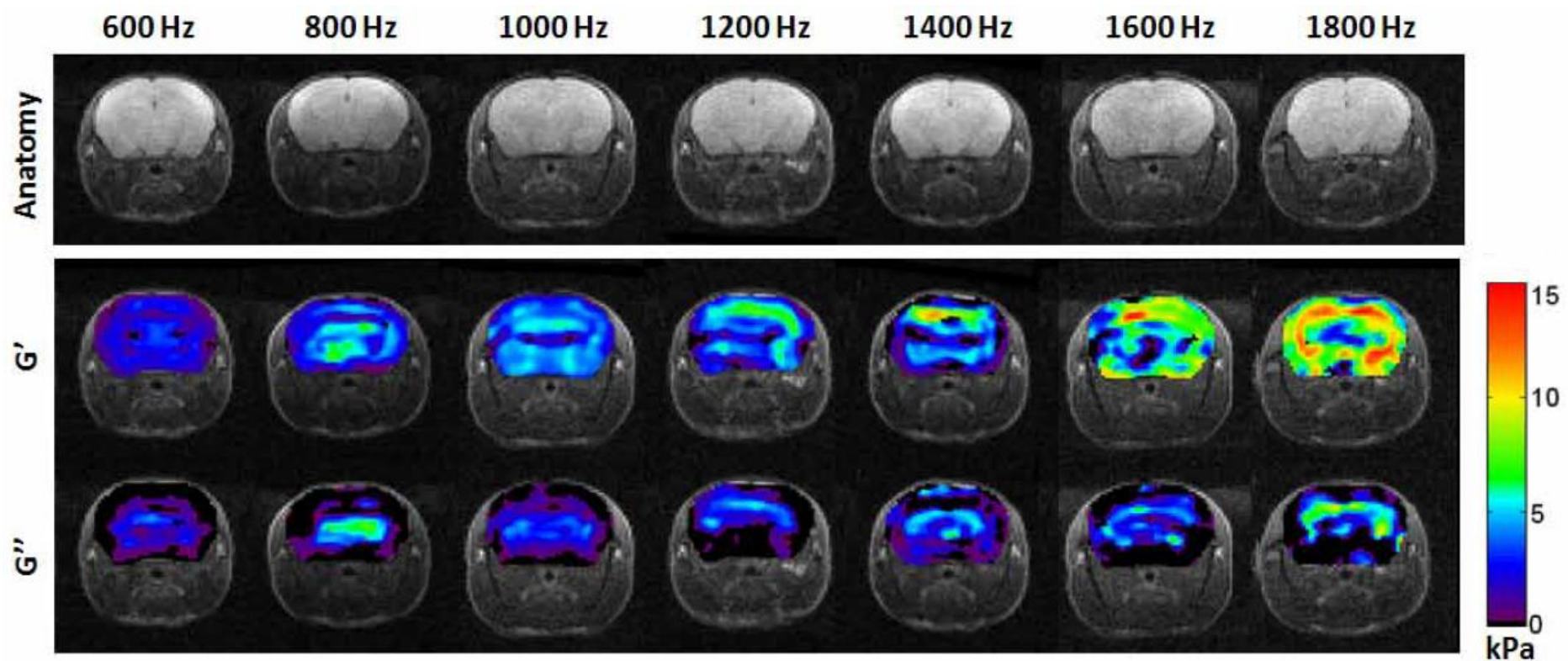


# MRE

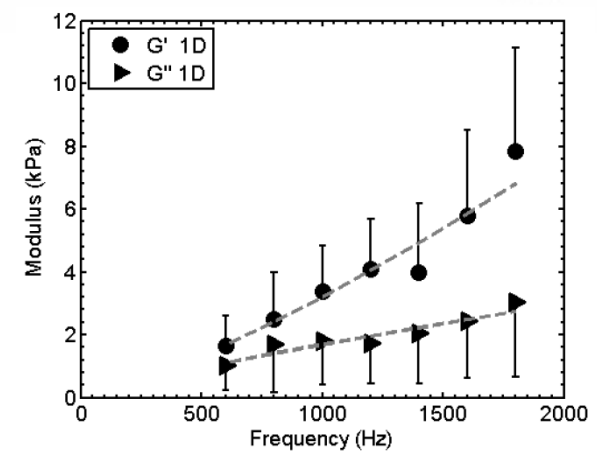
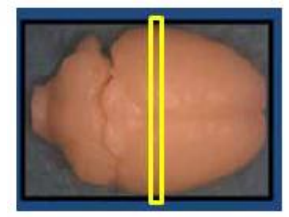
- Detection of mm-amplitude harmonic motion requires synchronous actuation



# Mouse brain

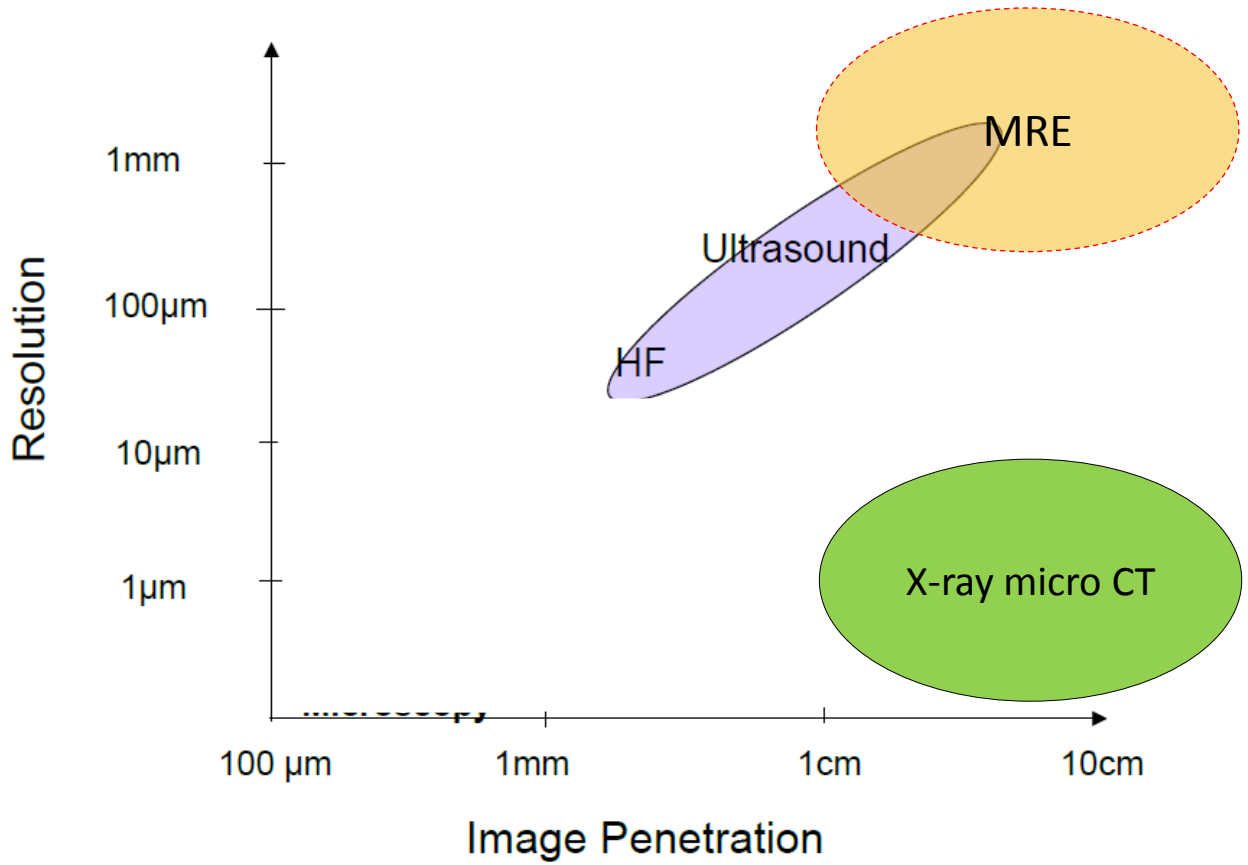


Voxel size:  $0.25 \times 0.25 \times 0.25 \text{mm}^3$



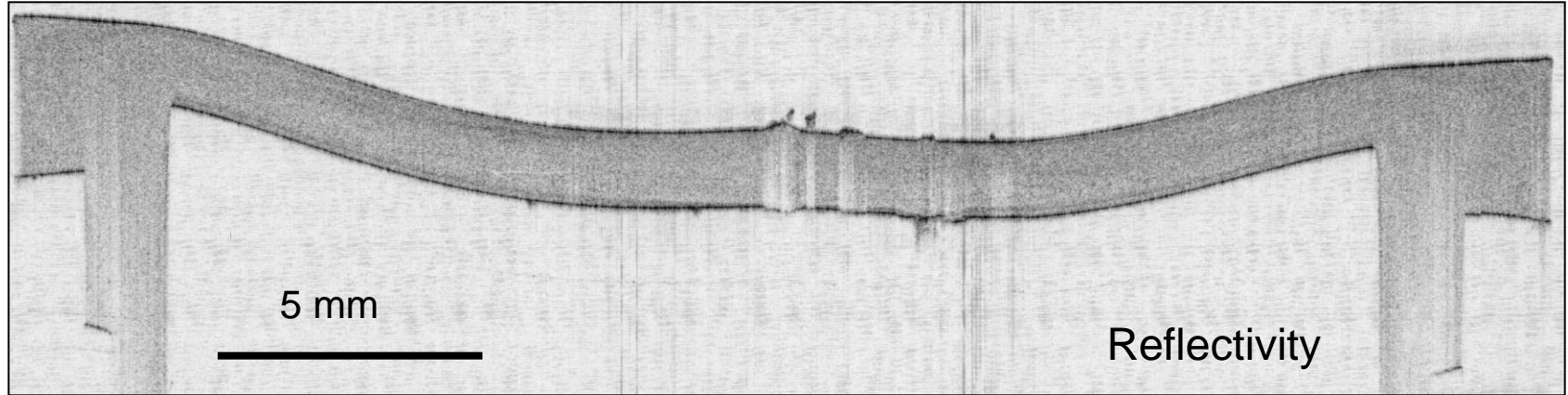
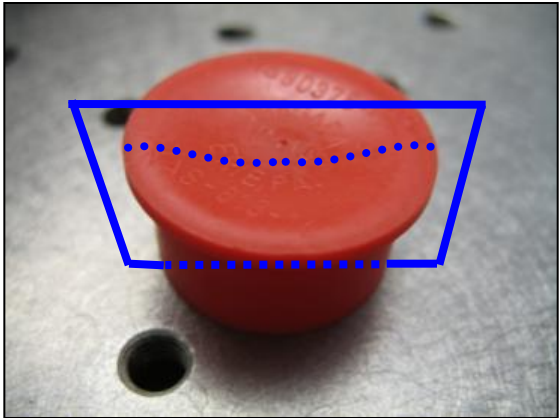


# Depth and resolution



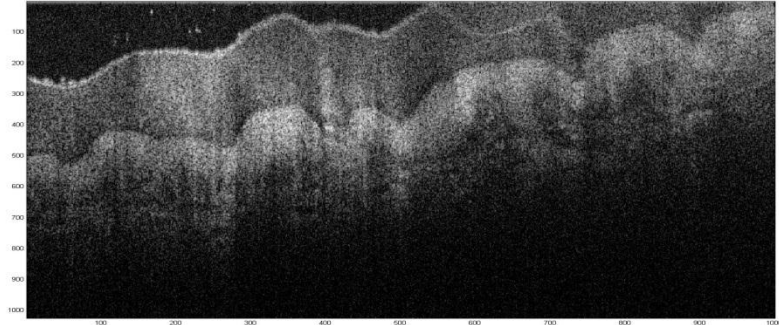
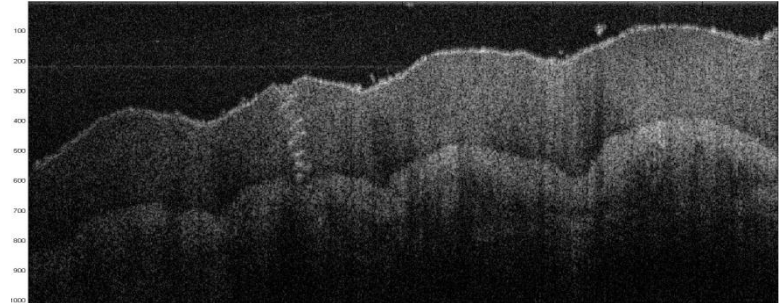
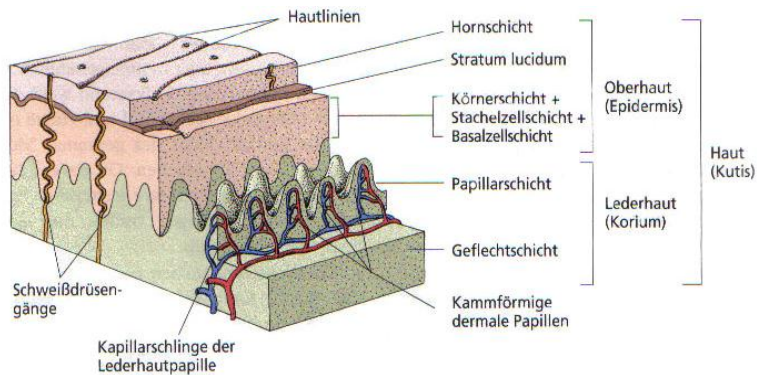
# Optical Coherence Tomography

