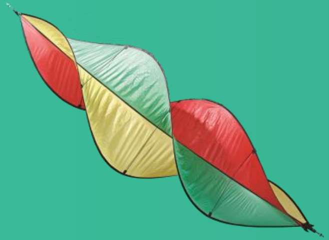
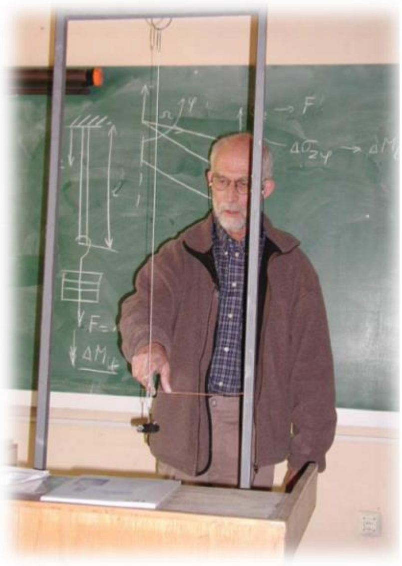


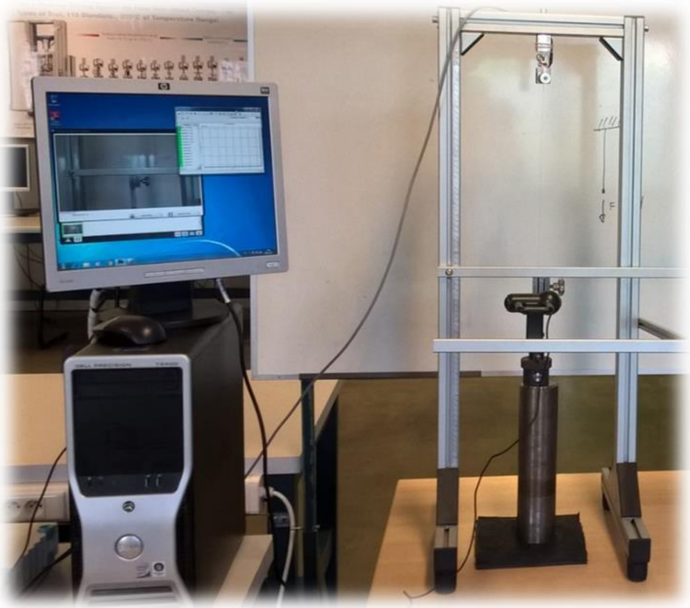
General introduction to stress reconstruction in metal plasticity

Sam Coppieters & Marco Rossi

The mystery of plastic deformation[1]



Stan Hoogenboon, Ankara 2002



[1] E.A. Tekkaya, JSTP International Prize Lecture “*The mystery of plastic deformation*”, ICTP 2014, Nagoya, Japan

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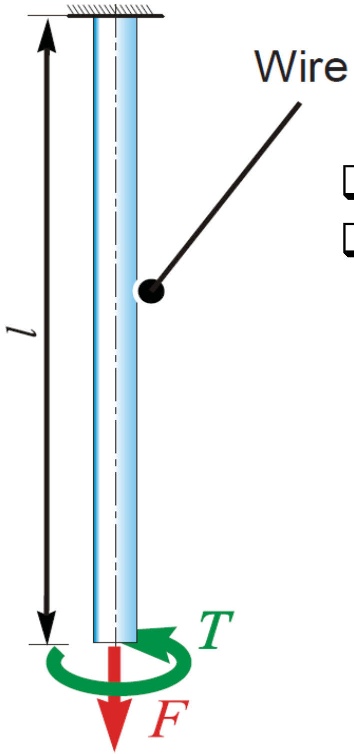
Experiment

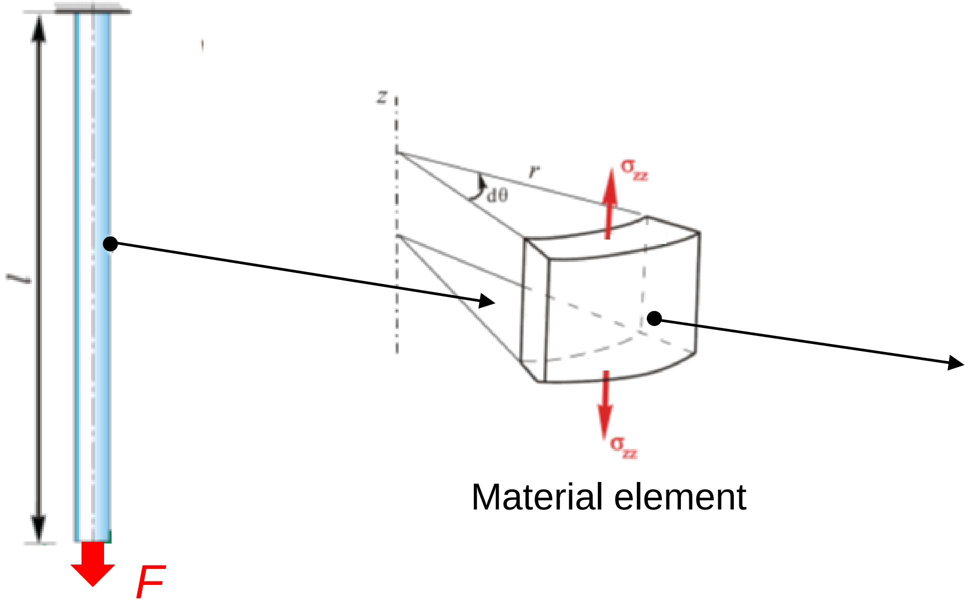


- Step 1: Attach weight until the wire elongates plastically
- Step 2: Rotate the weight

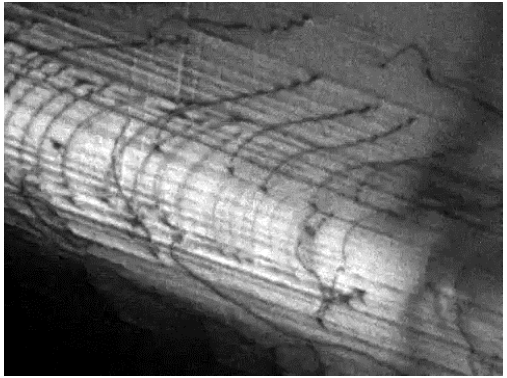
Experiment: Step 2

Analysis: Step 1

- Uniaxial tension causes yielding and plastic elongation of the wire



Material element



Movement of dislocations

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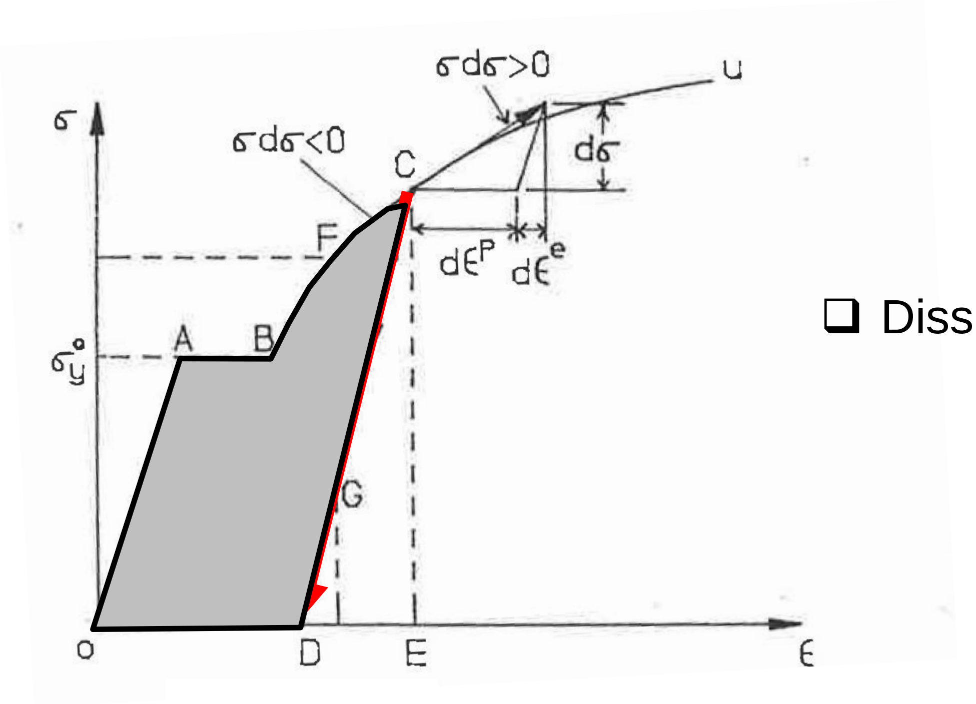
Uniaxial Tension:
Phenomenological macroscopic response of metals



- Dissipation of Energy

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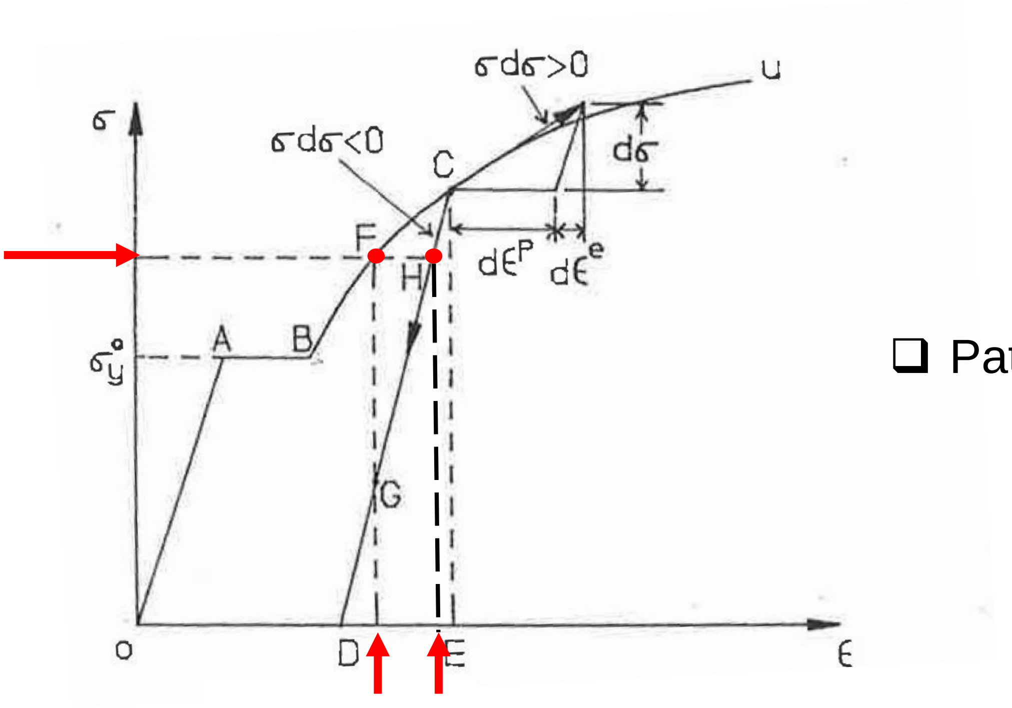
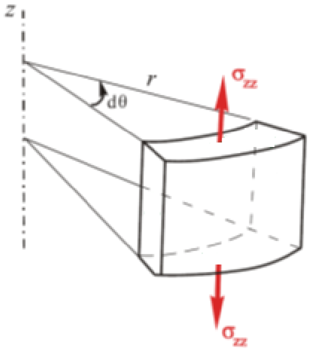
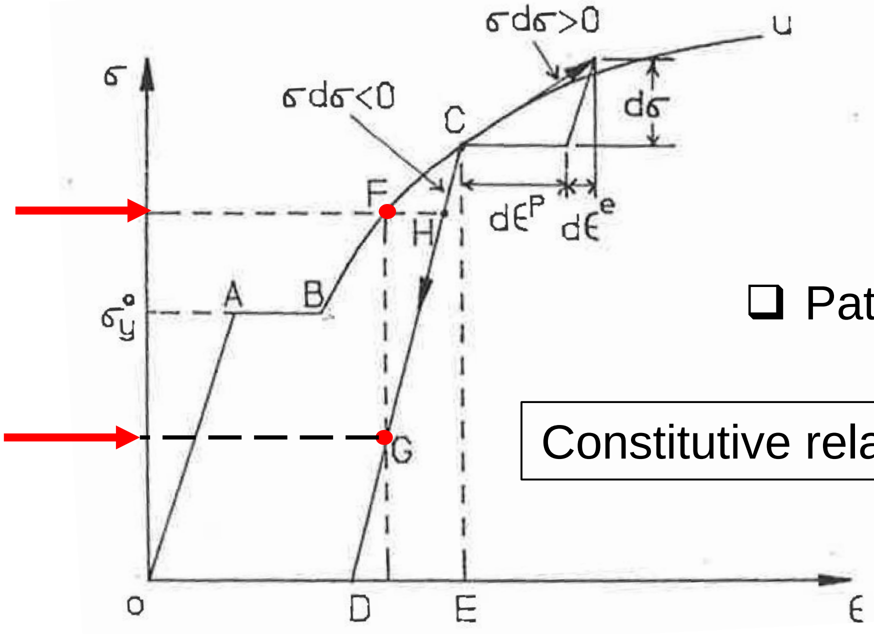
Uniaxial Tension:
Phenomenological macroscopic response of metals