polymer injection  
moulded part

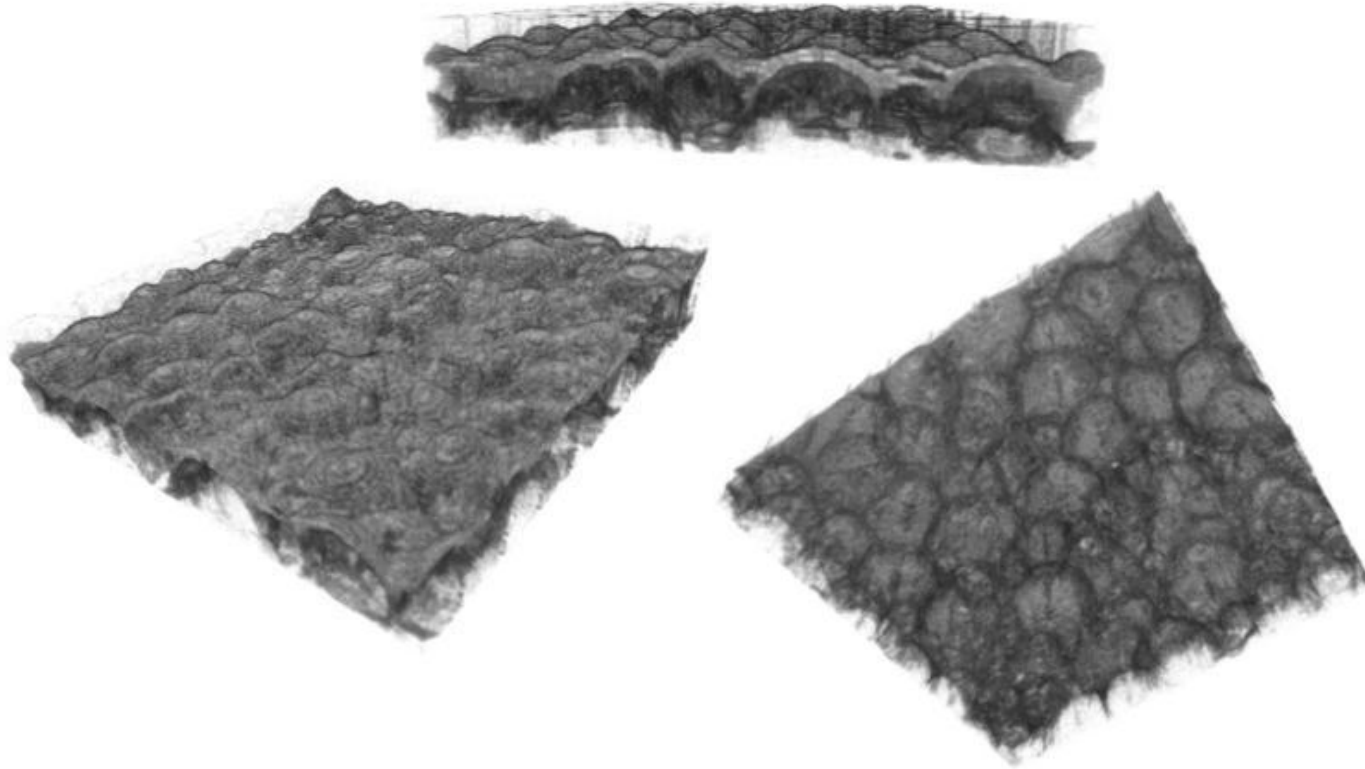


# Optical Coherence Tomography

human skin



# Optical Coherence Tomography



Polyolefin foams

1mm

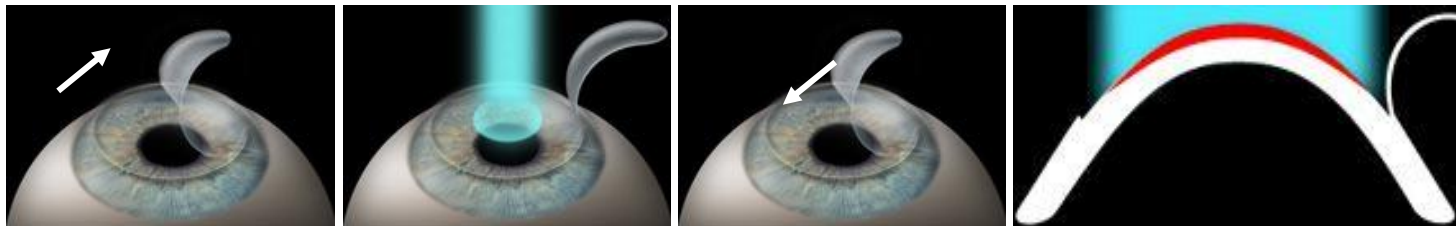
# Measurement of 3-D corneal displacements using DVC + OCT

# Motivation

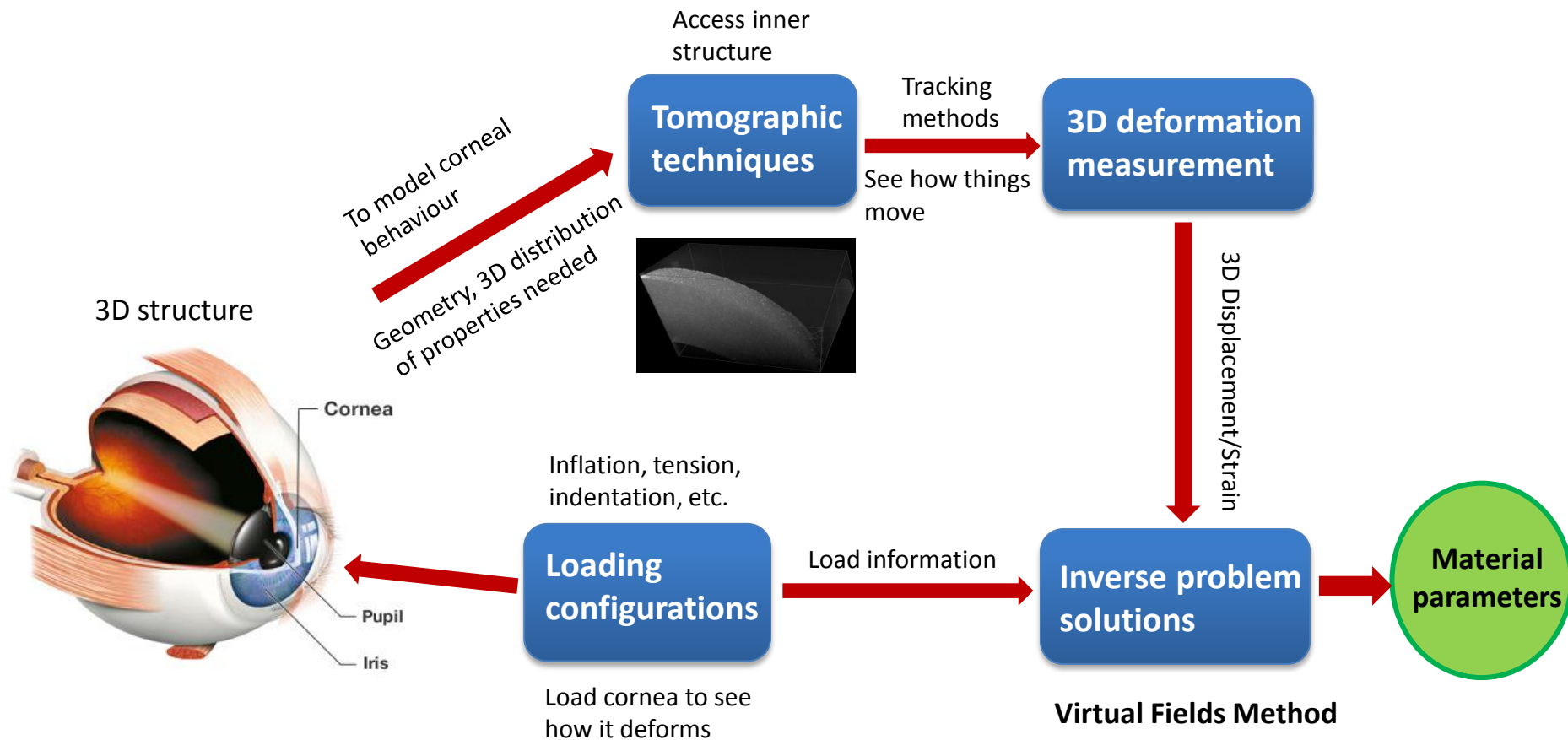
To use corneal mechanical response, rather than only corneal thickness, as the criteria to perform refractive surgery.

## Long term objective

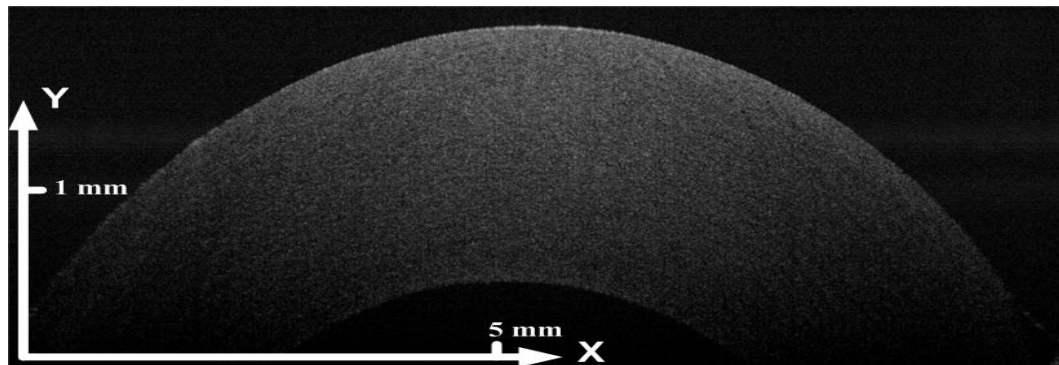
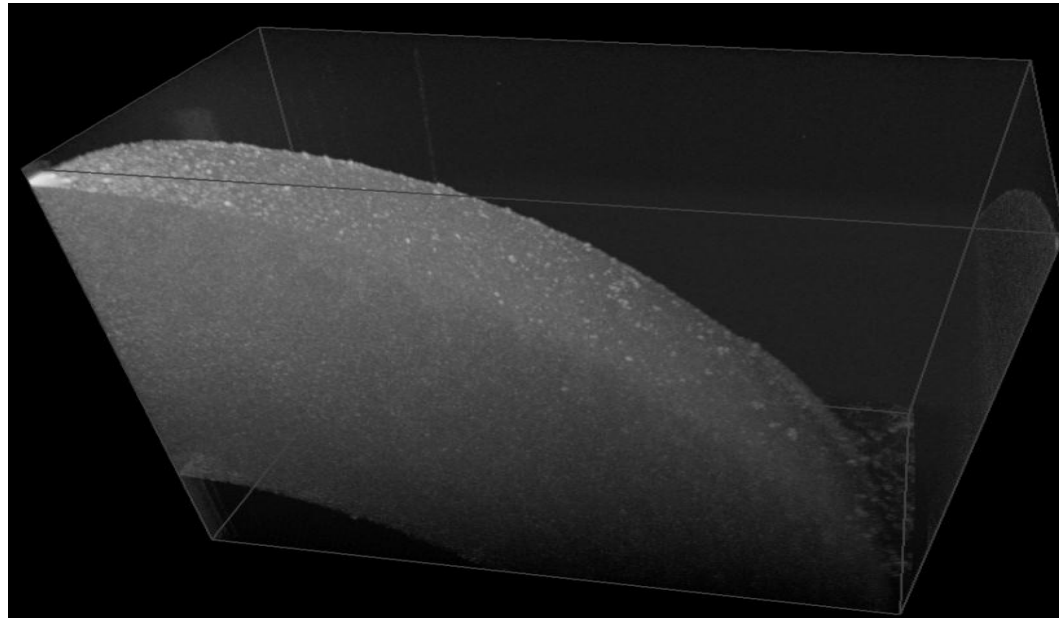
- Measure internal 3D deformation field
- Identify depth-resolved constitutive parameters
- Predict corneal mechanical behaviour during/after ablative surgery



# Methodology



# Optical coherence tomography



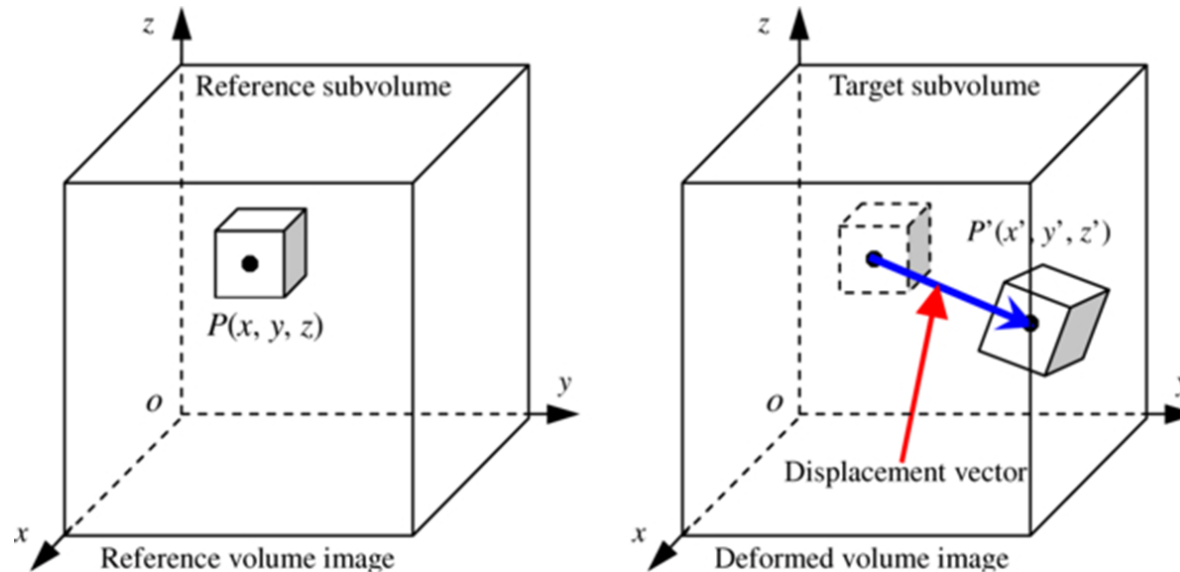
1024 × 512 × 1024 voxels data volume

Acquisition time ~ 3 minutes



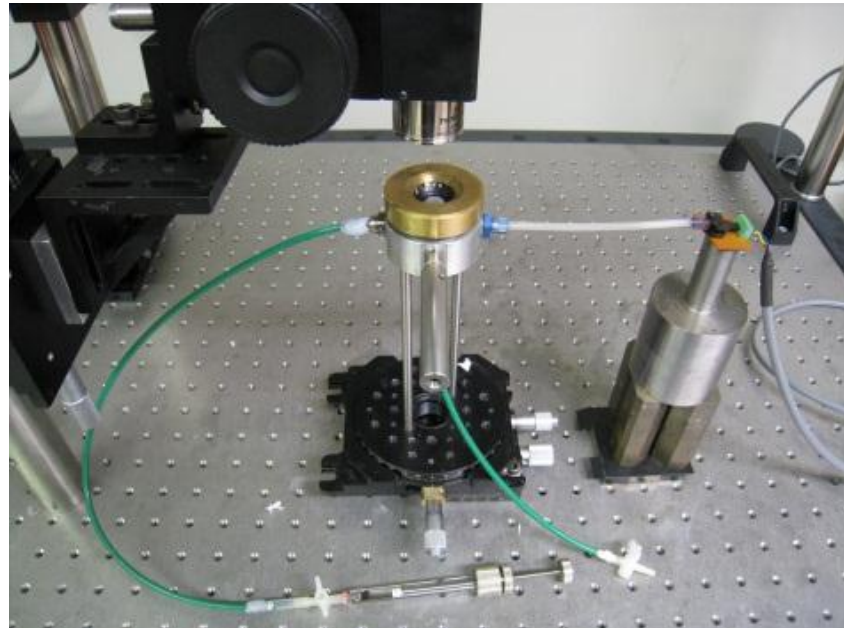
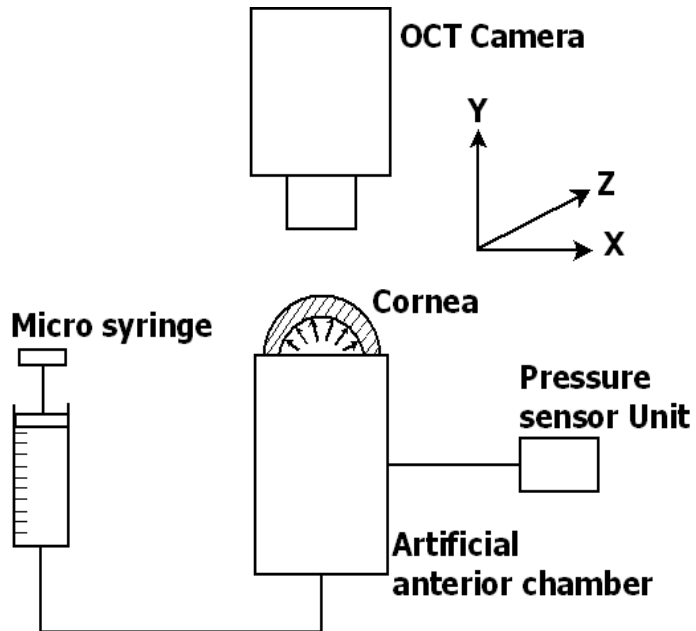
# 3D displacement fields

## Digital volume correlation



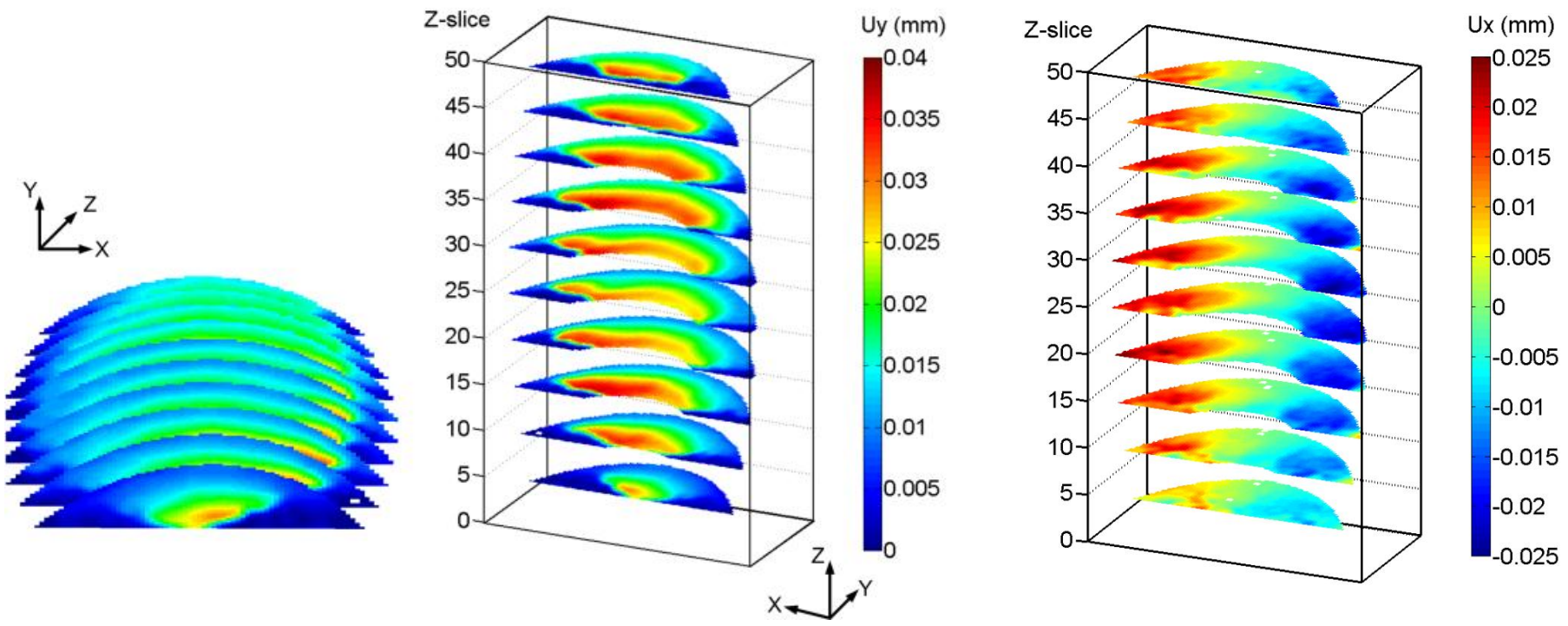
- Volume divided into sub-volumes.
- Displacement vector obtained from tracking and matching voxels between sub-volumes in reference and deformed states

# Inflation test



- Swept Source Optical Coherence Tomography system (Thorlabs OCS1300SS).
- Porcine corneas inflated from 2 to 2.5 kPa

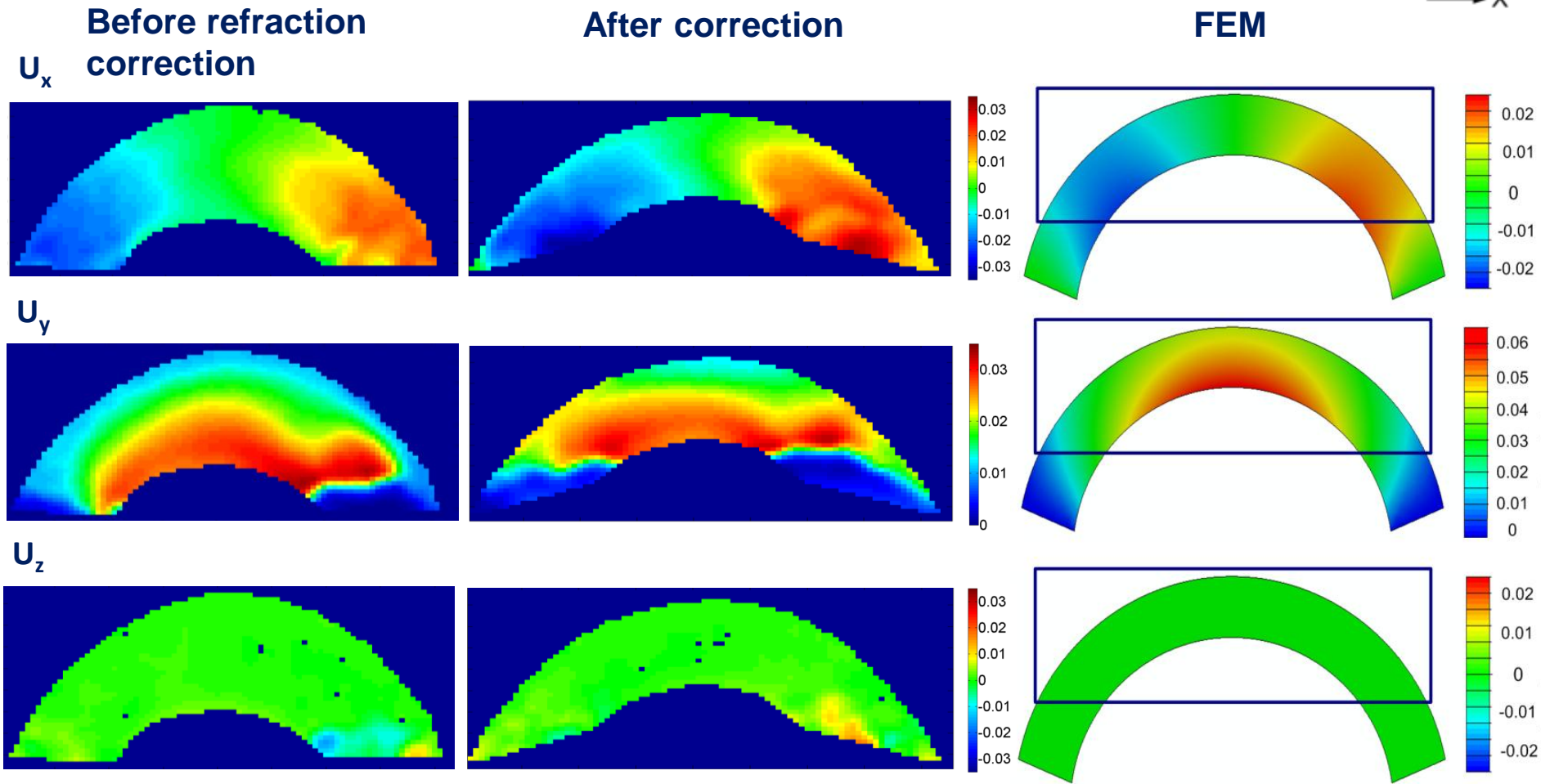
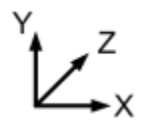
# Inflation test



- $24^3$ -voxel sub-volume, 50% overlap
- Inflated from 2 to 2.5 kPa

# Displacement

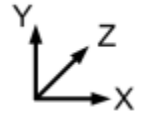
Displacement maps for a central z-slice for inflation from 2 to 2.5 kPa



$E=0.3 \text{ Mpa}; \nu=0.49$  (from literature)

# Strain

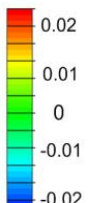
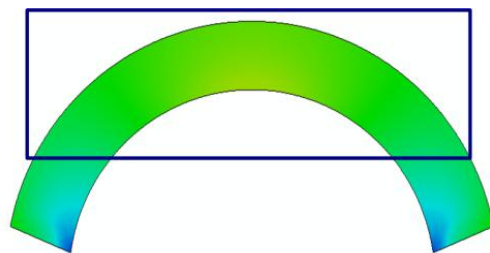
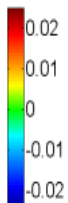
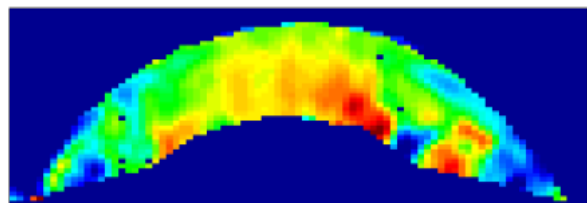
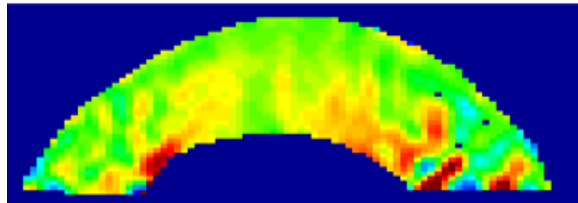
Strain maps for a central z-slice for inflation from 2 to 2.5 kPa



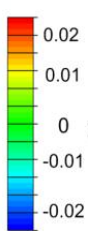
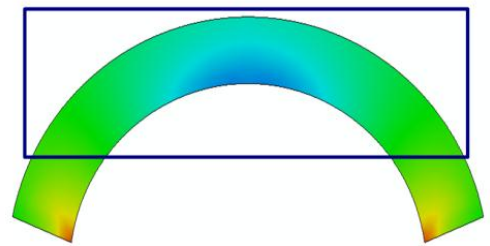
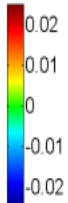
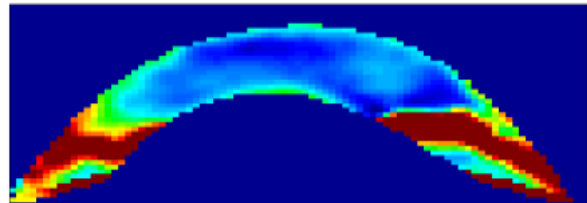
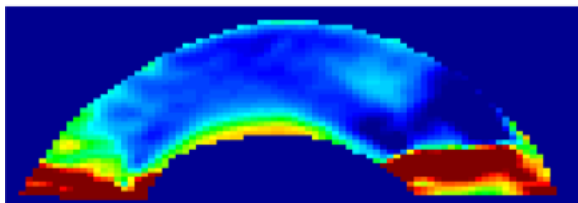
**Before refraction correction**  
 $\epsilon_{xx}$

**After correction**

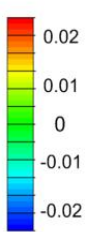
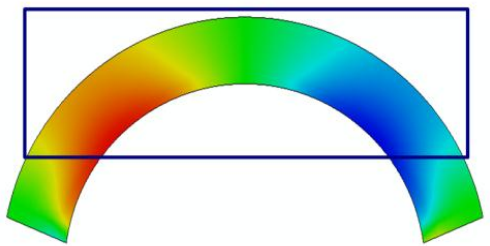
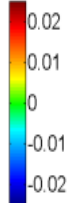
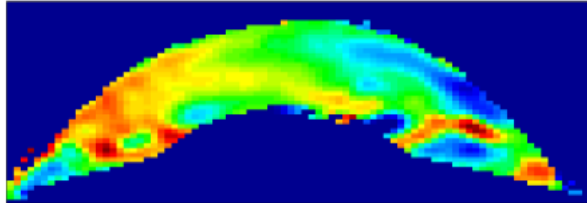
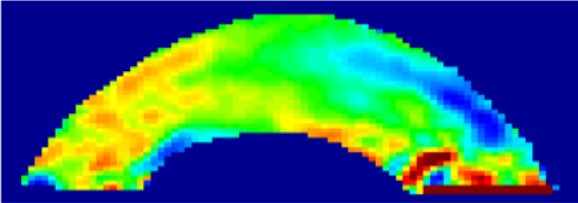
**FEM**