- ❑ Path Dependent

Uniaxial Tension:
Phenomenological macroscopic response of metals

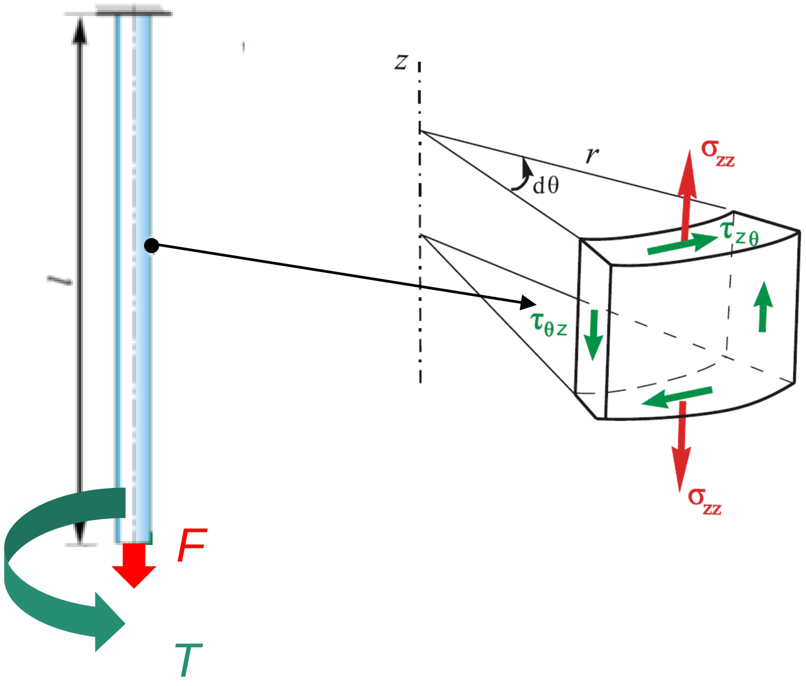




- Path Dependent

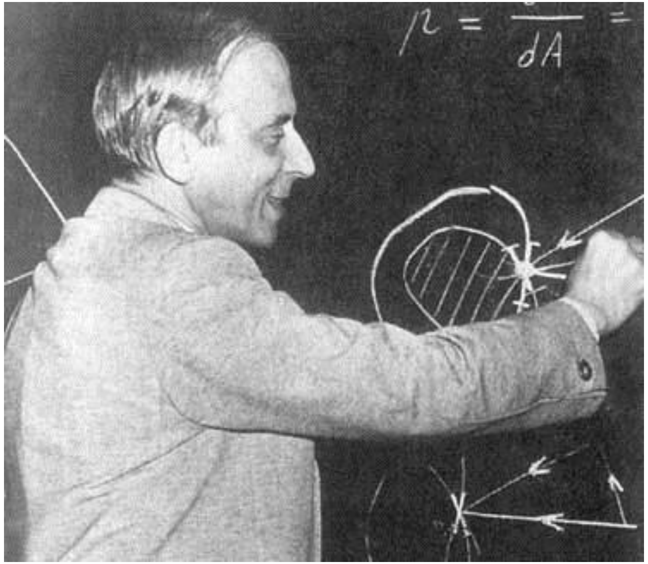
Constitutive relations in incremental form

Analysis: Step 2



Ingredient 1: Yield criterion

von Mises (1931)



$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{\frac{1}{2}}$$

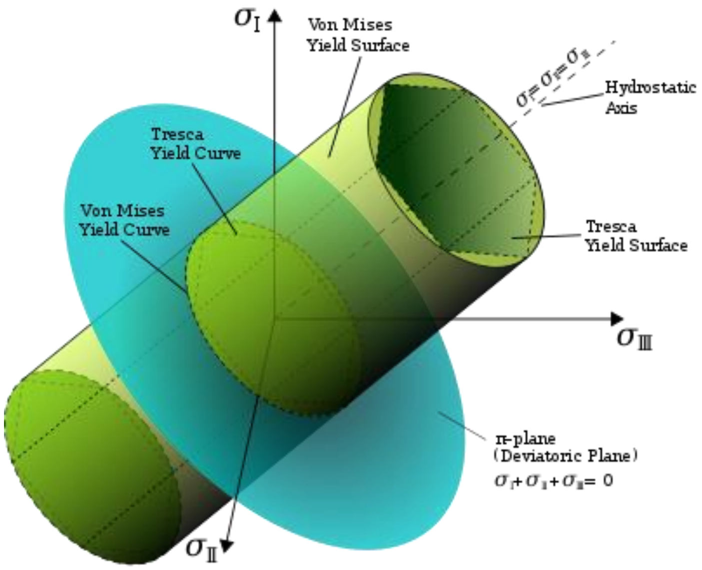
where are the dislocations?

$$\sigma_e = \left[\frac{3}{2}\boldsymbol{\sigma}':\boldsymbol{\sigma}'\right]^{\frac{1}{2}}$$

Deviatoric stress tensor = Shear = Slip = movement of dislocations

Ingredient 1: Yield criterion

von Mises (1931)



$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{\frac{1}{2}}$$

where are the dislocations?

$$\sigma_e = \left[\frac{3}{2}\boldsymbol{\sigma}':\boldsymbol{\sigma}'\right]^{\frac{1}{2}}$$

Deviatoric stress tensor = Shear = Slip = movement of dislocations

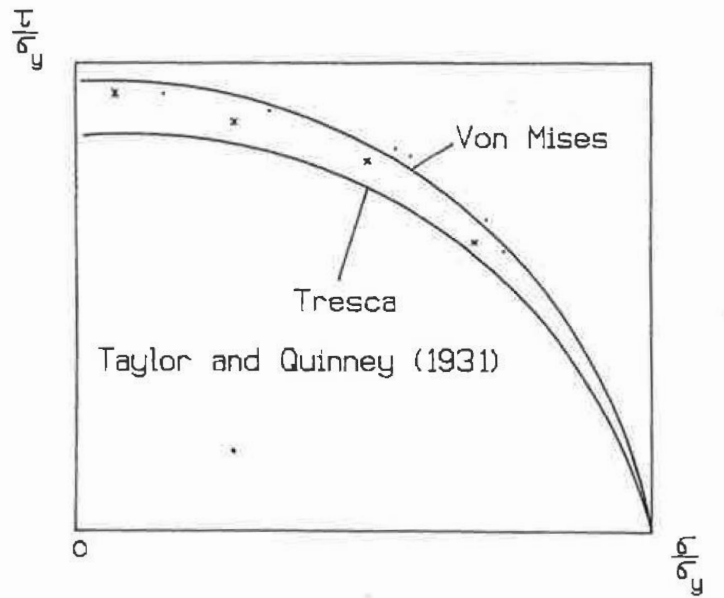
Ingredient 1: Yield Criterion

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+6(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2)\right]^{\frac{1}{2}}$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[2\,\sigma_{zz}{}^2 + 6\,\tau_{z\theta}^2)\right]^{\frac{1}{2}}$$

$$\sigma_Y = \frac{1}{\sqrt{2}}\left[2\,\sigma^2 + 6\tau^2)\right]^{\frac{1}{2}}$$

$$\left(\frac{\sigma}{\sigma_Y}\right)^2 + 3\left(\frac{\tau}{\sigma_Y}\right)^2 = 1$$



Taylor, G.O. and H. Quinney. 1931. The plastic distortion of metals. Phil. Trans. R. Soc., London A230:323

Ingredient 2: Flow Rule

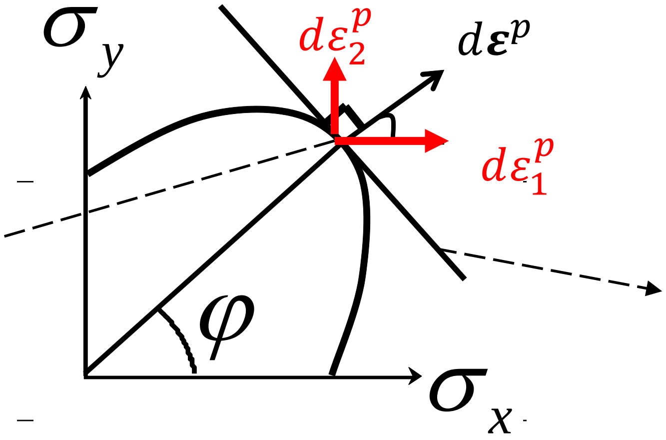
- Ingredient 1 = conditions to initiate yielding
- Ingredient 2 = direction of plastic flow
- Normality hypthesis = associated flow

Magnitude Direction

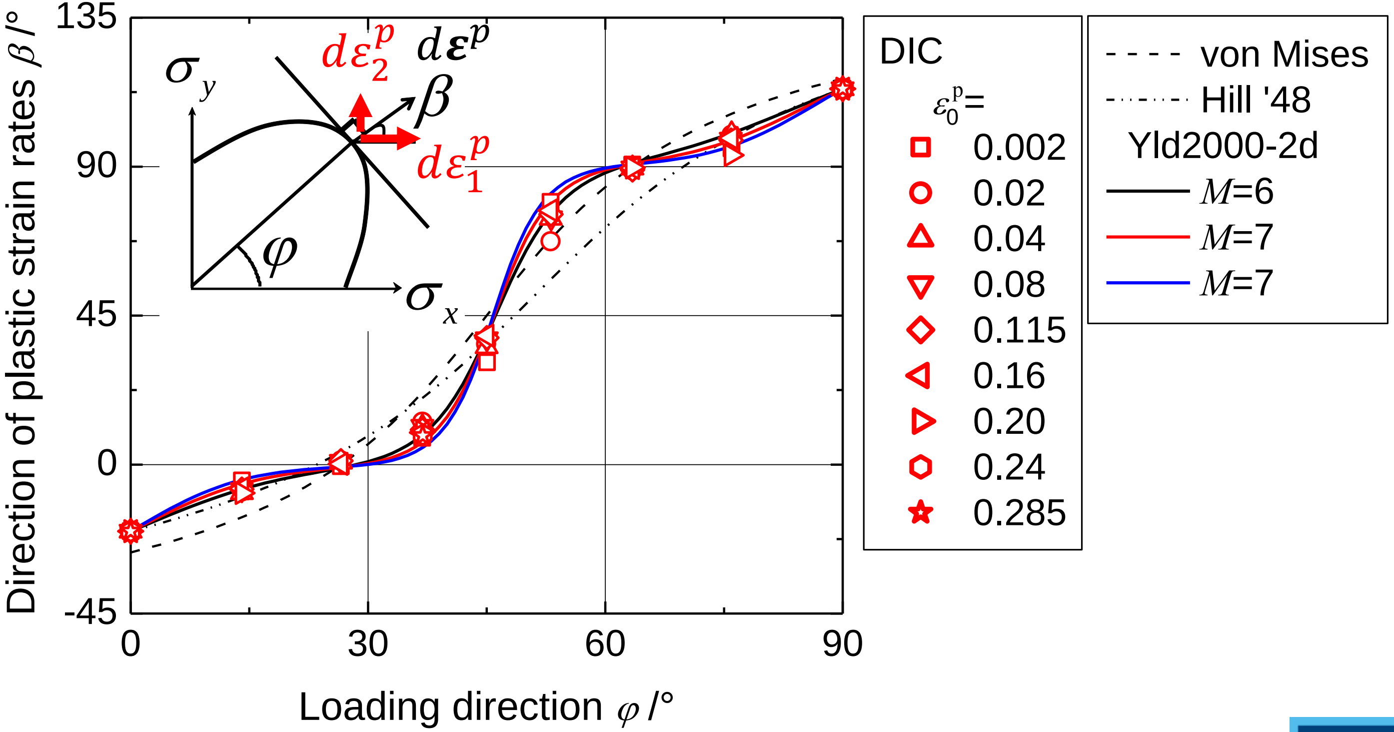
$$d\boldsymbol{\varepsilon}^p = d\lambda \frac{\partial f}{\partial \boldsymbol{\sigma}}$$