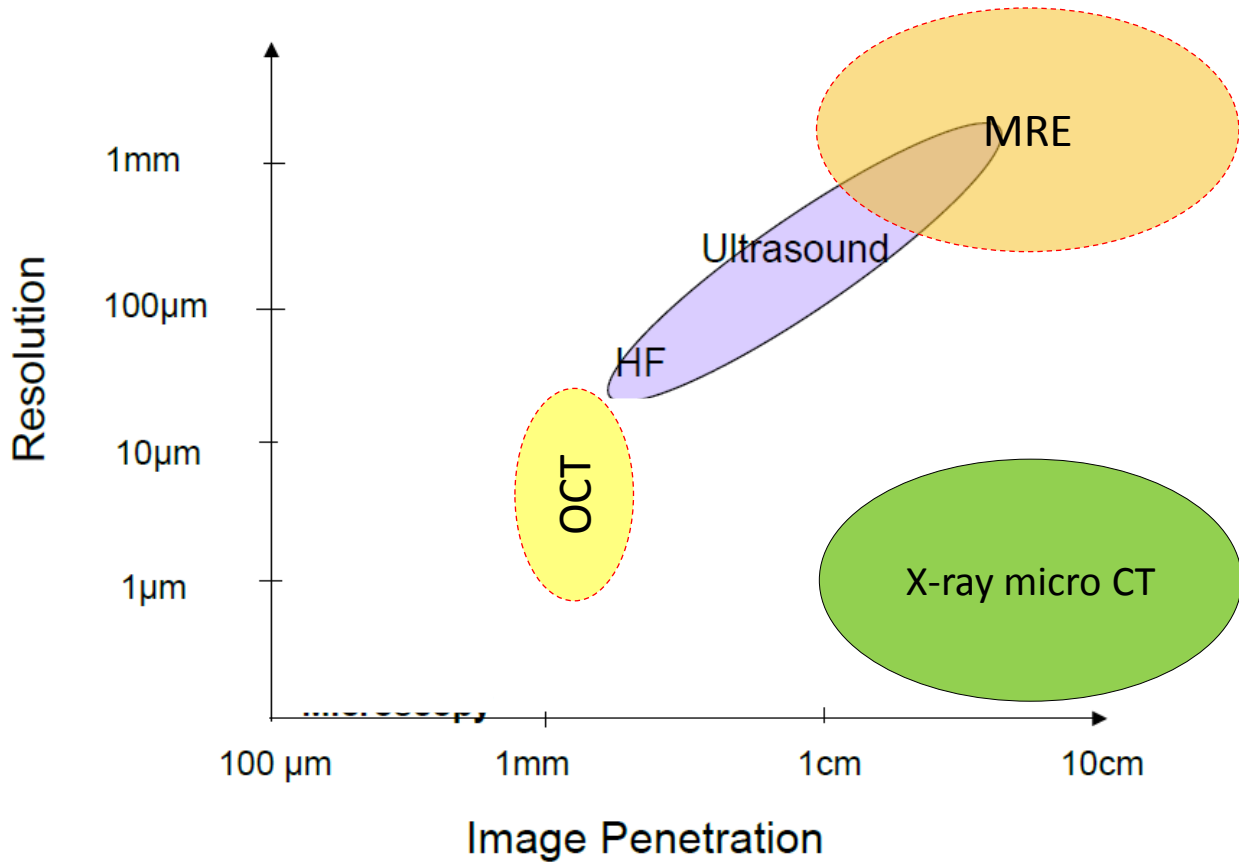
$\epsilon_{yy}$



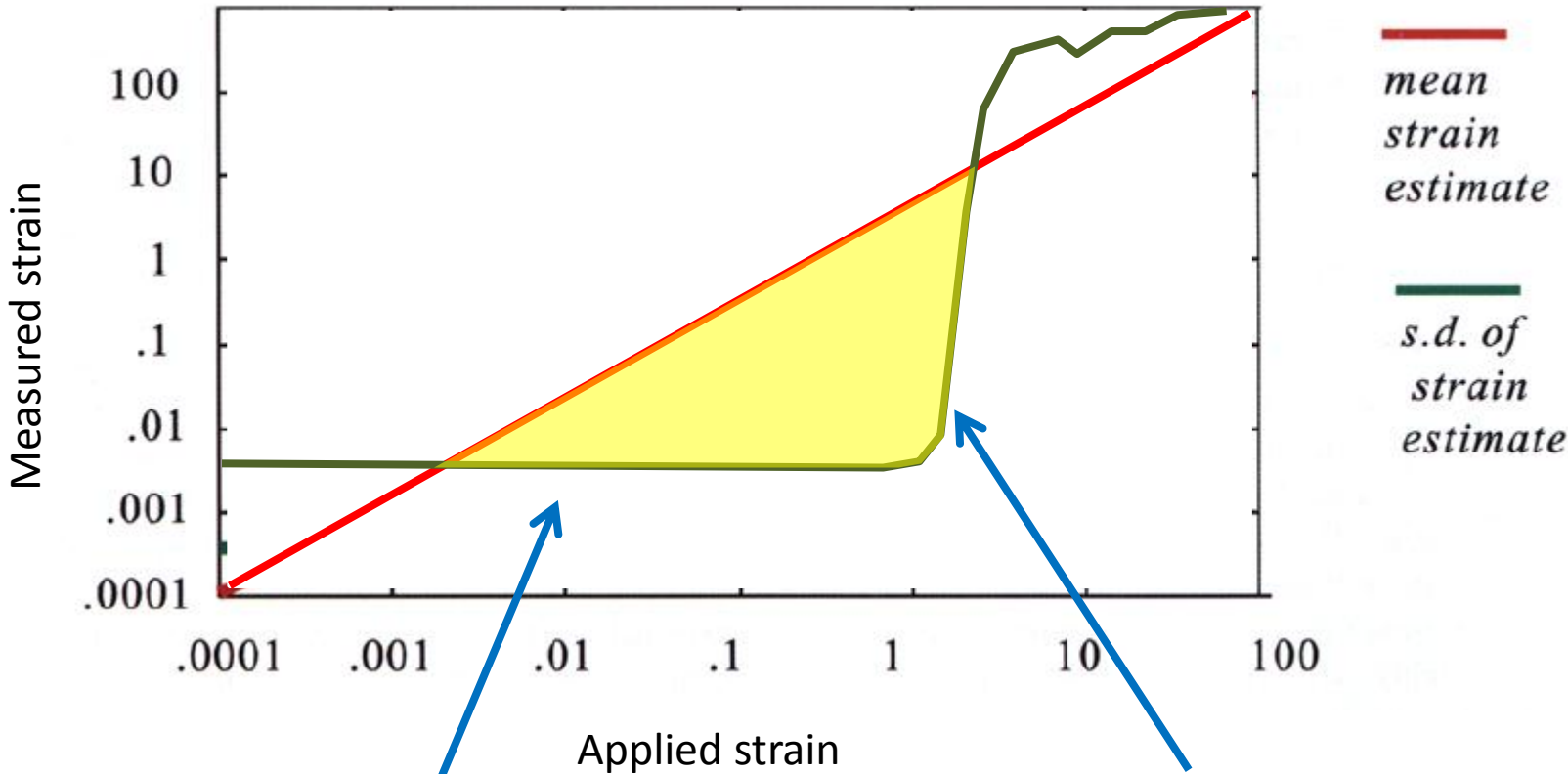
$\epsilon_{xy}$



# Depth and resolution



# OCT & DVC noise study



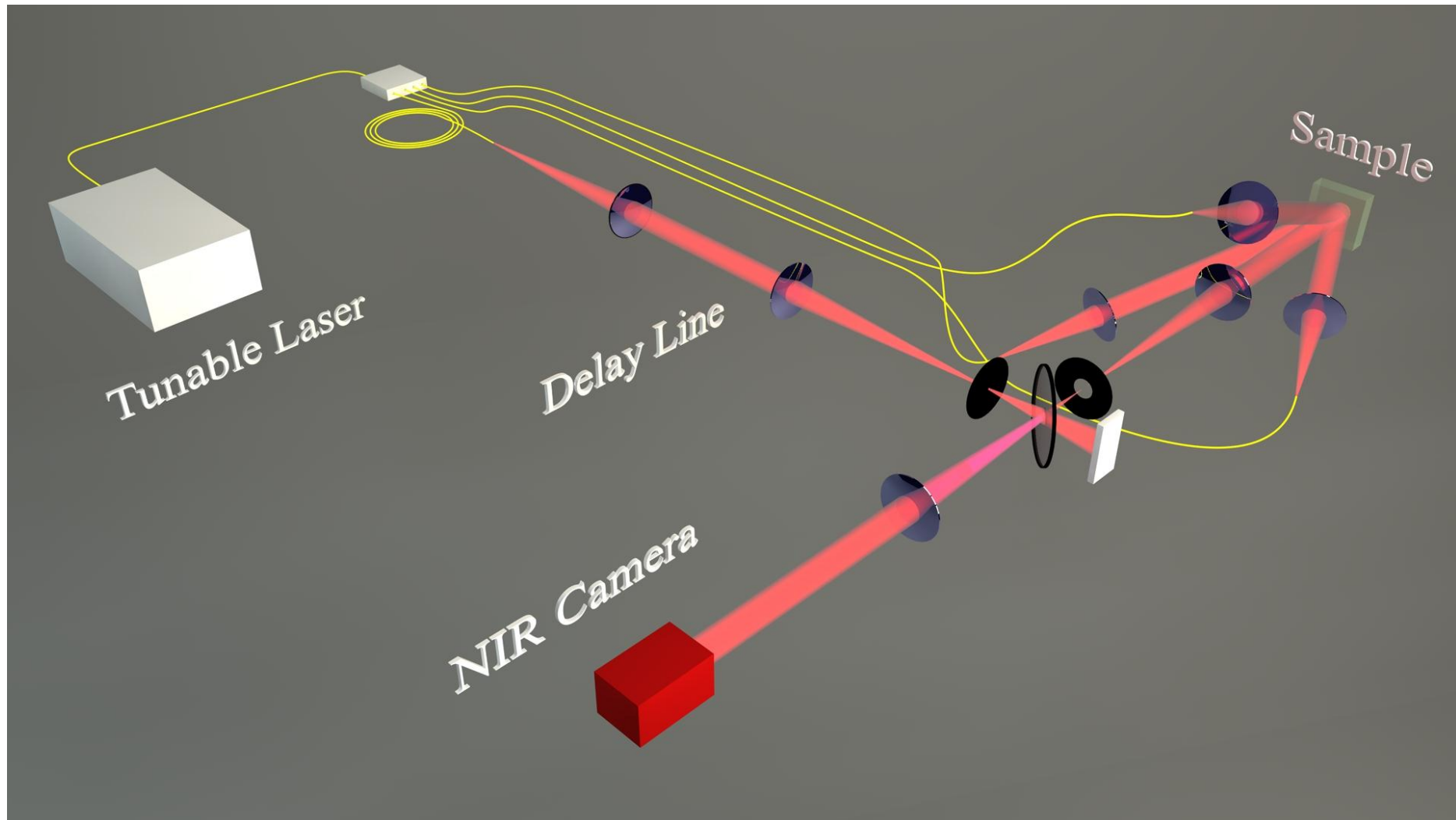
Systematic noise  
(intensity, electronic, phase jitter)  
strain~0.0005

Speckle decorrelation  
Strain sd~0.015

# **Wavelength scanning interferometry (phase detection) WSI**

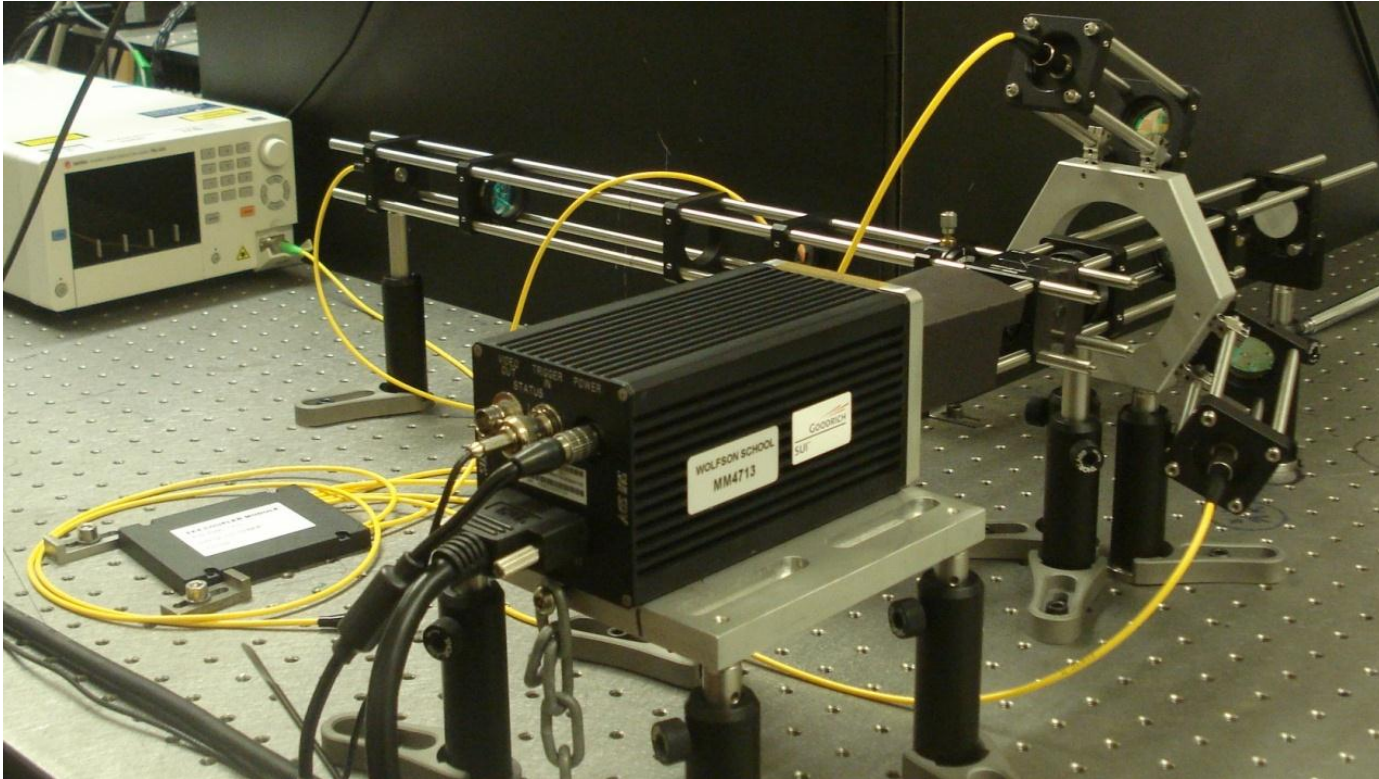


# WSI with multiple illumination directions



3 illumination directions with offset OPDs

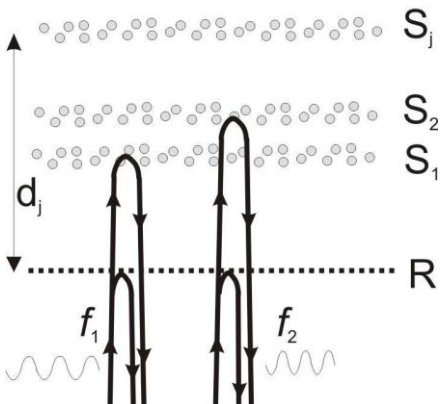
# WSI with multiple illumination directions



**Full paper:**

Chakraborty, S. and P.D. Ruiz, J. Opt. Soc. Am. A, 2012. **29**(9): p. 1776-1785.

# Opaque surface, one illumination beam



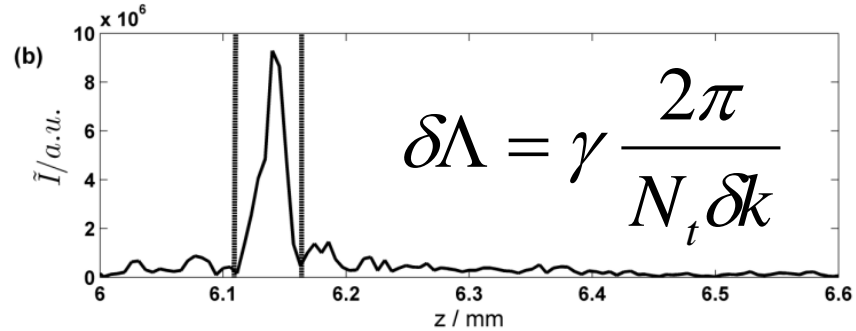
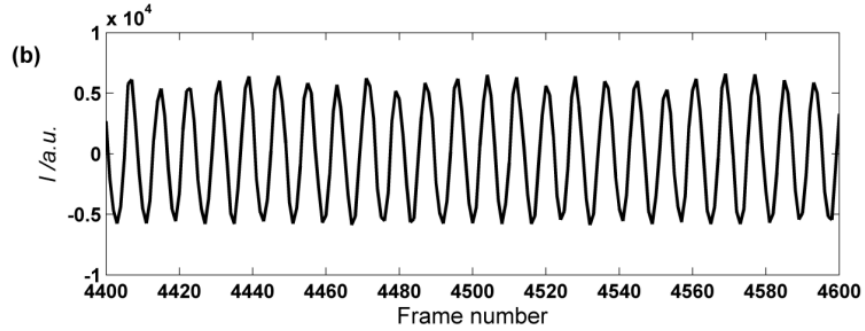
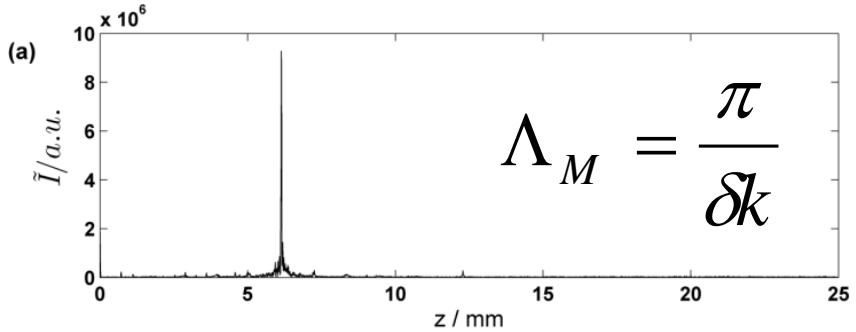
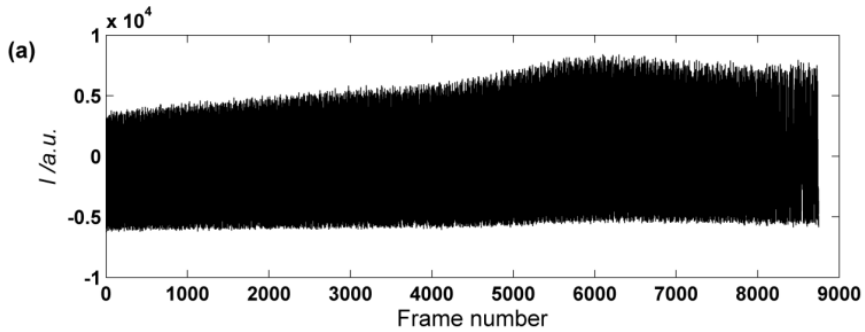
$$I(k) = I_0 + I_1 \cos[\phi(k)]$$