- ❑ Consistency condition
- ❑ von Mises Material

$$d\boldsymbol{\varepsilon}^p = \frac{3}{2} d\varepsilon^e \frac{\boldsymbol{\sigma}'}{\sigma_e}$$



Normality Hypothesis: Validation[1]



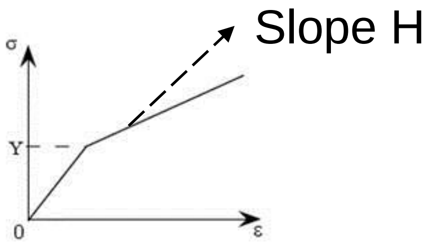
[1] Hakoyama et al., Material Modeling of High Strength Steel Sheet Using Multi-axial Tube Expansion Test with Optical Strain Measurement System, ESAFORM 2013

Ingredient 3: Strain hardening law

Assume linear strain hardening (Ingredient 3):

$$\sigma_e = \sigma_0 + H\varepsilon_e^p$$

$$\rightarrow d\varepsilon_e^p = \frac{d\sigma_e}{H}$$



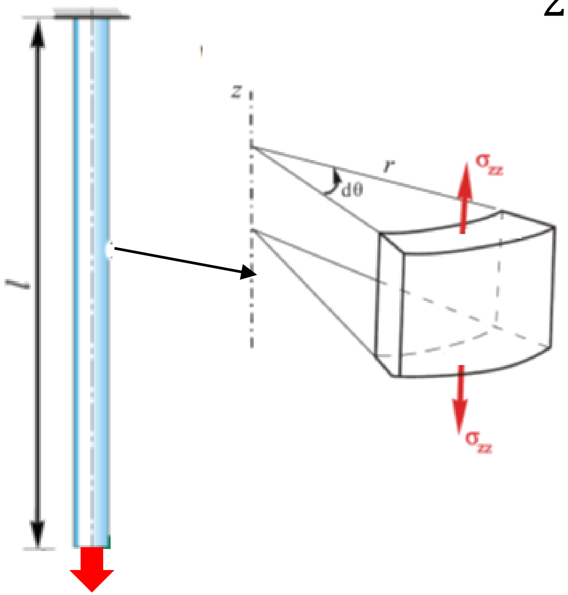
von Mises (Ingredient 1) and Flow rule (ingredient 2):

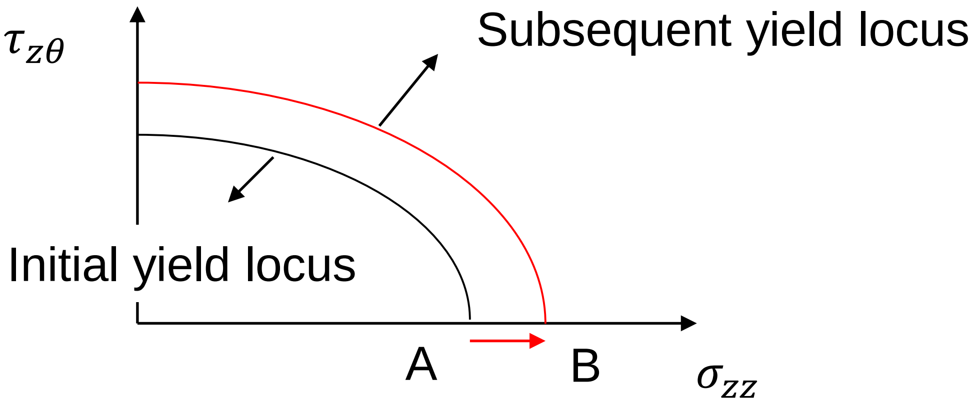
$$d\boldsymbol{\varepsilon}^p = \frac{3}{2} d\varepsilon_e^p \frac{\boldsymbol{\sigma}'}{\sigma_e}$$

$$d\boldsymbol{\varepsilon}^p = \frac{3}{2}\frac{d\sigma_e}{H\sigma_e}\boldsymbol{\sigma}'$$

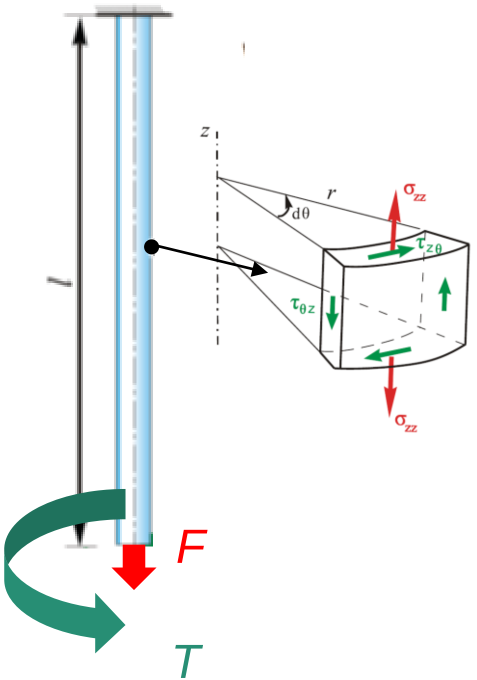
Analysis: step 1

$$d\varepsilon_{zz}^{pl} = \frac{3}{2}\frac{d\sigma_e}{H\sigma_e}\sigma' = \frac{3}{2}\frac{d\sigma_e}{H\sigma_e}\cdot\left(\sigma_{zz} - \frac{1}{3}\sigma_{zz}\right) = \frac{d\sigma_e}{H\sigma_e}\cdot\sigma_{zz} = \frac{d\sigma_e}{H}$$





Analysis: Step 2



$$(d\varepsilon_{zz}^{pl})_{\,step\ 2} = \frac{d\sigma_e}{H(\sigma_e)_{step\ 2}} \cdot \sigma_{zz}$$

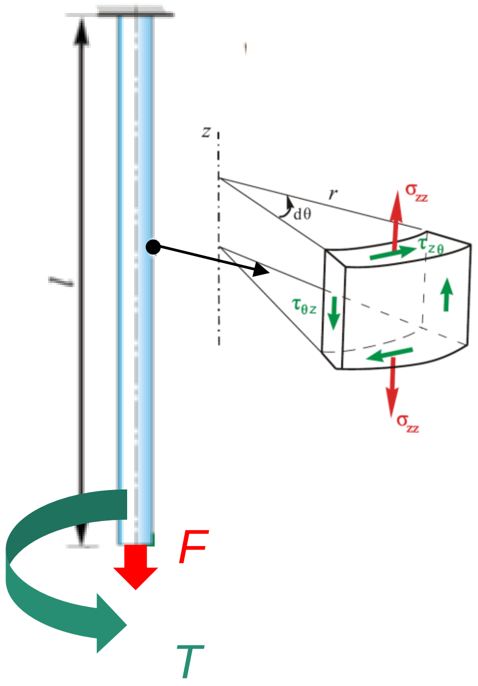
$$(\sigma_e)_{\,step\ 1} = \sigma_{zz}$$

$$(\sigma_e)_{\,step\ 2} = \frac{1}{\sqrt{2}}\left[2(\sigma_{zz})^2 + 6(\tau_{z\theta}^2)\right]^{\frac{1}{2}}$$

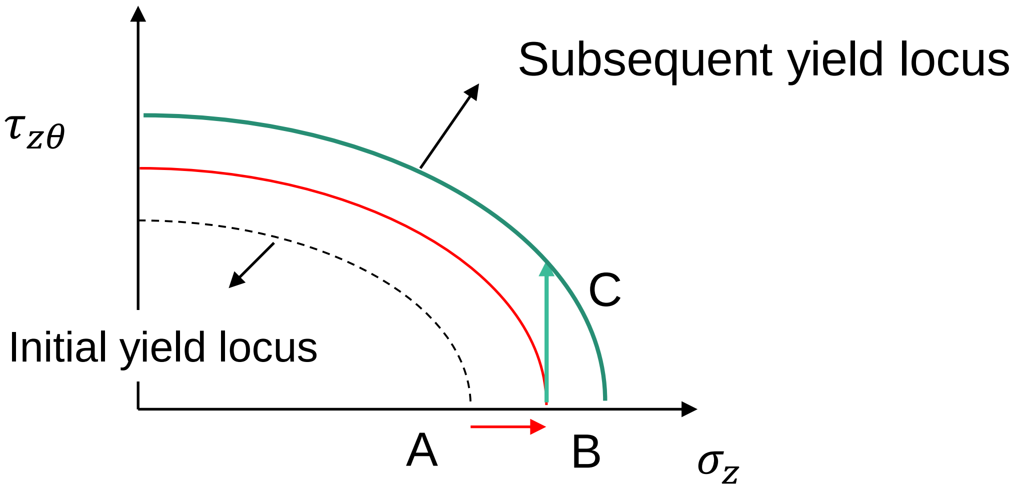
$$\rightarrow d\sigma_e \approx (\sigma_e)_{step\ 2} - (\sigma_e)_{step\ 1} > 0$$

$$\rightarrow (d\varepsilon_{zz}^{pl})_{step\ 2} > 0$$

Analysis: Step 2



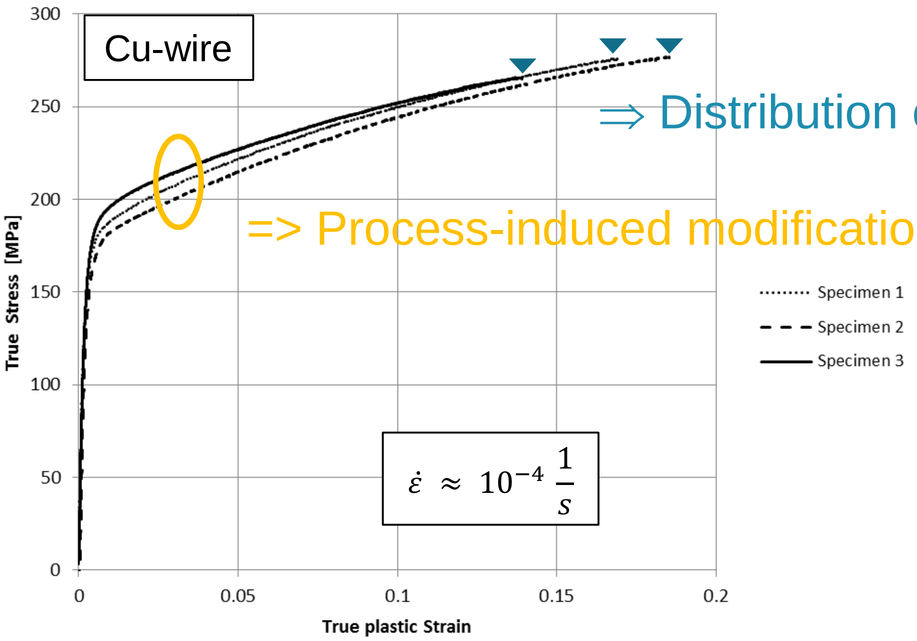
$$\rightarrow (d\varepsilon_{zz}^{pl})_{step\ 2} > 0$$



Computational J2 plasticity

✓ von Mises + *strain hardening behavior*

$d_0 = 0.265\ mm$

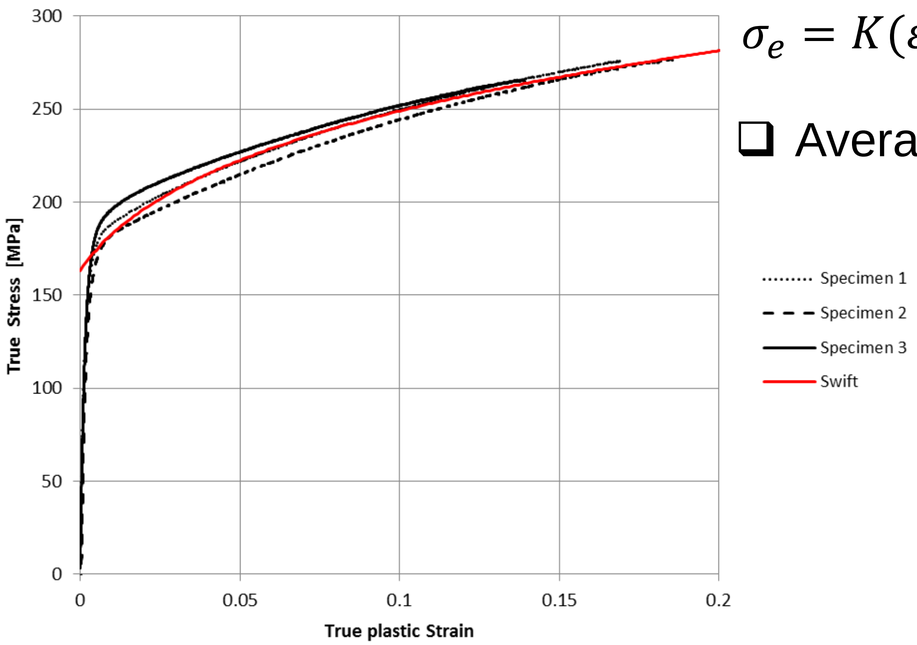


⇒ Distribution of defects

=> Process-induced modifications/defects

Computational J2 plasticity

✓ von Mises + *strain hardening behavior: Swift*



$$\sigma_e = K(\varepsilon_0 + \varepsilon_{eq}^{pl})^n$$

- ❑ Average behavior

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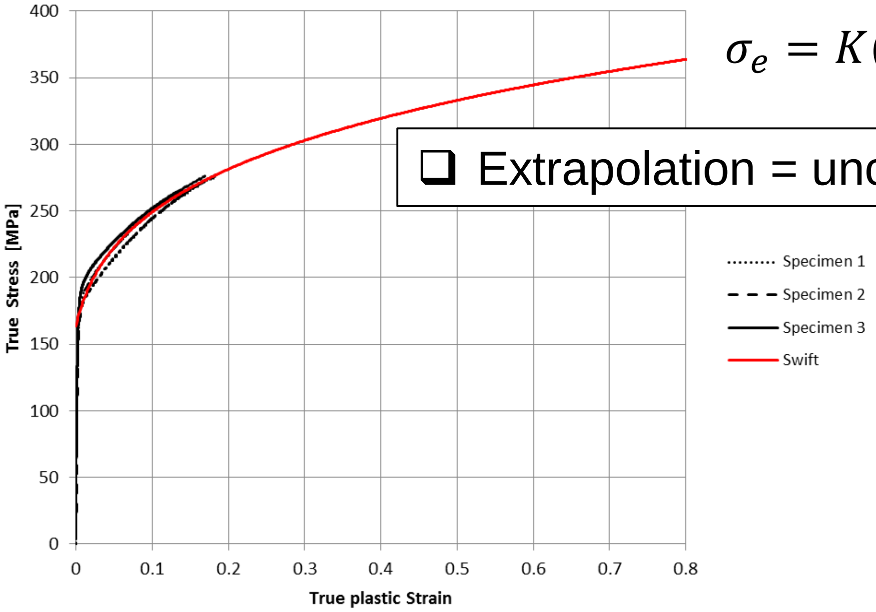
Computational J2 plasticity

✓ von Mises + *strain hardening behavior*



$$\sigma_e = K(\varepsilon_0 + \varepsilon_{eq}^{pl})^n$$

❑ Extrapolation = uncertainty

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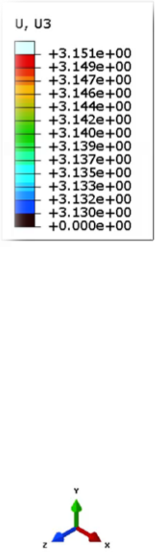
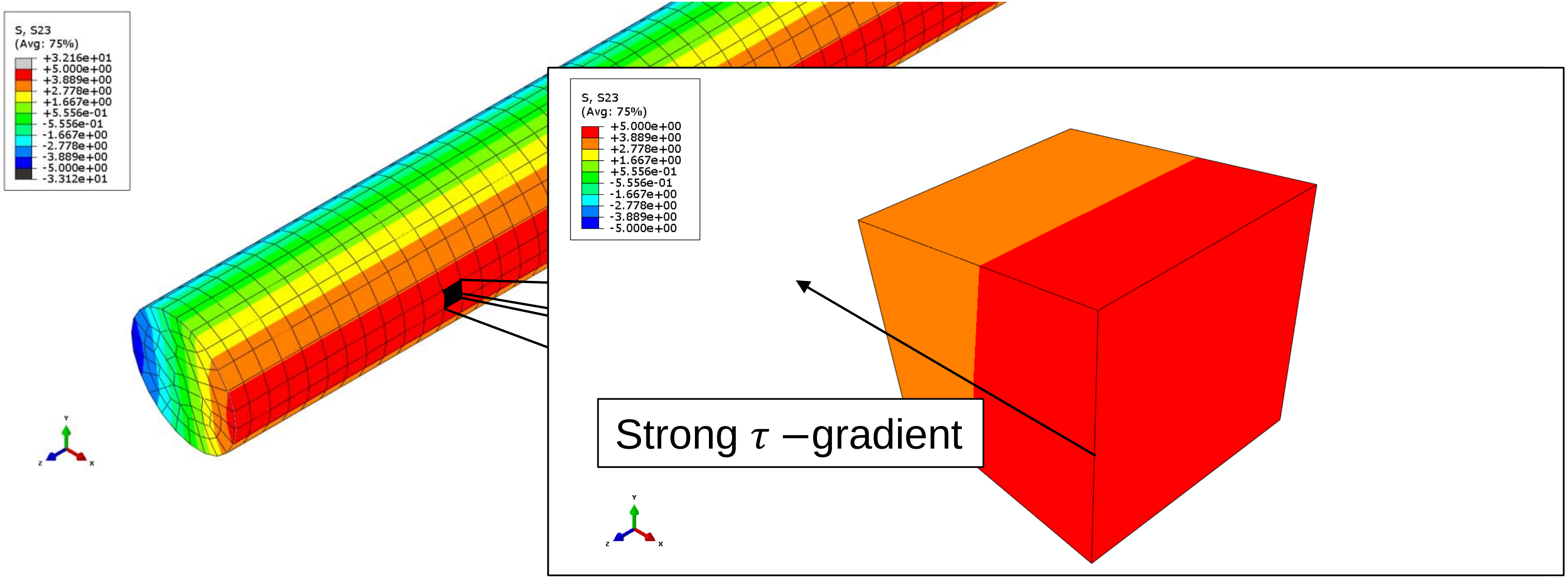
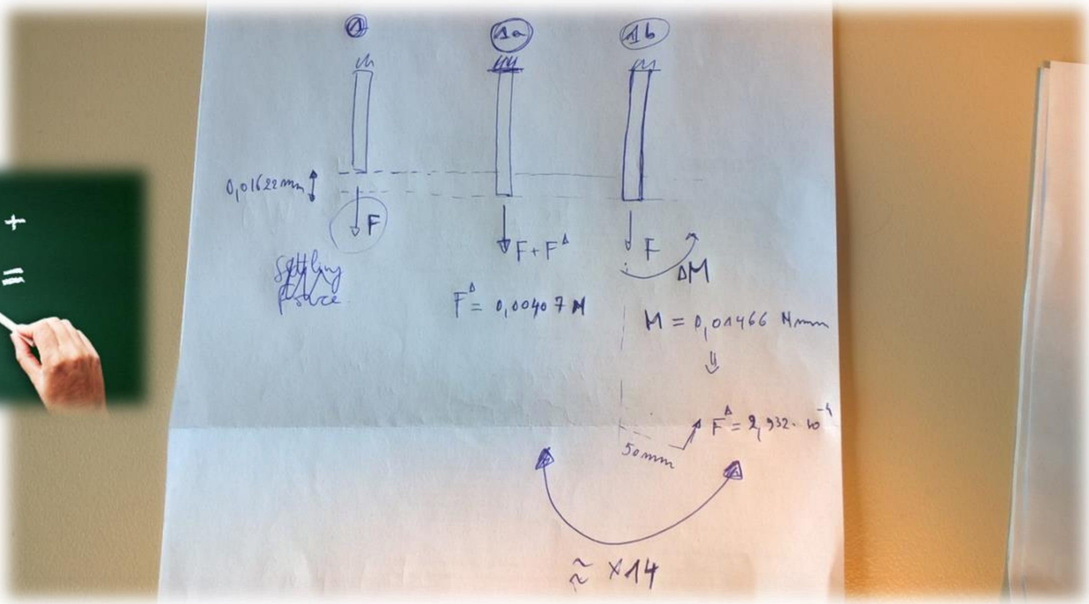
FE Analysis: Step 2



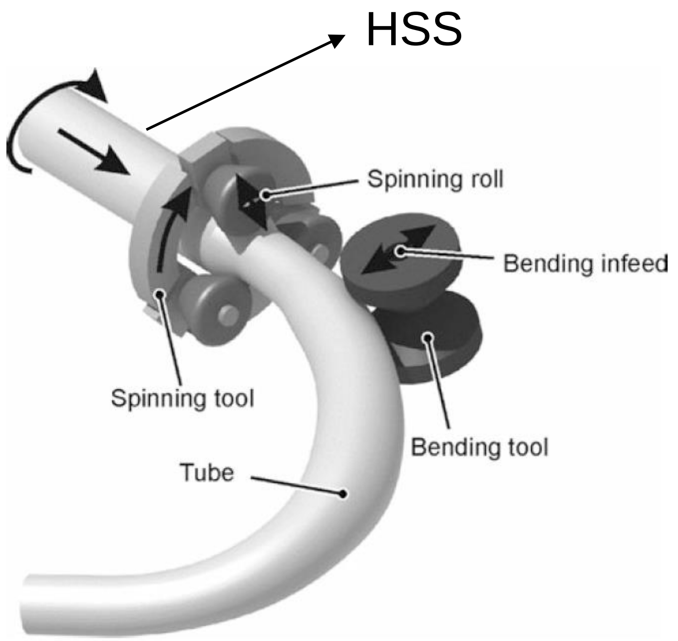
FE Analysis: improve understandig



FE Analysis: support innovation



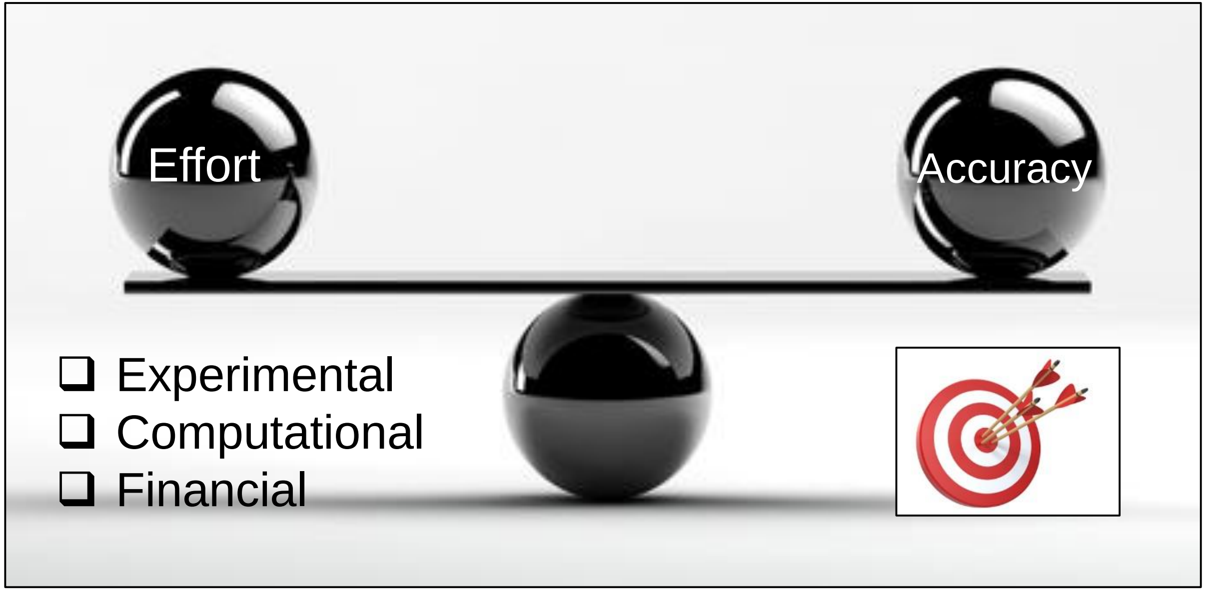
Industrial application[1]