$$k(t) = k_c + \delta k t$$

$$\phi_j(t) = \phi_{sj} + k_c \Lambda_j + \delta k \Lambda_j t$$

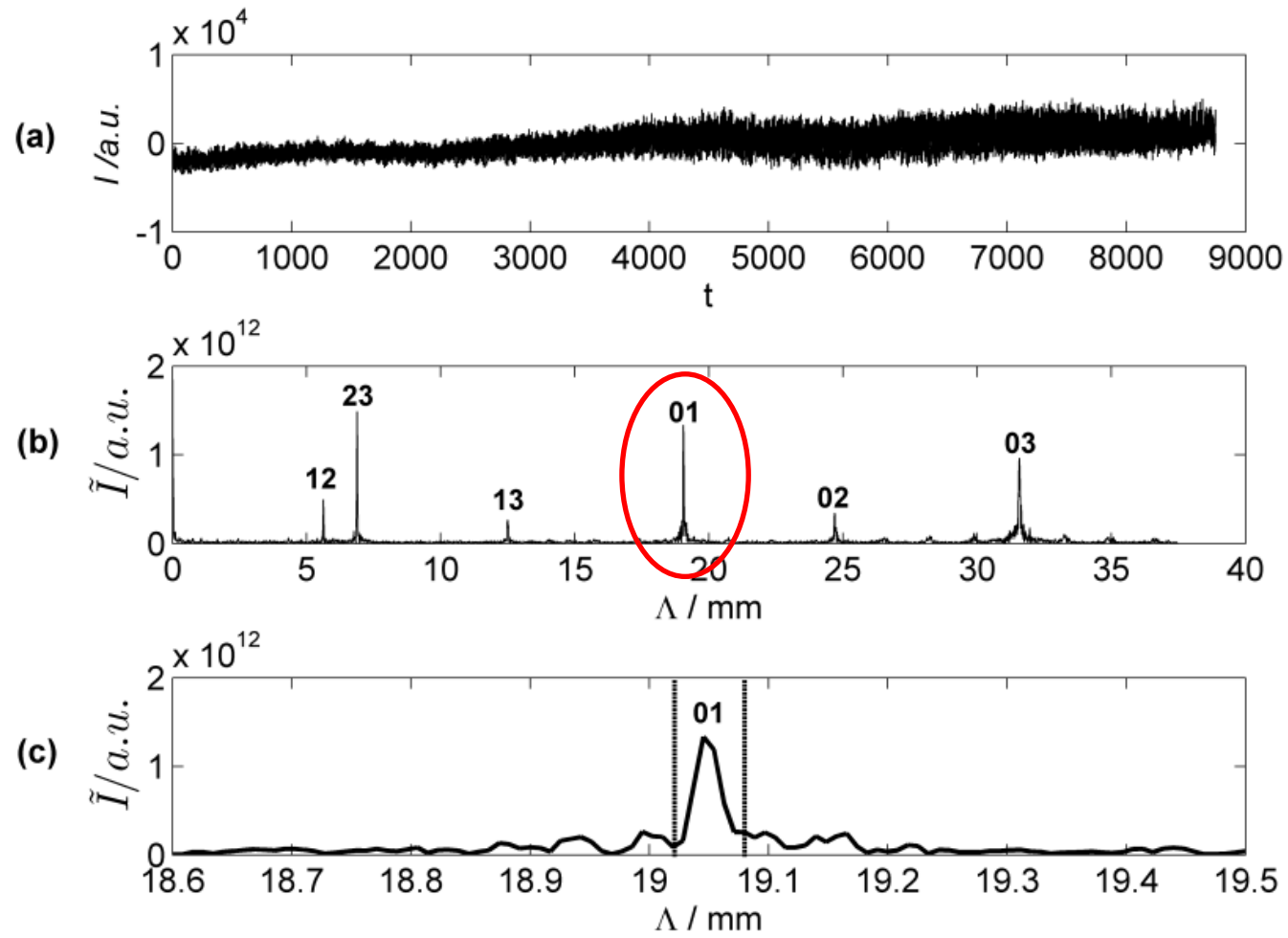
$$f_j = \frac{N \delta k}{2\pi} \Lambda_j$$

Frequency encodes depth!



# WSI with multiple illumination directions

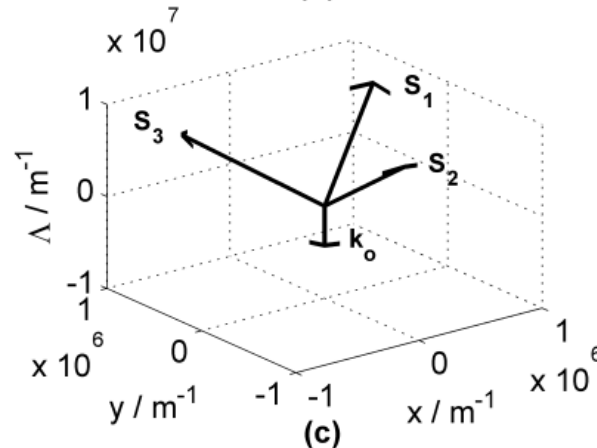
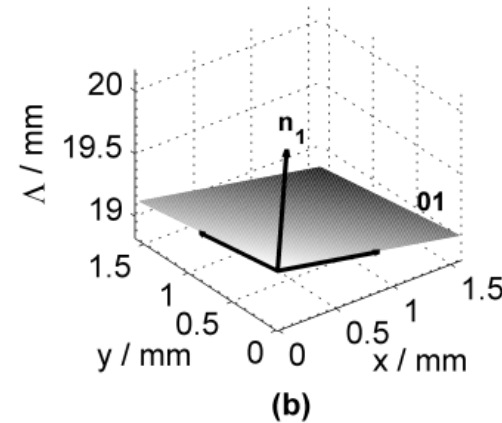
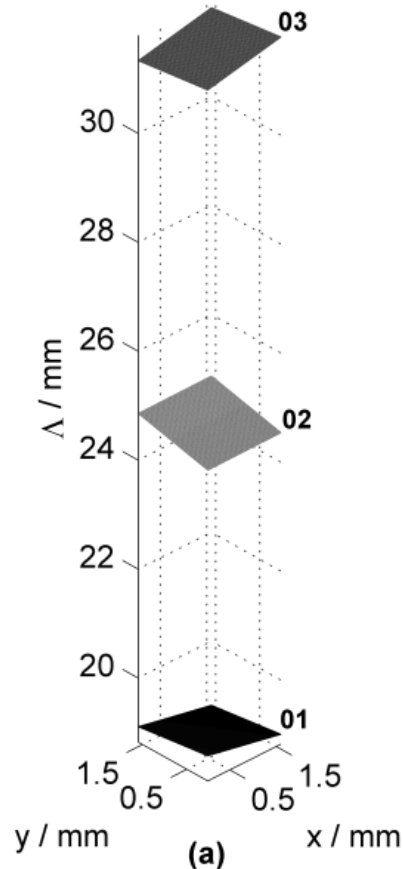
Signal for the case of an **opaque** surface



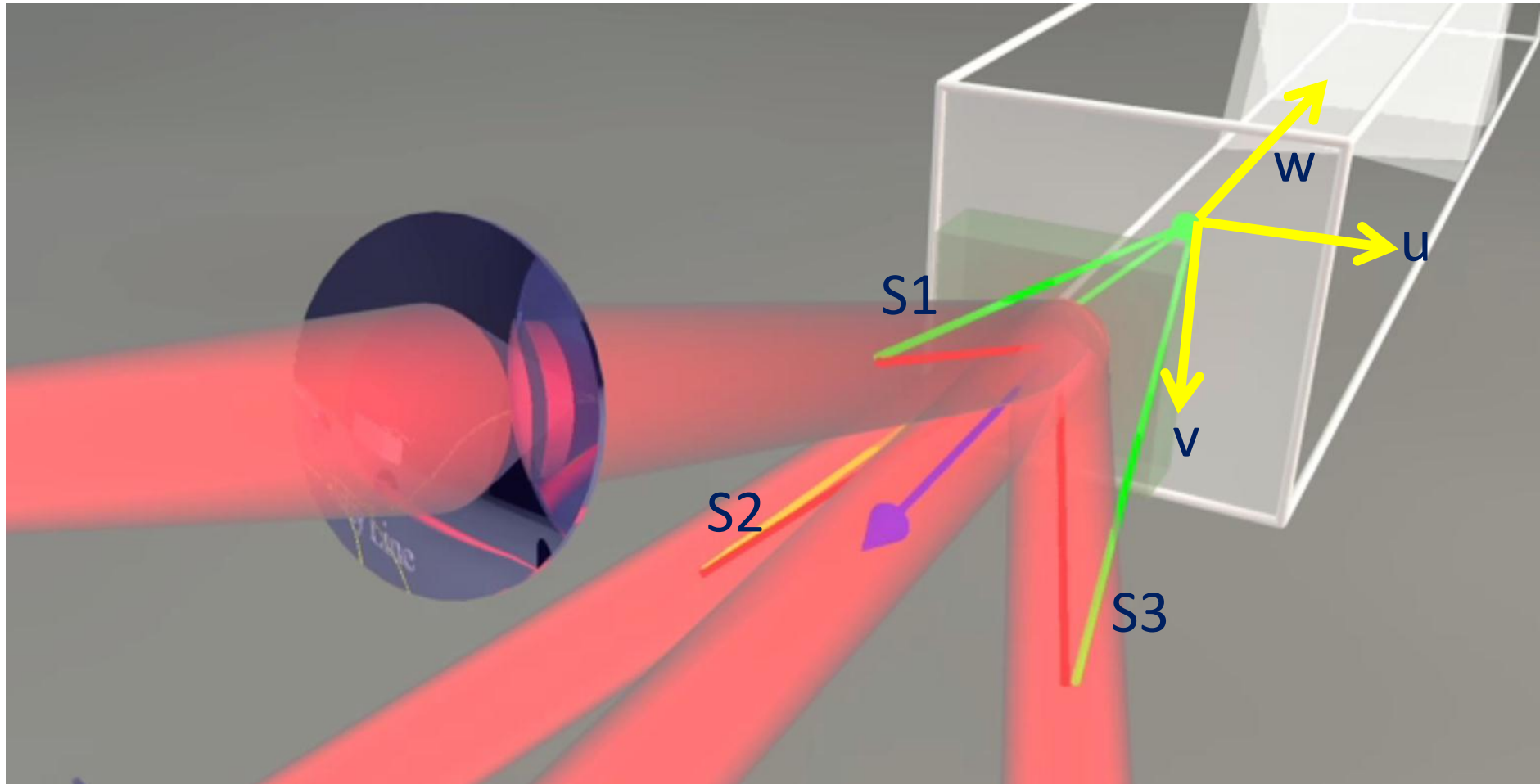
Theoretical  $\delta\Lambda=68 \mu\text{m}$ ; measured  $\delta\Lambda = 70 \mu\text{m}$

# Evaluation of the Sensitivity matrix

- Flat opaque scattering surface used as datum
- Record full scan and perform pixel-wise FFT
- Orientation of the reconstructed surfaces for each illumination is used to evaluate illumination and sensitivity vectors



# Vector transformation to find $u$ , $v$ , $w$



# 2) phase unwrapping and displacements

$$\begin{pmatrix} \phi_1^U(x, y, \Lambda) \\ \phi_2^U(x, y, \Lambda) \\ \phi_3^U(x, y, \Lambda) \end{pmatrix} = \frac{2\pi}{\lambda_c} \begin{pmatrix} \sin \theta_1 \cos \xi_1 & \sin \theta_1 \sin \xi_1 & 1 + \cos \theta_1 \\ \sin \theta_2 \cos \xi_2 & \sin \theta_2 \sin \xi_2 & 1 + \cos \theta_2 \\ \sin \theta_3 \cos \xi_3 & \sin \theta_3 \sin \xi_3 & 1 + \cos \theta_3 \end{pmatrix} \begin{pmatrix} u(x, y, \Lambda) \\ v(x, y, \Lambda) \\ w(x, y, \Lambda) \end{pmatrix}$$

measured

Known from calibration

calculated

$$\Phi = S \cdot \Delta r$$

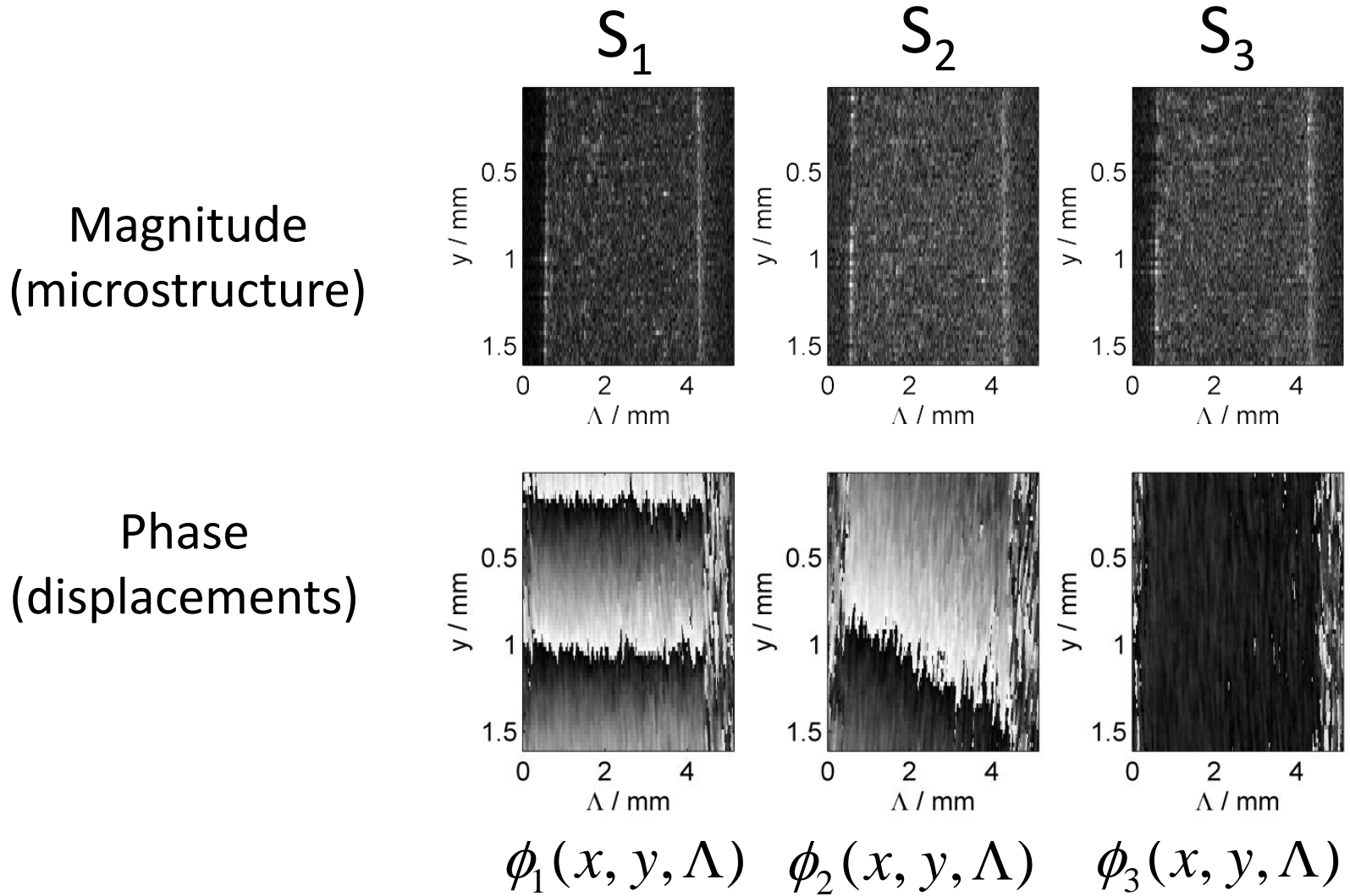
$u$ ,  $v$  and  $w$  are obtained by inverting the sensitivity matrix

**Unwrapping algorithm:**

Salfity, M.F., et al., Applied Optics, 2006. **45**(12): p. 2711-2722.

# Volume reconstruction: 1) Re-registration

- The complex volumes associated to all 3 sensitivity vectors are re-registered to a common coordinate system.



Magnitude  
(microstructure)

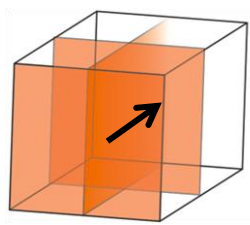
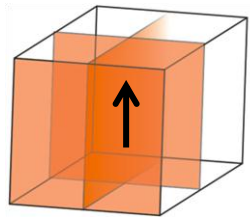
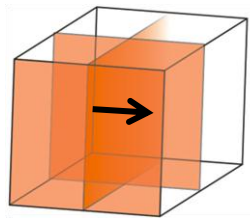
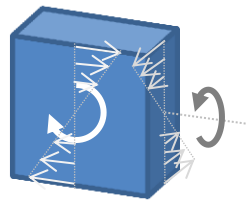
Phase  
(displacements)



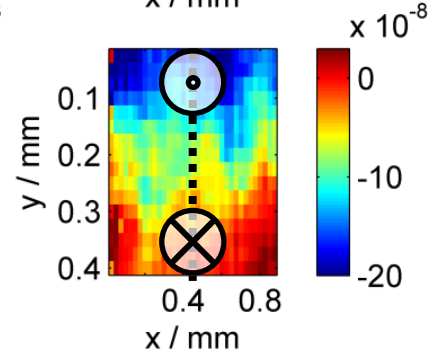
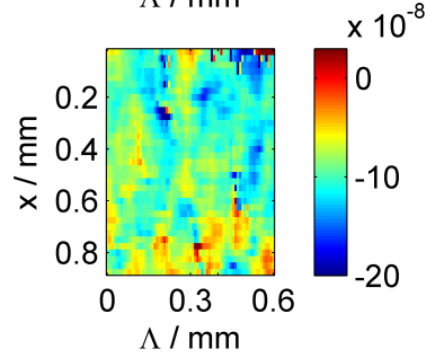
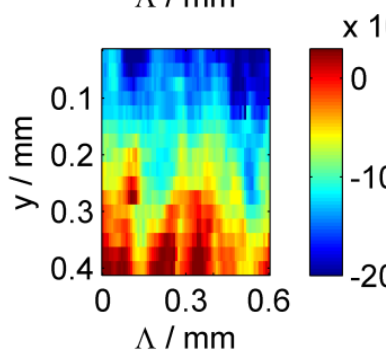
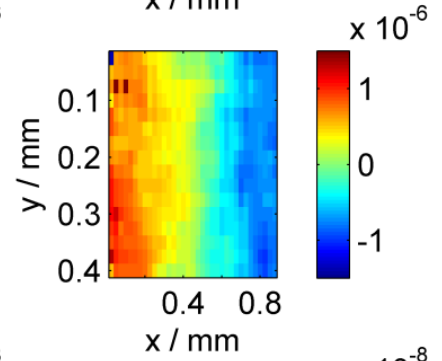
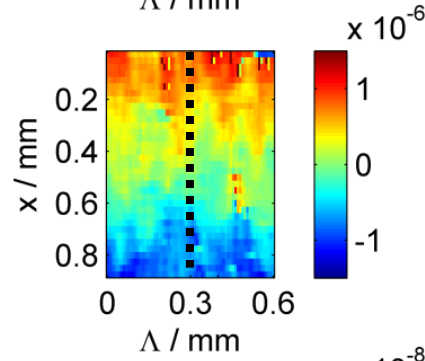
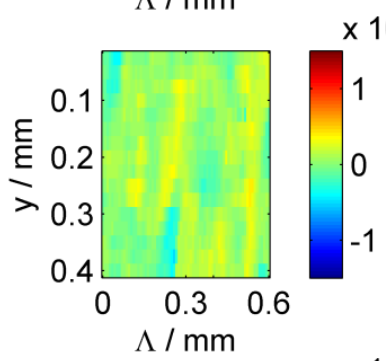
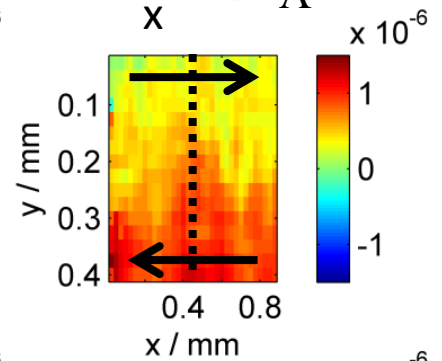
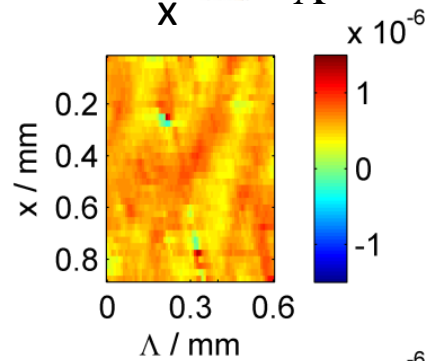
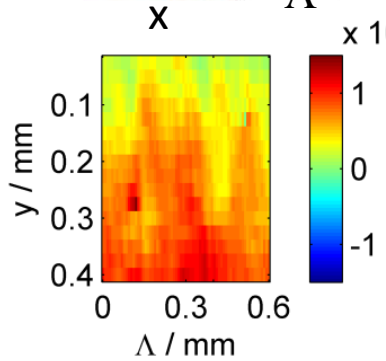
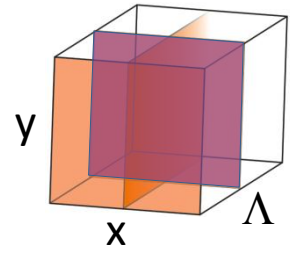
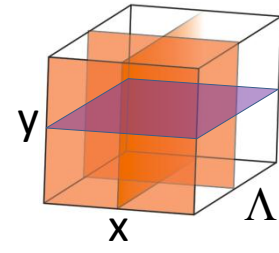
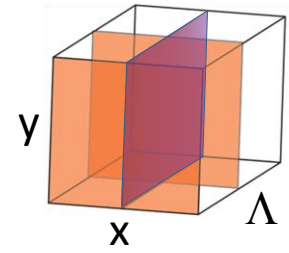
# Validation of $u(x, y, z)$ ; $v(x, y, z)$ and $w(x, y, z)$