Reduction of bending moment

[1]Becker et al., Fundamentals of the incremental tube forming process, CIRP Annals – Manufacturing Technology 63 (2014) 253-256

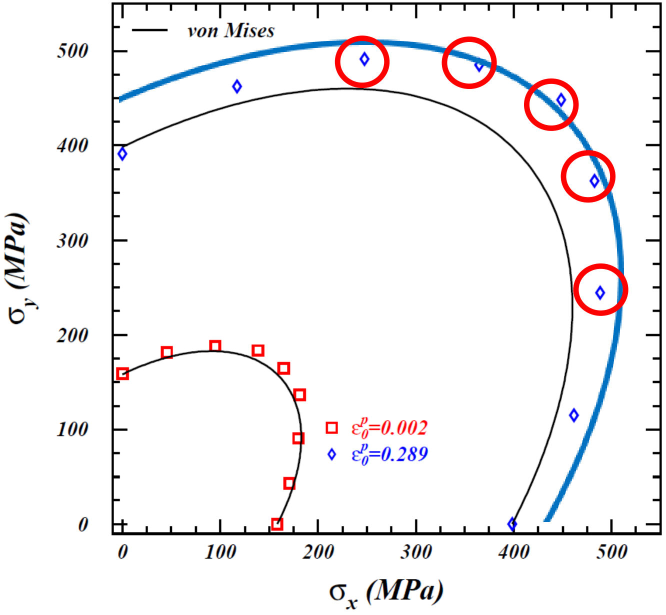
Effort and predictive accuracy



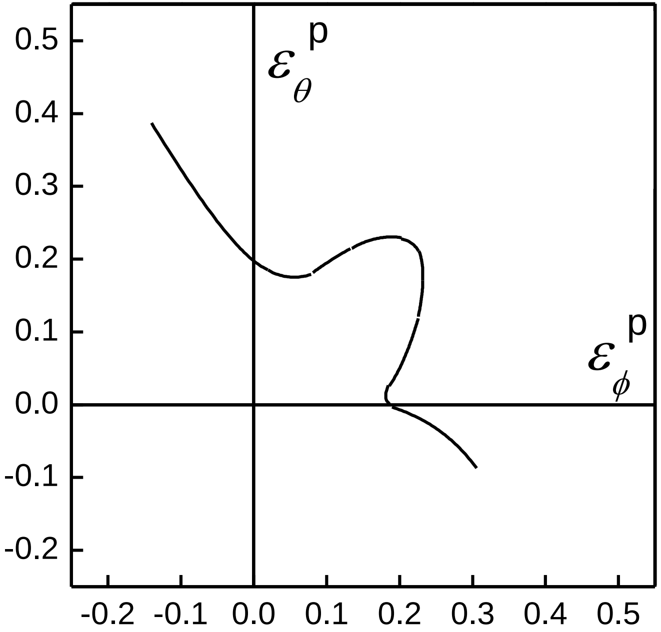
Engineering practice: ***keep things simple***

Plasticity

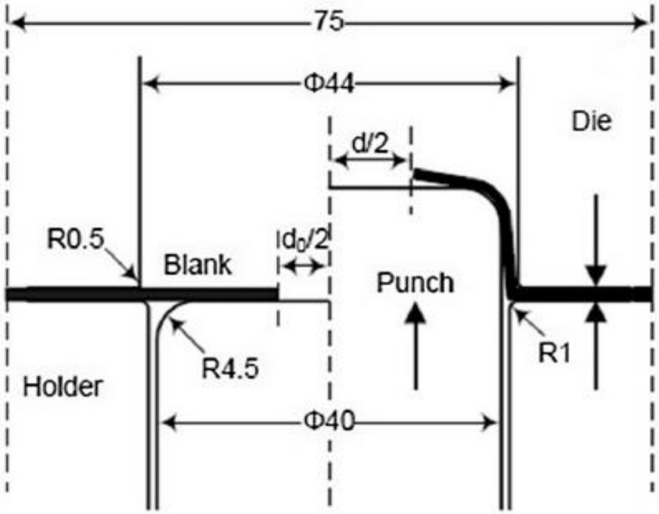
R. von Mises (1913)



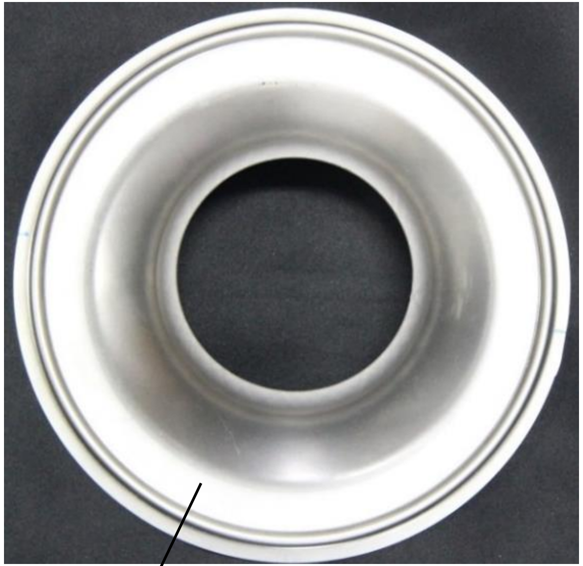
Ductile failure criterion



…but **not simpler:** Fracture Prediction HET



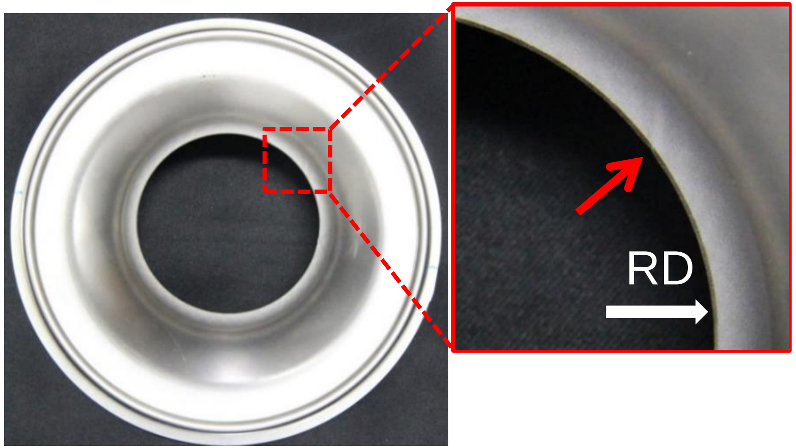
Hole **E**xpansion **T**est



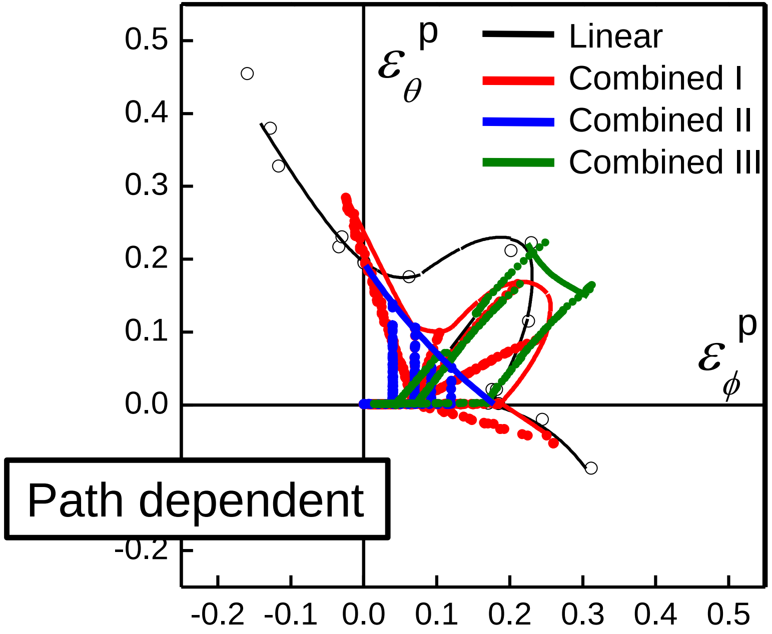
High Strength Metal

…but **not simpler:** Fracture Prediction HET[1,2]

Plasticity



Ductile failure criterion



[1] Benchmark Problem NUMISHEET 2018
[2] Hakoyama, Nakano and Kuwabara, Fracture Prediction of Hole Expansion Forming Using Forming Limit Stress Criterion, ESAFORM Conference 2017, Dublin.

General Approach

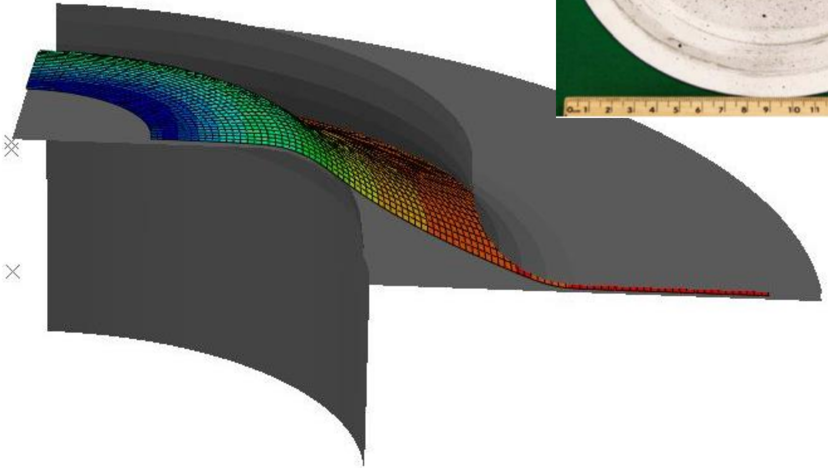
Experimental Data

↓

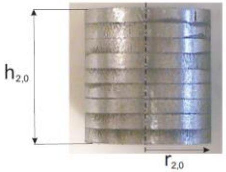
Material modeling
(Macroscopic model)

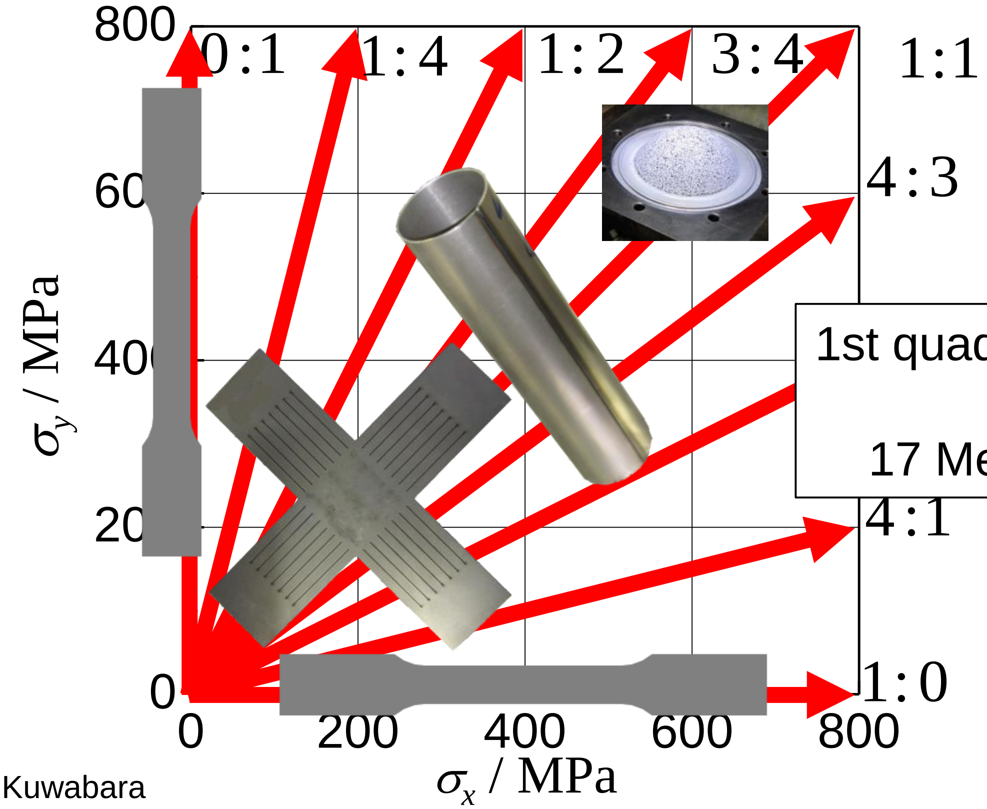
↓

Finite element analysis &
Fracture prediction
(Macroscopic model)



Stress-controlled material testing*

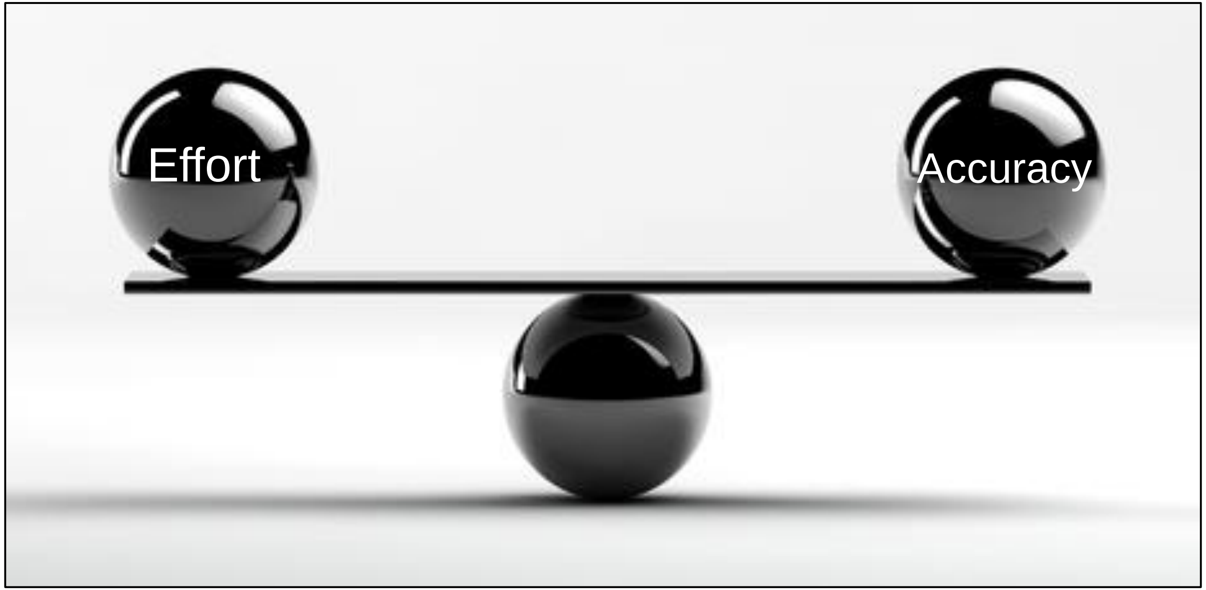




1st quadrant of stress space
=
17 Mechanical tests

*See presentation prof. Kuwabara

Effort and predictive accuracy



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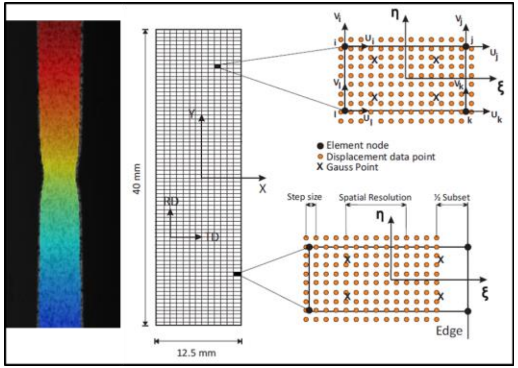
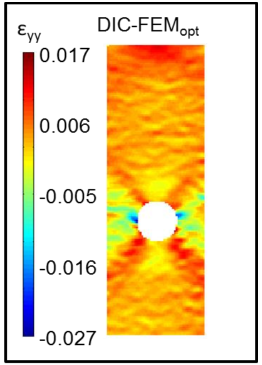
Minimize time-to-market

MGI[1] = effort to discover, manufacture, and <u>deploy advanced materials</u> twice as fast, at a fraction of the cost.

Strategic Goal: Integrate Experiments & Computation

- ✓ ***Reducing traditional testing by leveraging an existing material test***
- ✓ ***Enhance the synergy between experiments and computational methods***





[1]Materials Genome Initiative https://www.mgi.gov/

General Approach

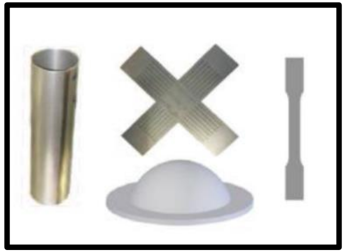
Experimental Data

↓

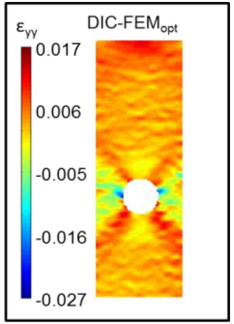
Material modeling
(Macroscopic model)

↓

Finite element analysis &
Fracture prediction
(Macroscopic model)



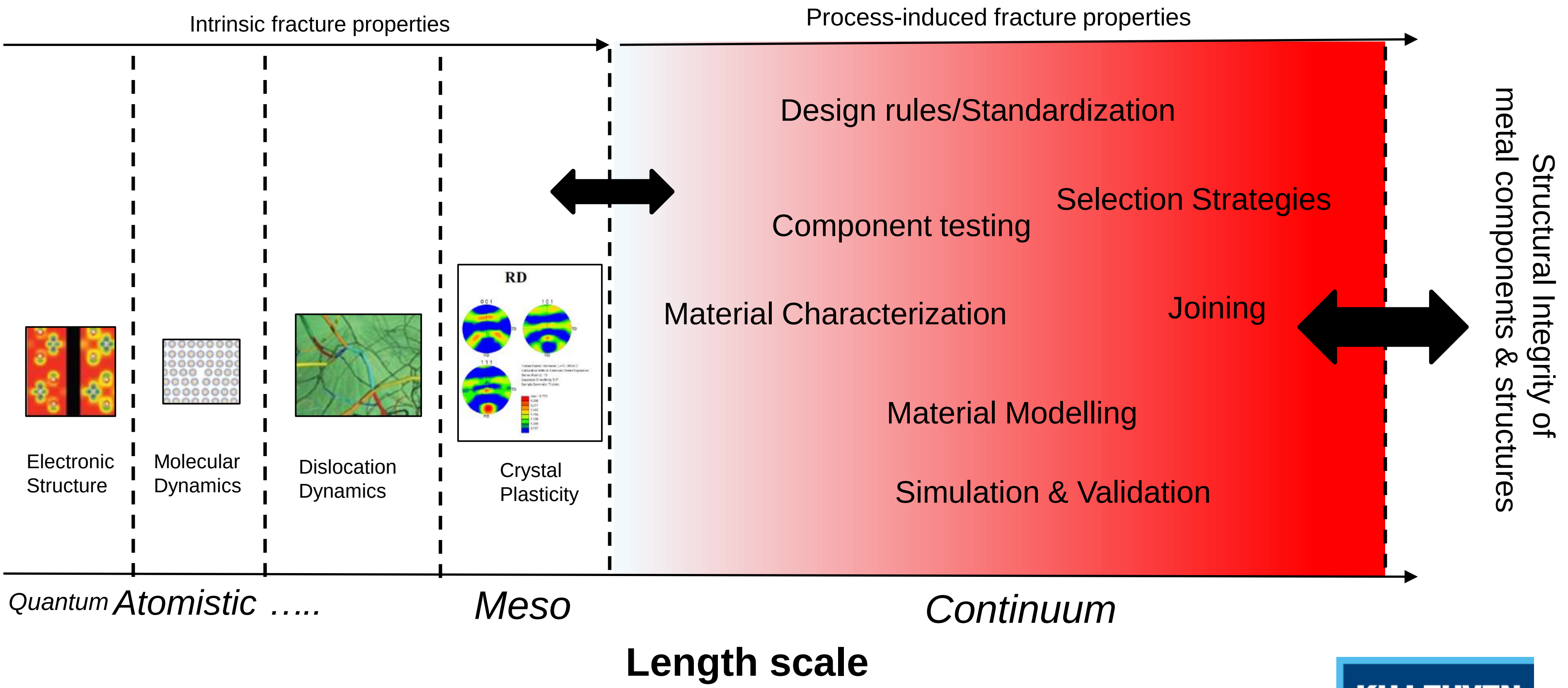
A



B

A. Stress-controlled Material Testing
B. Inverse Material Identification

Multi-Scale Approach



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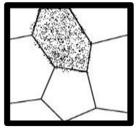
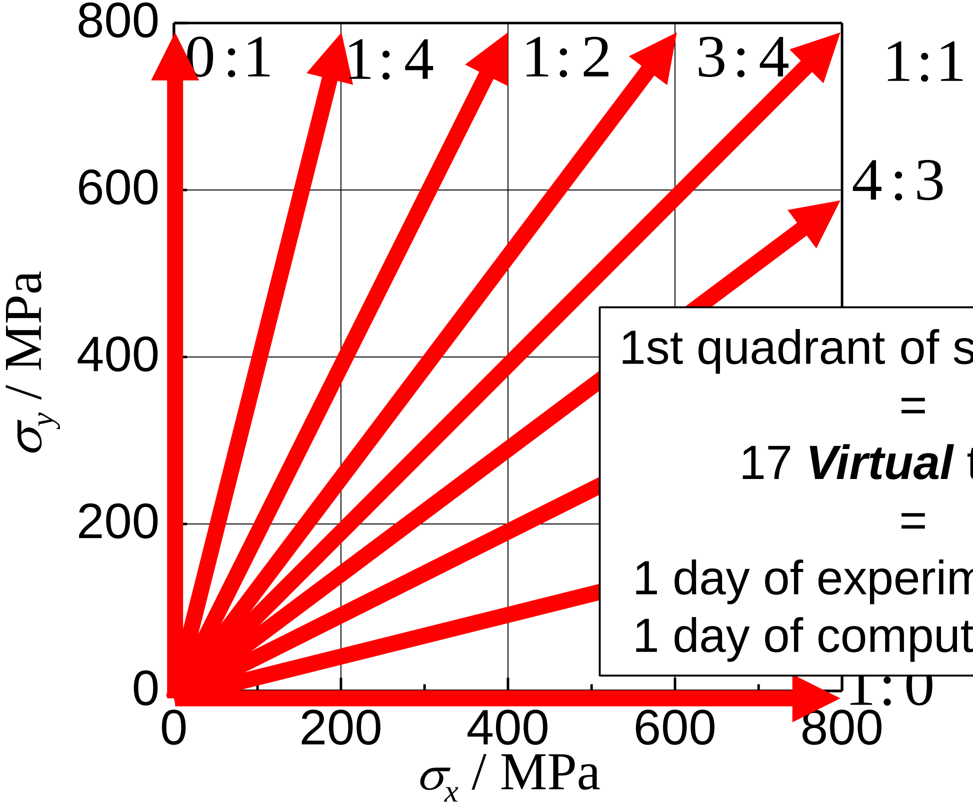
Mircostructurally-informed constitutive modeling[1]



Virtual Material testing



1st quadrant of stress space
=
17 ***Virtual*** tests
=
1 day of experimental effort
1 day of computational effort

Mircostructurally-informed constitutive modeling[1]