In-plane rotation + rotation +

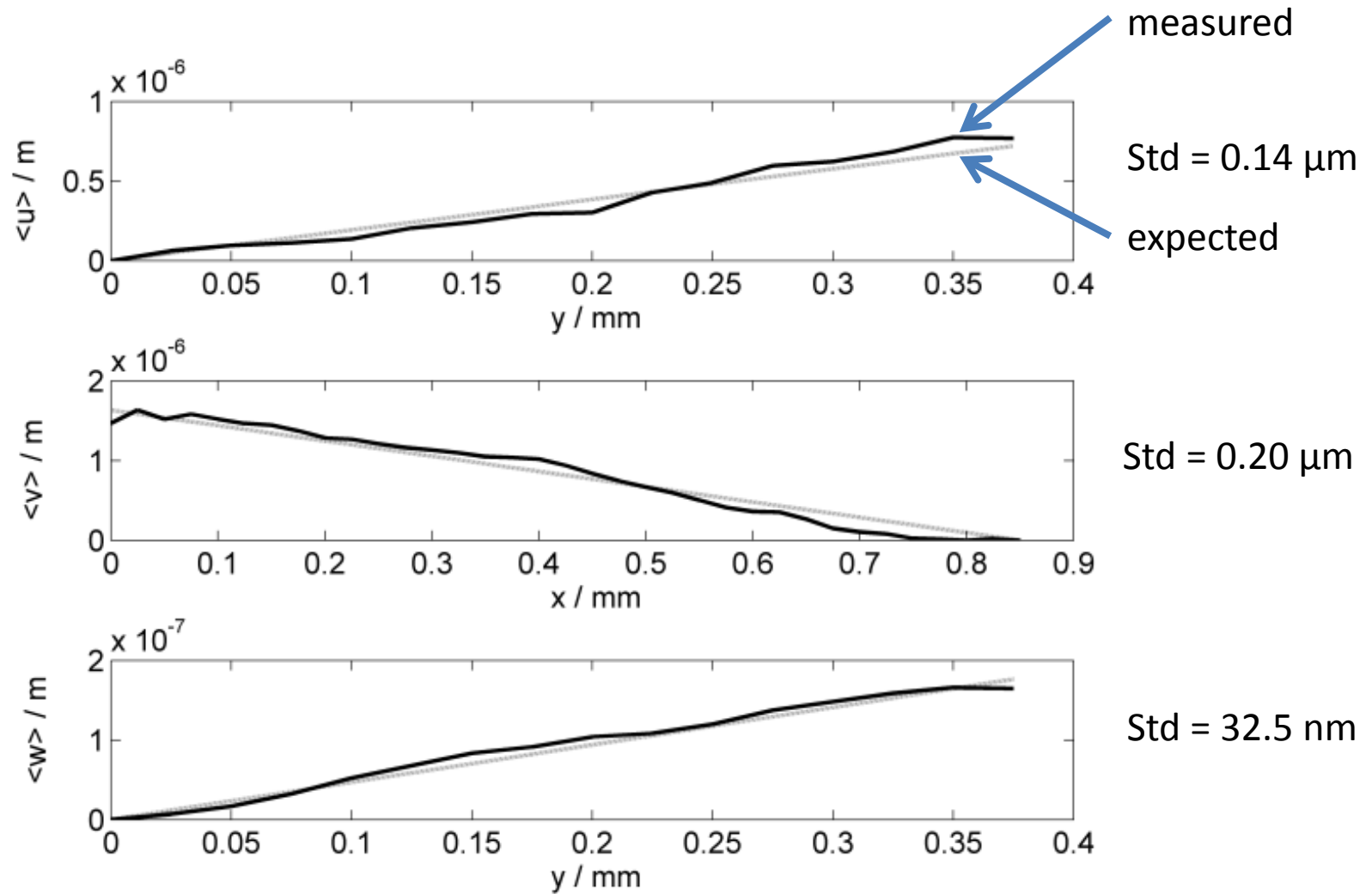
Out-of-plane tilt



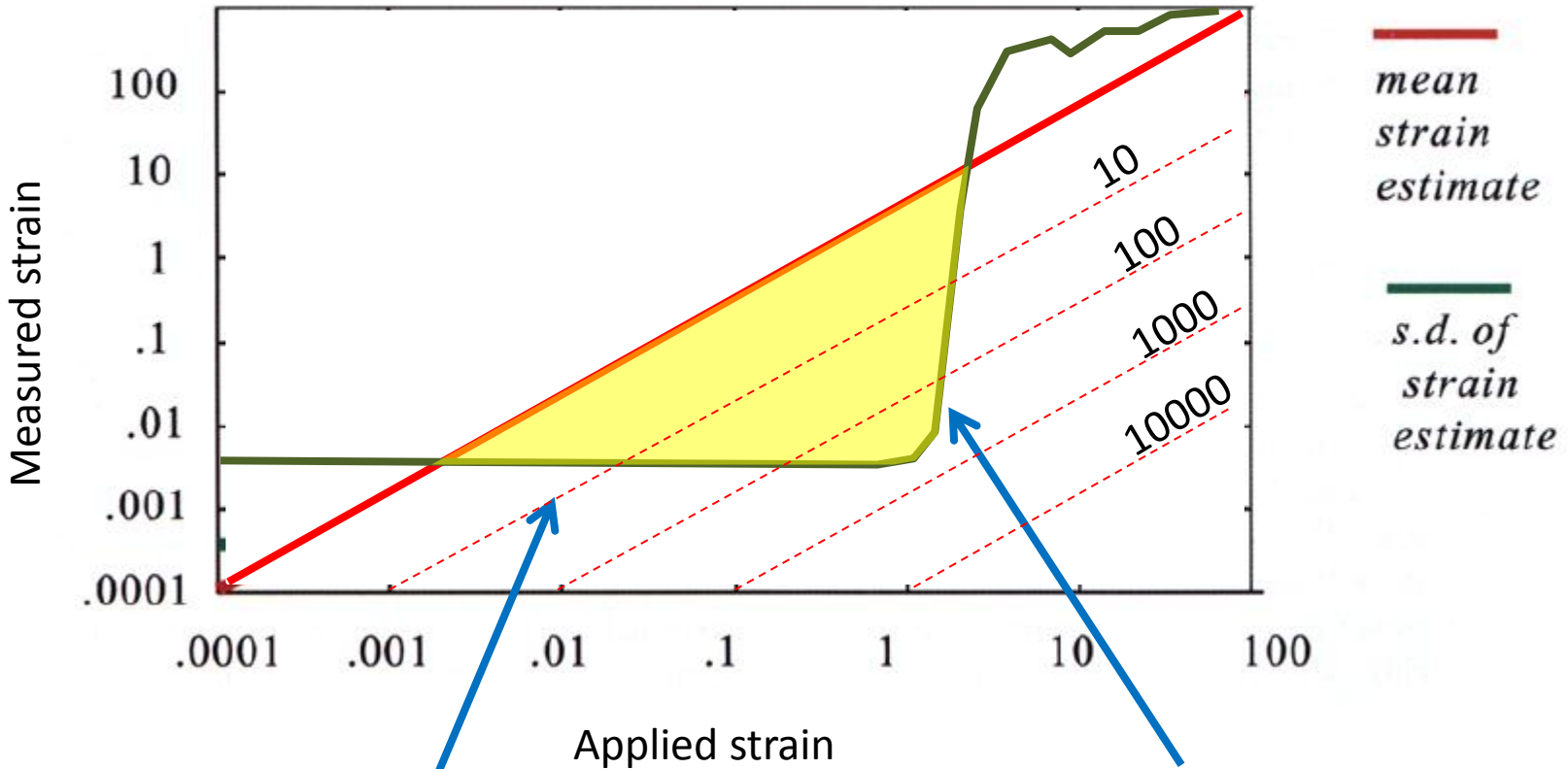
Volume shown  $\sim 0.2 \text{ mm}^3$



# Validation results



# OCT & DVC noise study

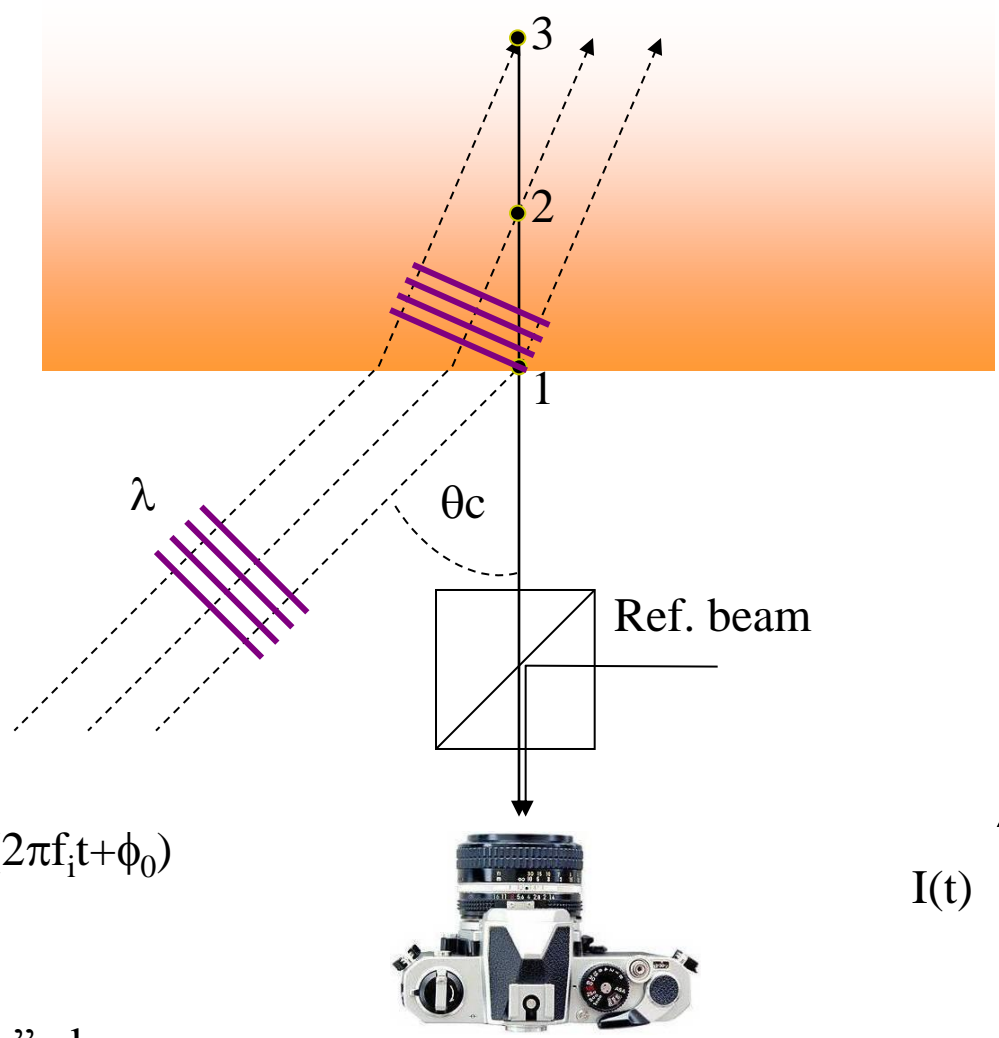


Systematic noise  
(intensity, electronic, phase jitter)  
strain~0.0005

Speckle decorrelation  
Strain sd~0.015

# **Tilt Scanning Interferometry (phase detection) TSI**

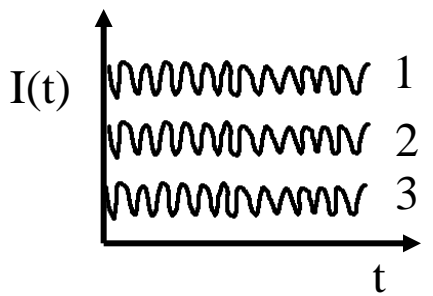
# Tilt Scanning Interferometry: Principle



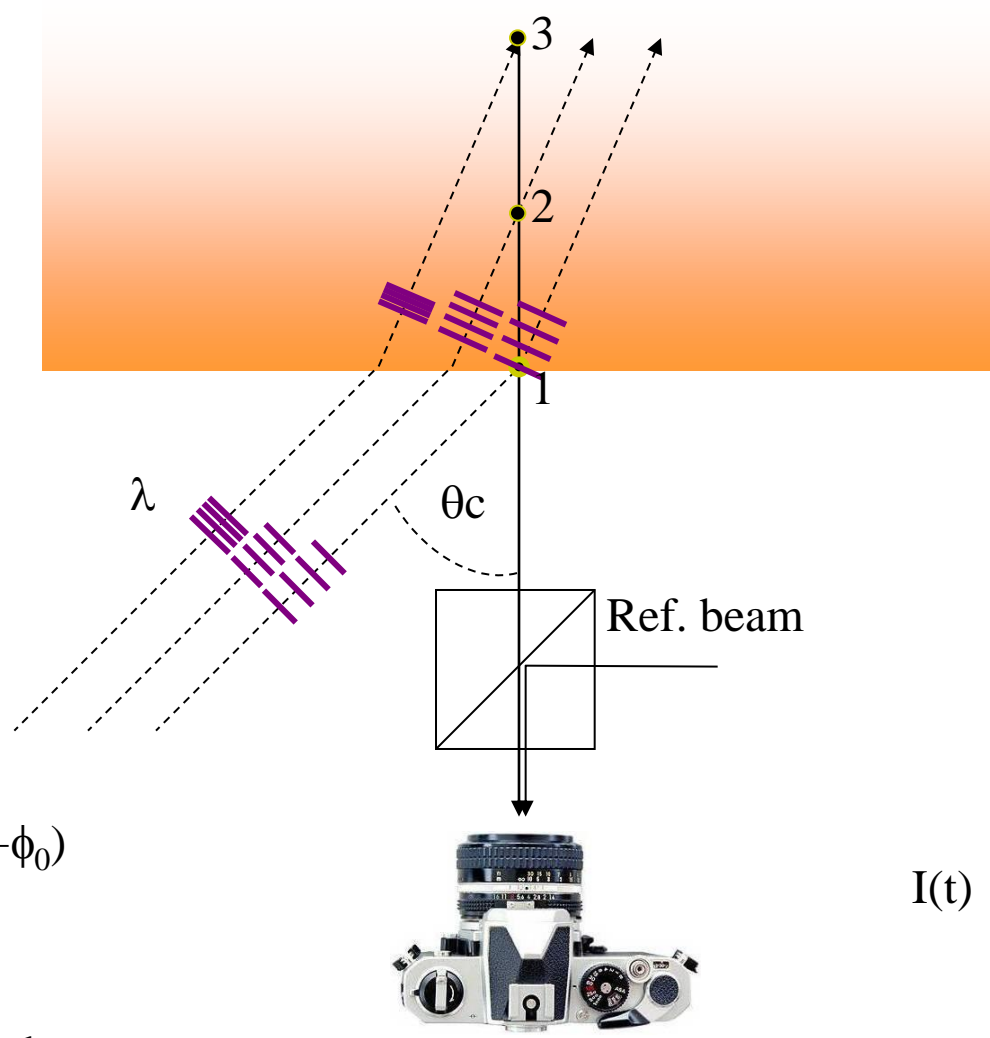
$$I_i(t) = I_0 + I_M \cos(2\pi f_i t + \phi_0)$$

$$f_1 = f_2 = f_3$$

Effect of a "piston" phase



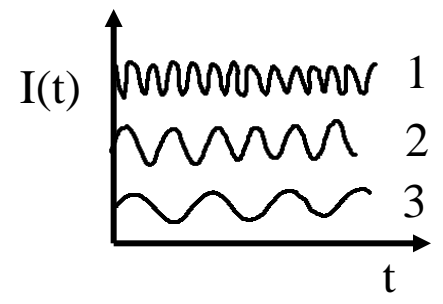
# Tilt Scanning Interferometry: Principle



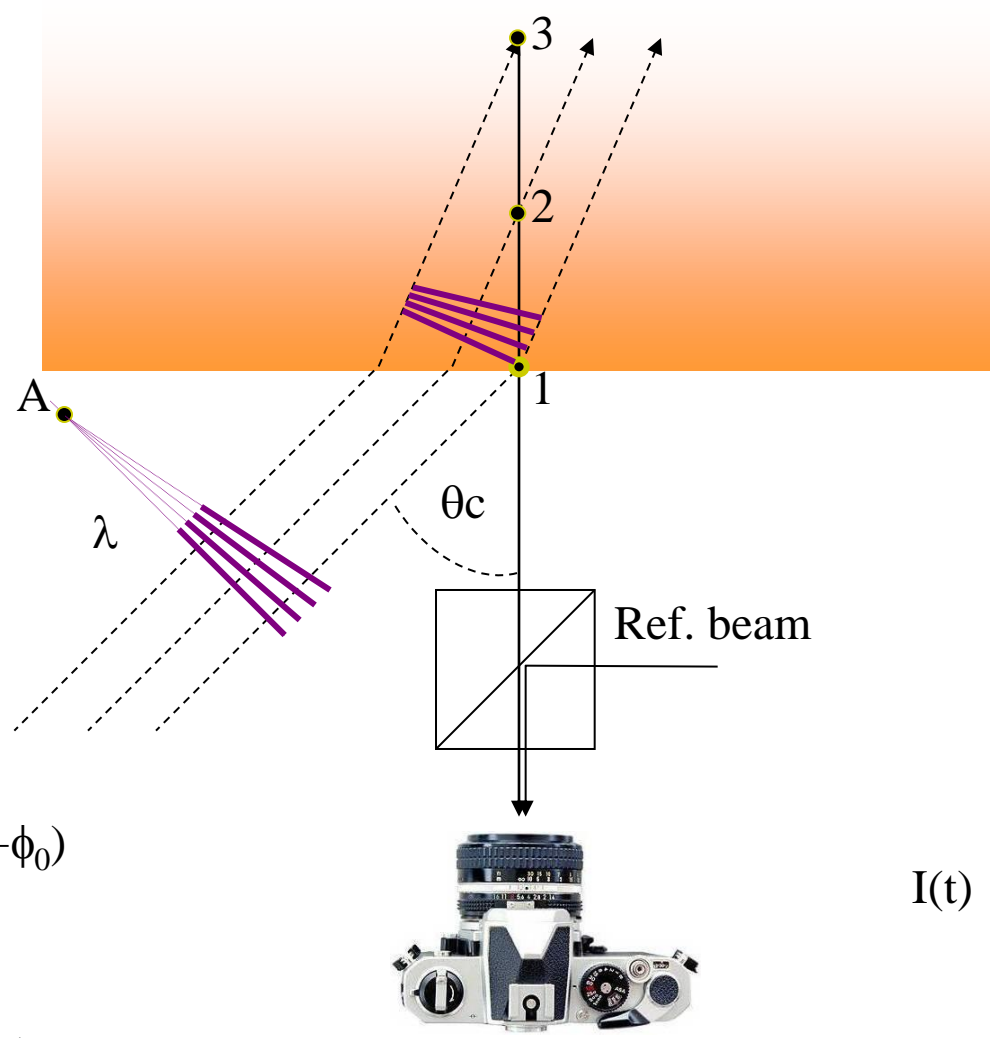
$$I_i = I_0 + I_M \cos(2\pi f_i t + \phi_0)$$

$$f_1 > f_2 > f_3$$

Piecewise “piston” phase



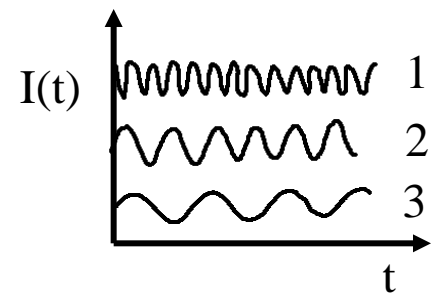
# Tilt Scanning Interferometry: Principle



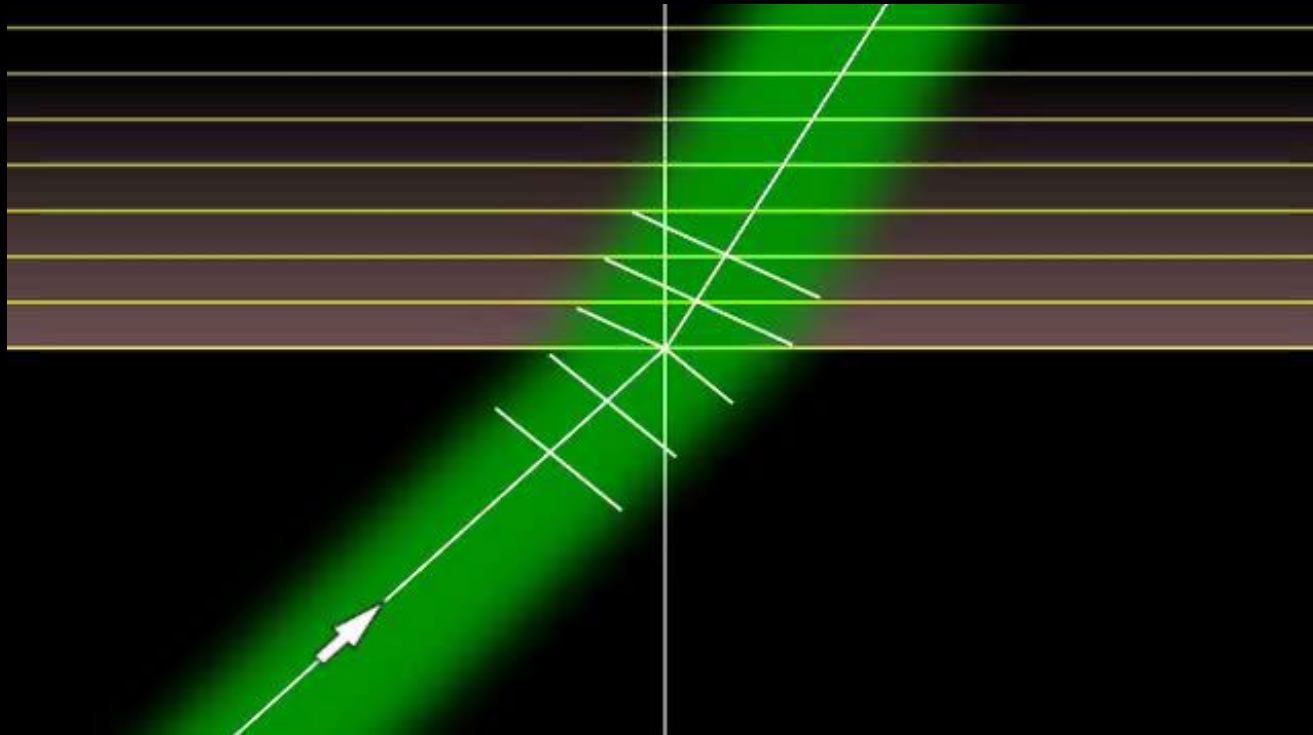
$$I_i = I_0 + I_M \cos(2\pi f_i t + \phi_0)$$

$$f_1 > f_2 > f_3$$

Wavefront tilt about A

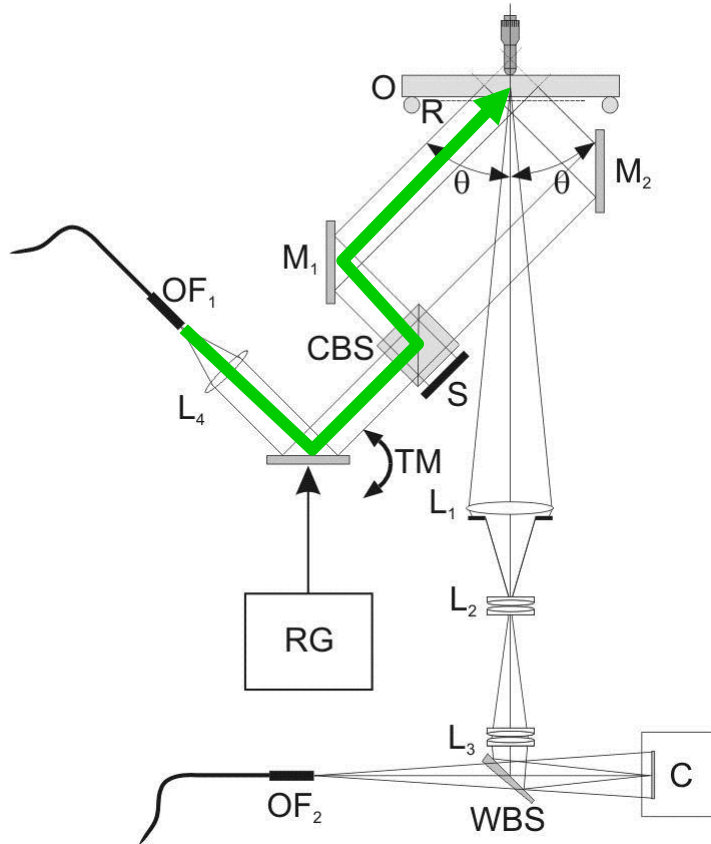


# Tilt Scanning Interferometry: Principle



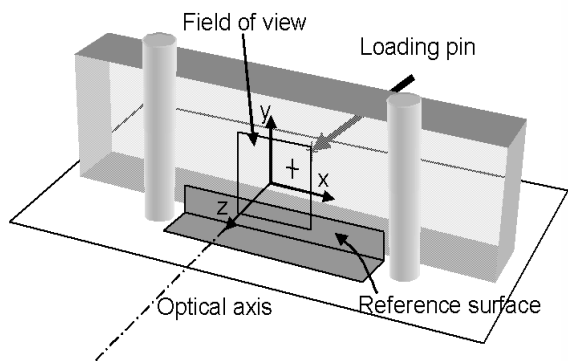


# Tilt scanning interferometry: Setup

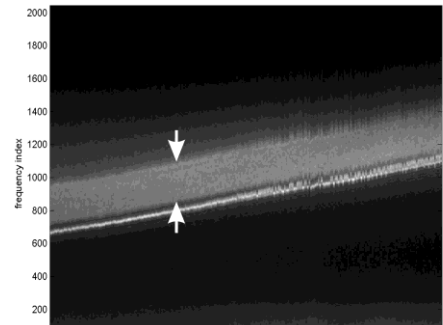




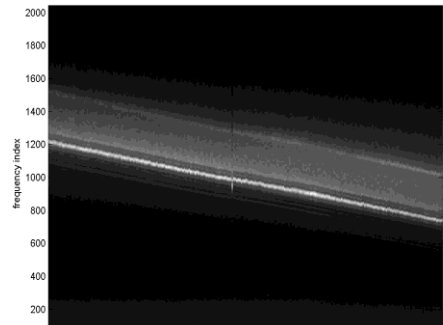
# Wrapped phase maps



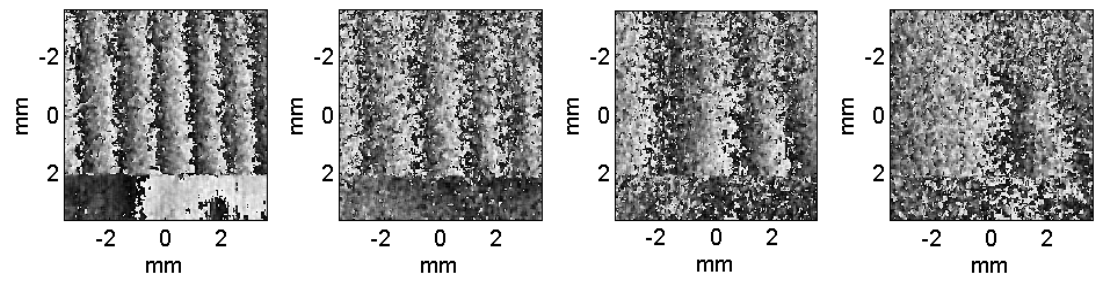
Left illumination



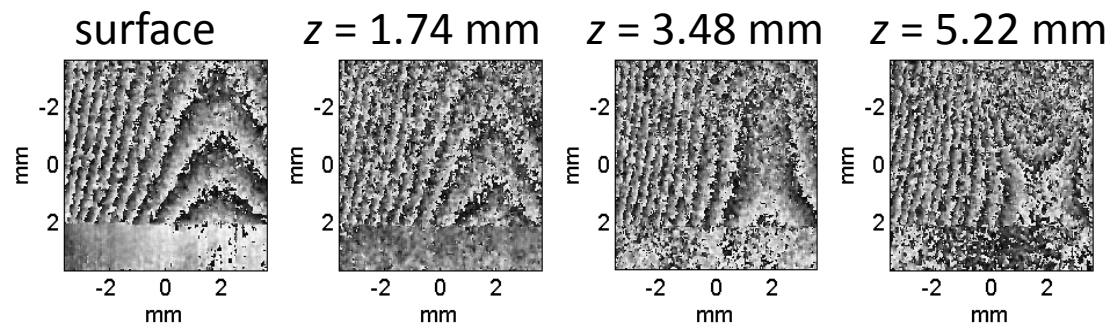
Right illumination



Horizontal  
in-plane  
displacement  
component

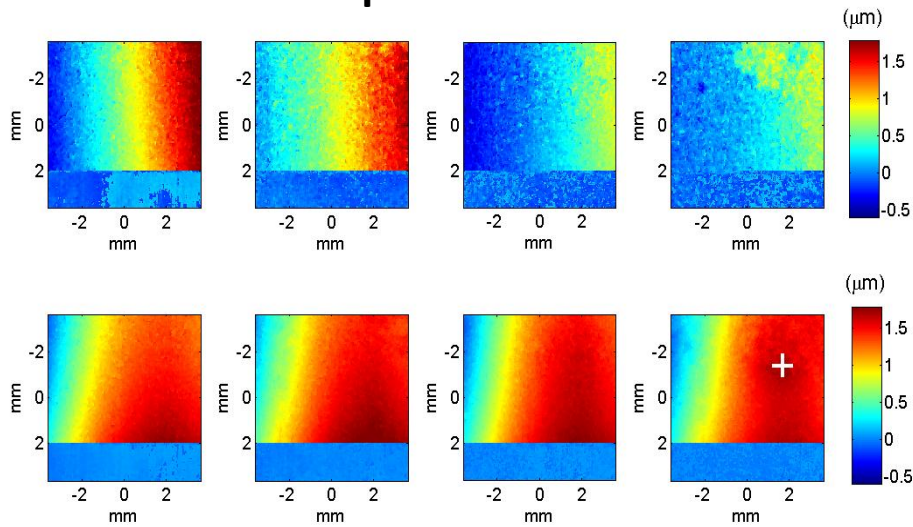


Out-of-plane  
displacement  
component



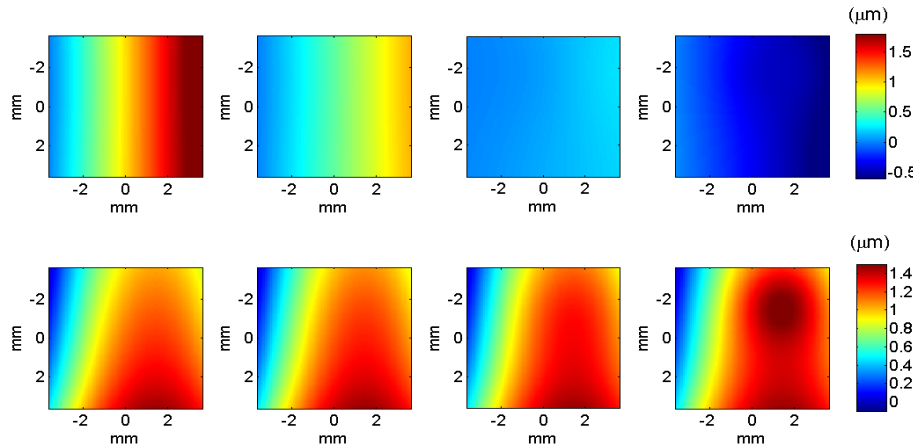
# Displacement fields – experimental vs FEA

## Experiment

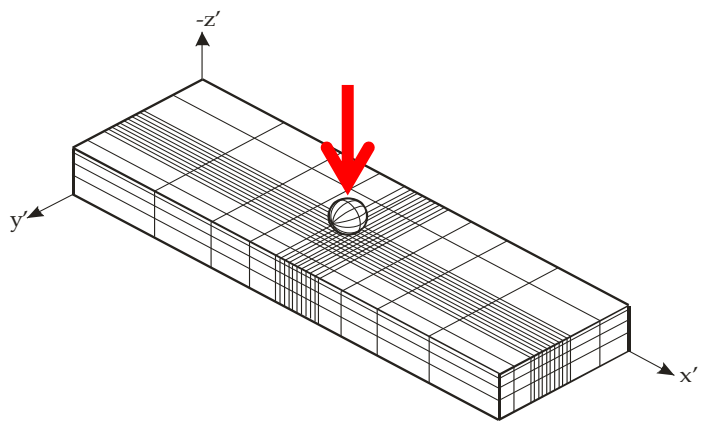


Horizontal  
in-plane  
displacement  
component

Out-of-plane  
displacement  
component

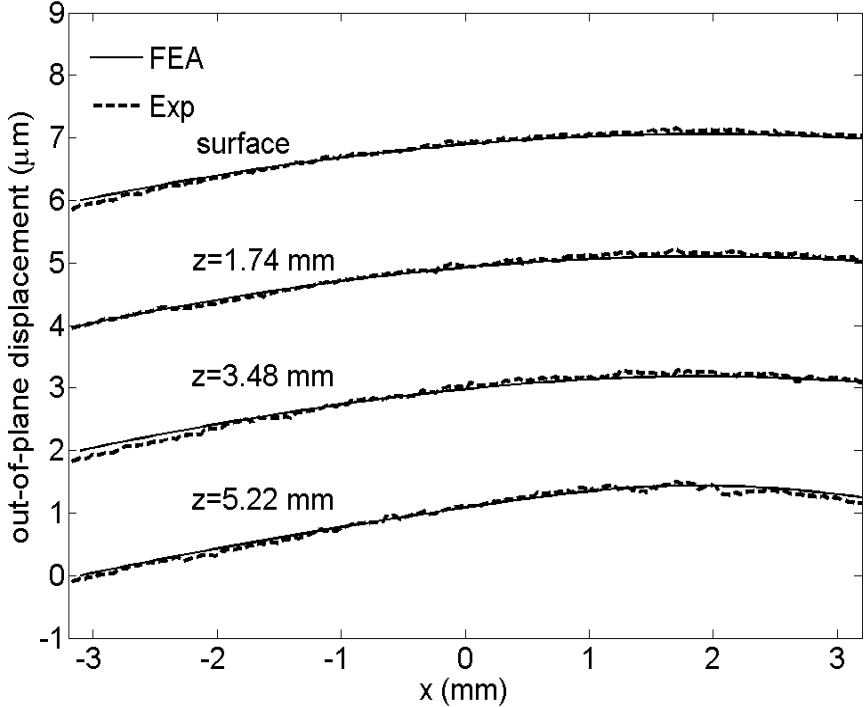


## Finite Element Analysis

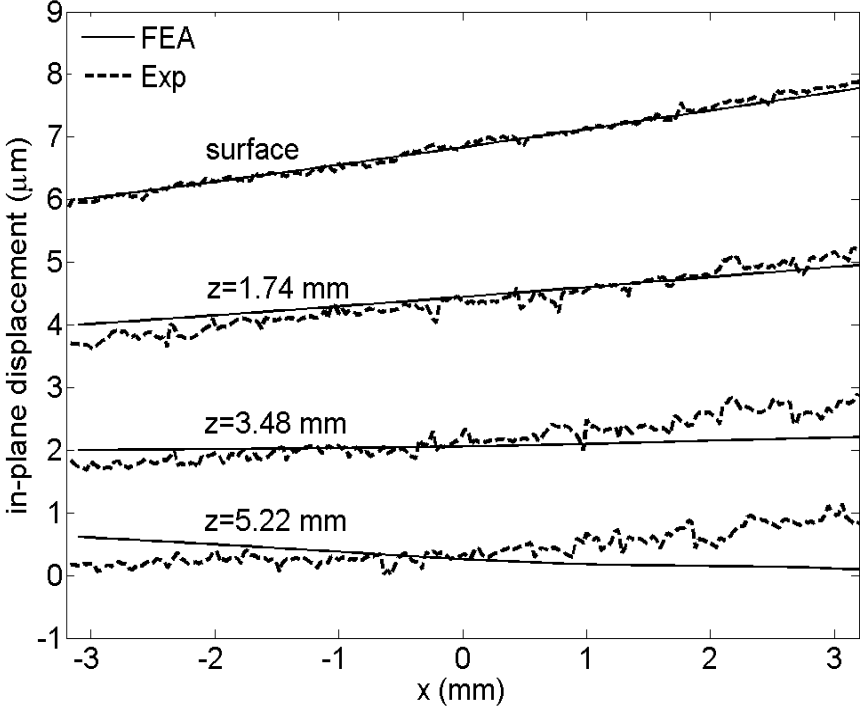


# Horizontal cross-sections through contact point

### Out-of-plane displacement



### In-plane displacement



# Summary of measurement techniques

Technique	Materials	Minimum size of gauge volume	Displacement/strain sensitivity	Acquisition time
3D strain gauge	polymer model	$> 1 \text{ mm}^3$	$\sim 10\text{e-}06$	$\sim 1 \mu\text{s}$ (1 point)
Ultrasound	Tissues, metals, composites	$\sim 1 \text{ mm}^3$	$\sim 10\text{e-}03$	$\sim$ and below 1 s 3-D US
MRE	Proton rich water-fat carbon possible?	$(250 \mu\text{m})^3$	$< 1 \mu\text{m}$	4 ms/slice (spin tagging) Few minutes/slice (phase contrast)
Xray CT+DVC	foams, ceramics, granular materials, composites, bone	$(1\text{-}10 \mu\text{m})^3$ (structure) $(10\text{-}100 \mu\text{m})^3$ (displacement) $(30\text{-}300 \mu\text{m})^3$ (strain)	$\sim 10\text{e-}04 - 10\text{e-}06$	tens of minutes
OCT Phase contrast	optically translucent	$< (10 \mu\text{m})^3$	$< 1 \mu\text{m}$	$10 \mu\text{s}$ A-scan (Spectral domain)
OCT + DVC	optically translucent	$\sim (50 \mu\text{m})^3$	$\sim 10\text{e-}04$	$\sim 10$ minutes per scan (full volume)

**Thank you for your attention!**