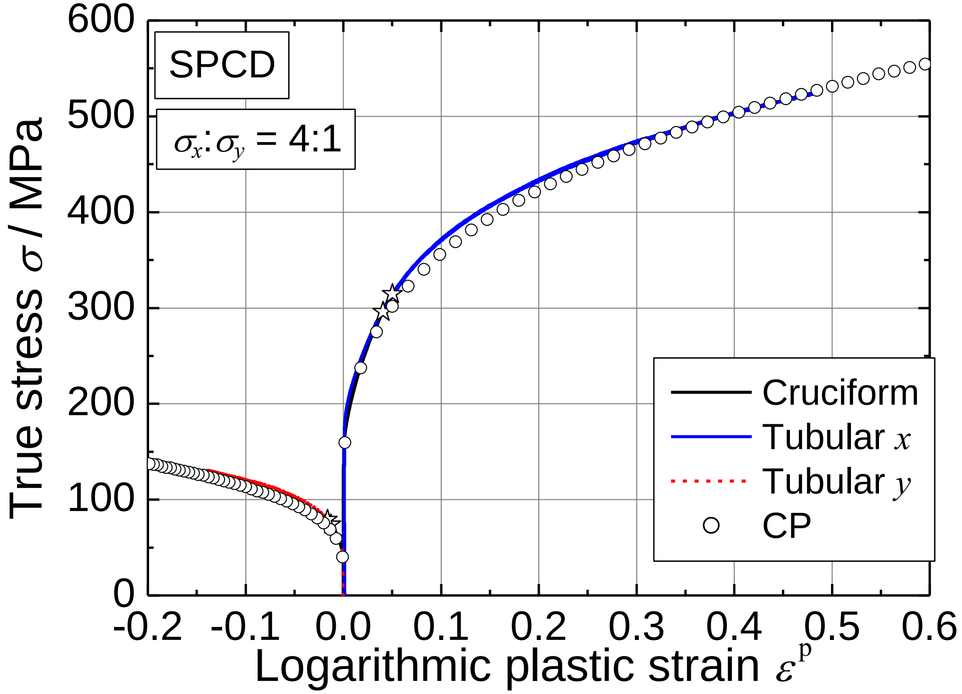
Microstructure-based plasticity model
(Virtual material test)

↓

Material modeling
(Macroscopic model)

↓

Finite element analysis
(Macroscopic model)



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Mircostructurally-informed constitutive modeling

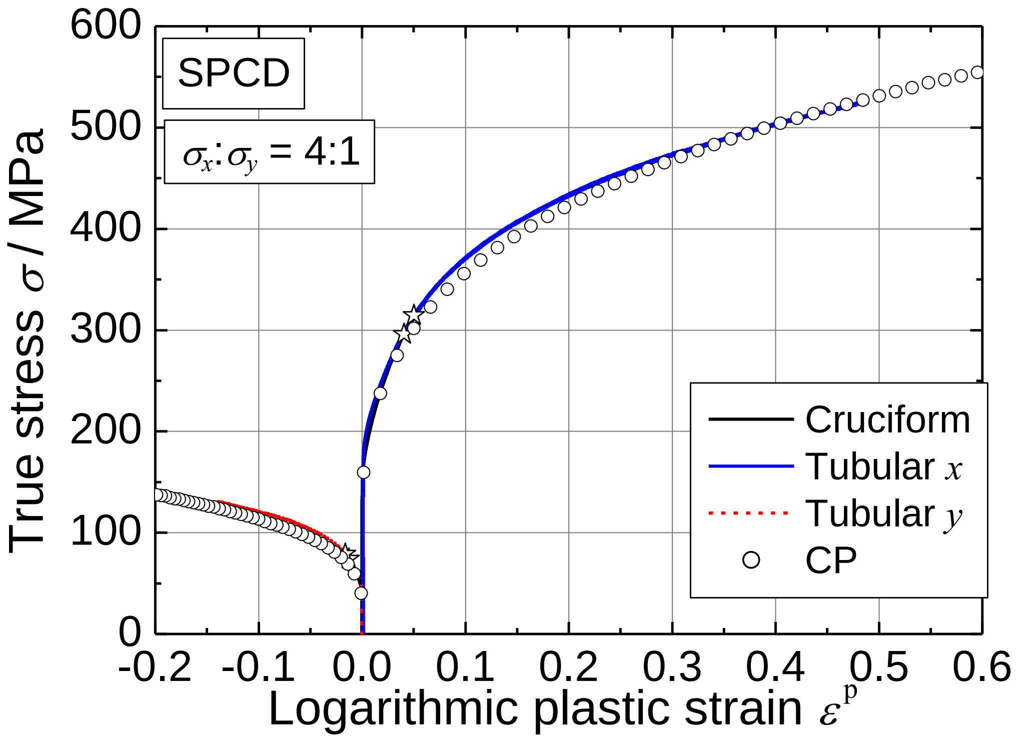
Microstructure-based plasticity model
(Virtual material test)

↓

Material modeling
(Macroscopic model)

↓

Finite element analysis
(Macroscopic model)



Mircostructurally-informed constitutive modeling

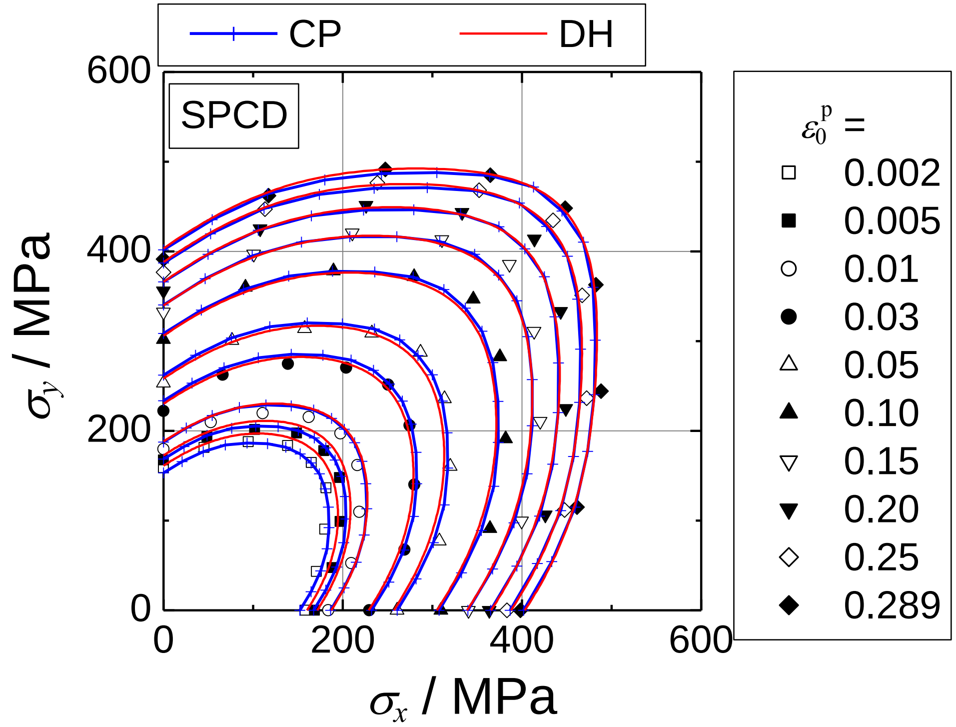
Crystal plasticity simulation
(Virtual tensile test)

↓

Material modeling
(Macroscopic model)

↓

Finite element analysis
(Macroscopic model)



Mircostructurally-informed constitutive modeling

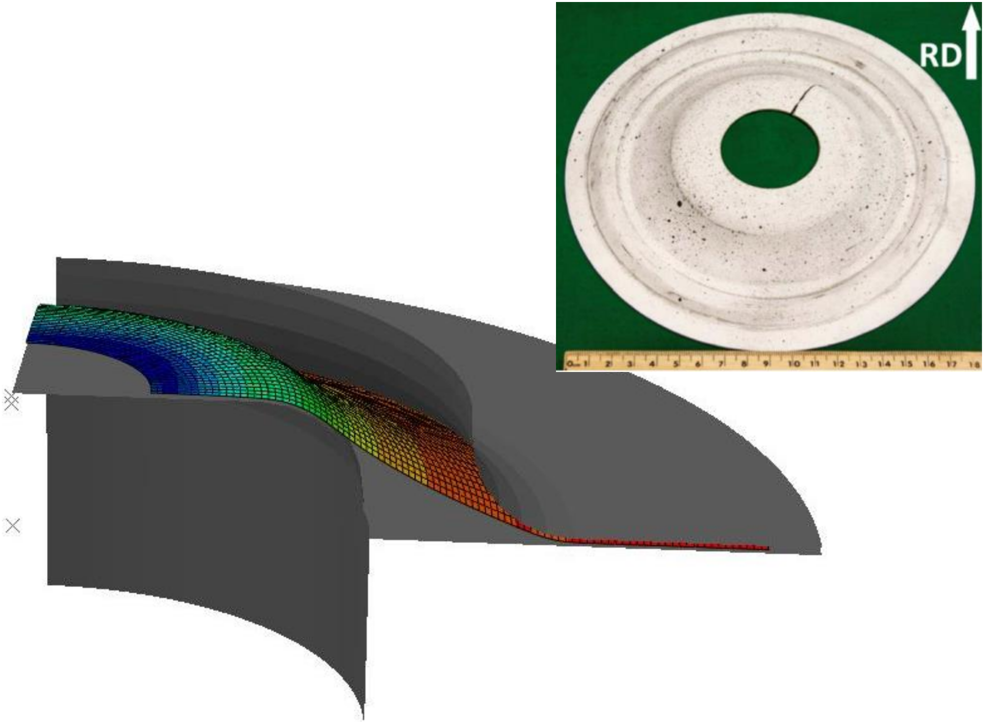
Crystal plasticity simulation
(Virtual tensile test)

↓

Material modeling
(Macroscopic model)

↓

Finite element analysis &
Fracture prediction
(Macroscopic model)



General Approach

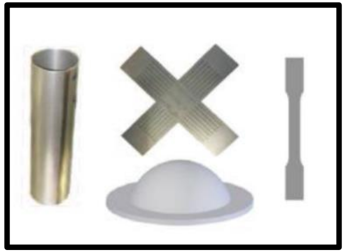
Experimental Data

↓

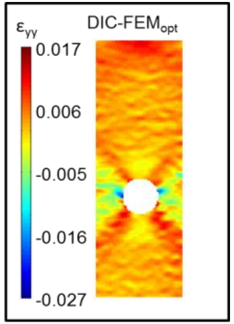
Material modeling
(Macroscopic model)

↓

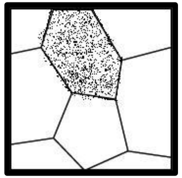
Finite element analysis &
Fracture prediction
(Macroscopic model)



A



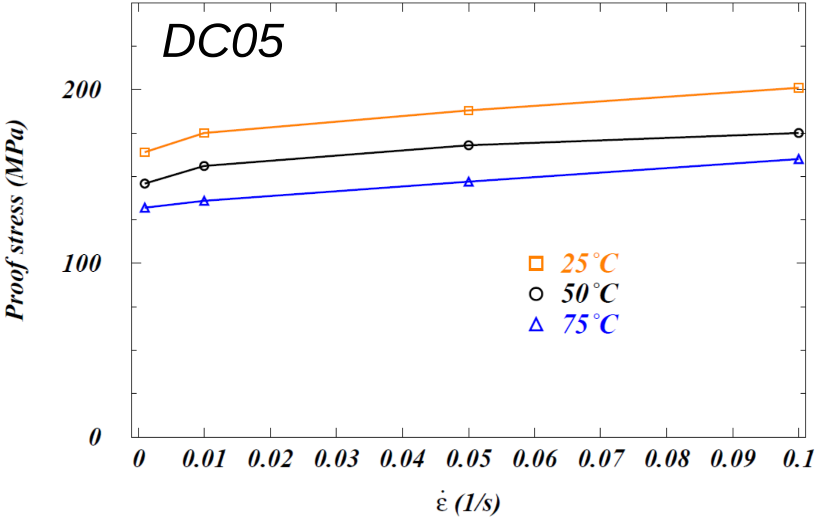
B



C

A. Stress-controlled Material Testing
B. Inverse Material Identification
C. Virtual Material Testing

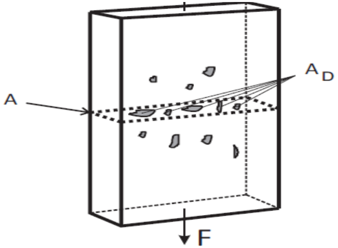
Plastic material response
=
the result of synergistic effects

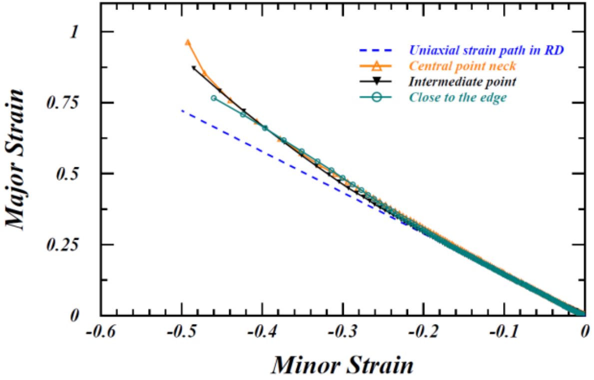


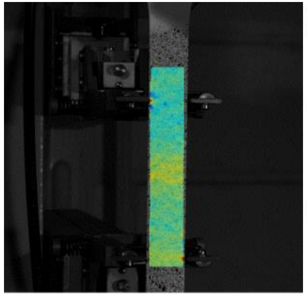
- ❑ Loading conditions
- ❑ Environmental conditions
- ❑ Microstructure
- ❑ …

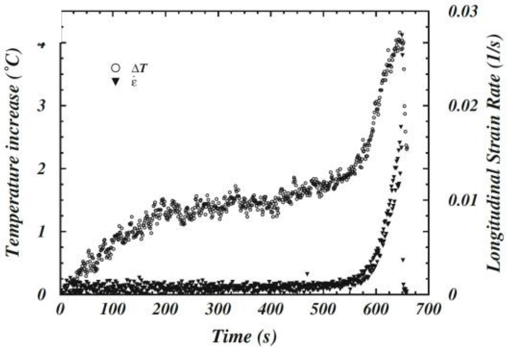
Synergistic effects during diffuse necking

Damage evolution









Large plastic strains,
Biaxial stress state
& strain path change

Strain rate & Temperature increase

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Plastic material response can be complex…

- Differential work hardening
- Bauschinger effect
- Strain rate sensitivity
- Kinematic hardening
- Ductile Damage
- …

Plastic material response can be complex…

- Differential work hardening
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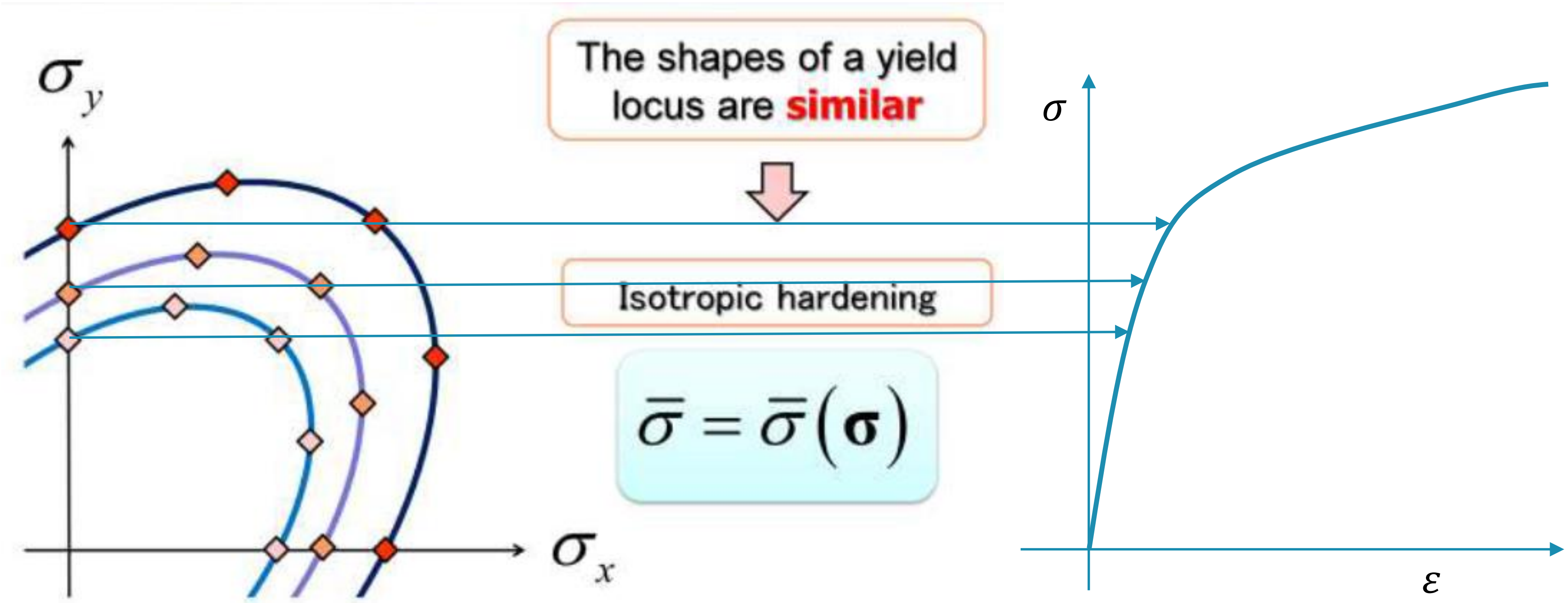
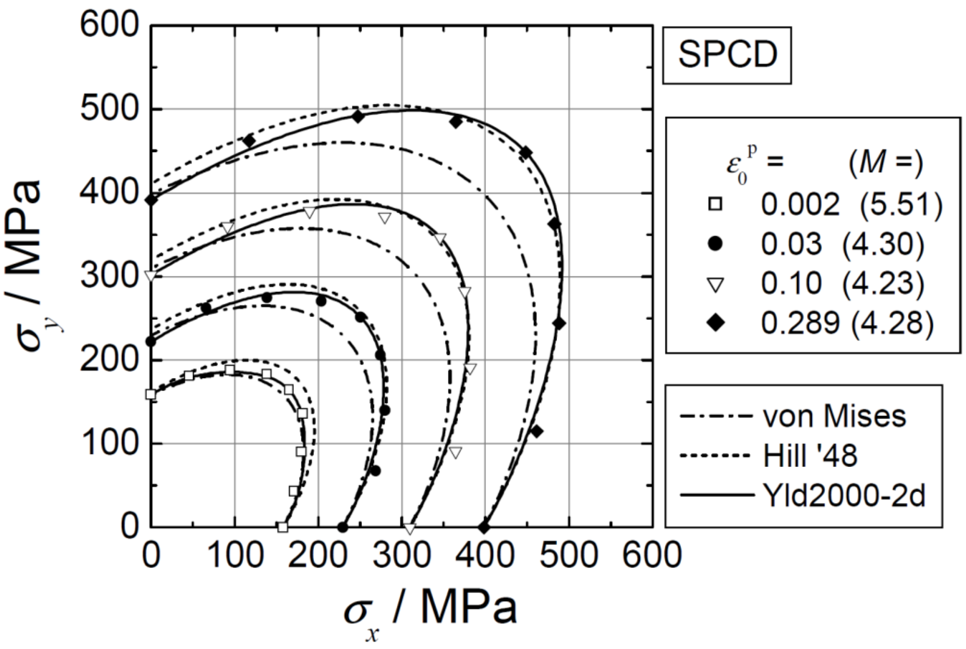
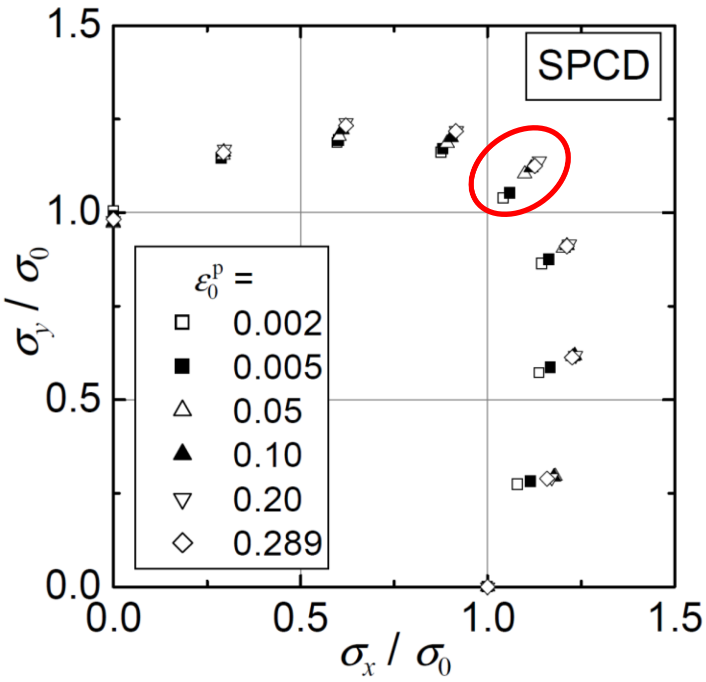
Isotropic work hardening



Differential work hardening





Problem statement

Experimental Data

↓

Material modeling
(Macroscopic model)

↓

Finite element analysis
(Macroscopic model)

Problems

- ❑ Many models exist

Anisotropic Yield Functions

- ❑ Hill 1948
- ❑ Hill 1990
- ❑ Gotoh
- ❑ Barlat Yld2000-2d
- ❑ Barlat Yld2004
- ❑ Yoshida
- ❑ BBC2005
- ❑ BBC2008
- ❑ Vegter Spline
- ❑ Karafills-Boyce
- ❑ CPB2006
- ❑ …

Coupled Damage Models

Model - Author	Yield function
Gurson - [7]	$\Phi = \left(\frac{\sigma_{eq}}{\sigma_0}\right)^2 + 2f\cosh\left(\frac{3p}{2\sigma_0}\right) - 1 - f^2 = 0$
G&T - [8]	$\Phi = \left(\frac{\sigma_{eq}}{\sigma_0}\right)^2 + 2q_1 f\cosh\left(q_2\frac{3p}{2\sigma_0}\right) - 1 - q_1^2 f^2 = 0$
S&W - [119]	$\Phi = \left(\frac{\sigma_{eq}}{\sigma_0}\right)^2 + \left(2 - \frac{1}{2}\log(f)\right) f\cosh\left(\frac{3p}{2\sigma_0}\right) - 1 - f(1 + \log(f)) = 0$
VAR - ([109, 112–114])	$\Phi = \left(\frac{\sigma_{eq}}{\sigma_0}\right)^2 \left(1 + \frac{2}{3}f\right) + f\left(\frac{3p}{2\sigma_0}\right)^2 - (1-f)^2 = 0$
MVAR - ([111])	$\Phi = \left(\frac{\sigma_{eq}}{\sigma_0}\right)^2 \left(1 + \frac{2}{3}f\right) + f\left(\frac{1-f}{\sqrt{f}\log(1/f)}\right)^2 \left(\frac{3p}{2\sigma_0}\right)^2 - (1-f)^2 = 0$
G&S - ([118])	$\Phi = \left(\frac{\sigma_{eq}}{\sigma_0}\right)^2 \left(1 + \frac{2}{3}f\right) + 2f\cosh\left(\frac{3p}{2\sigma_0}\right) - 1 - f^2 = 0$
GVAR - ([117])	$\Phi = \left(\frac{\sigma_{eq}}{\sigma_0}\right)^2 \left(1 + \frac{2}{3}\alpha_g f\right) + 2q_1 f\cosh\left(q_2\frac{3p}{2\sigma_0}\right) - 1 - q_1^2 f^2 = 0$

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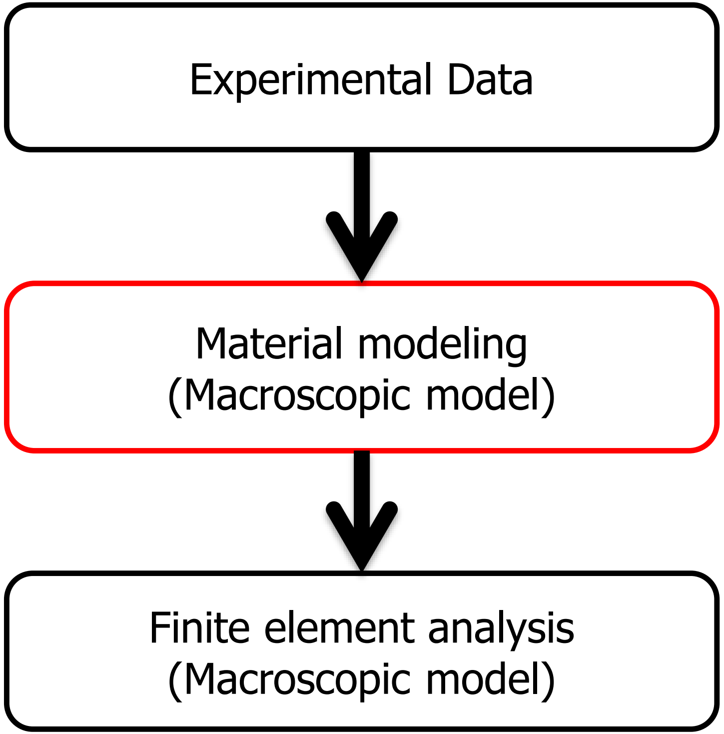
Problem statement



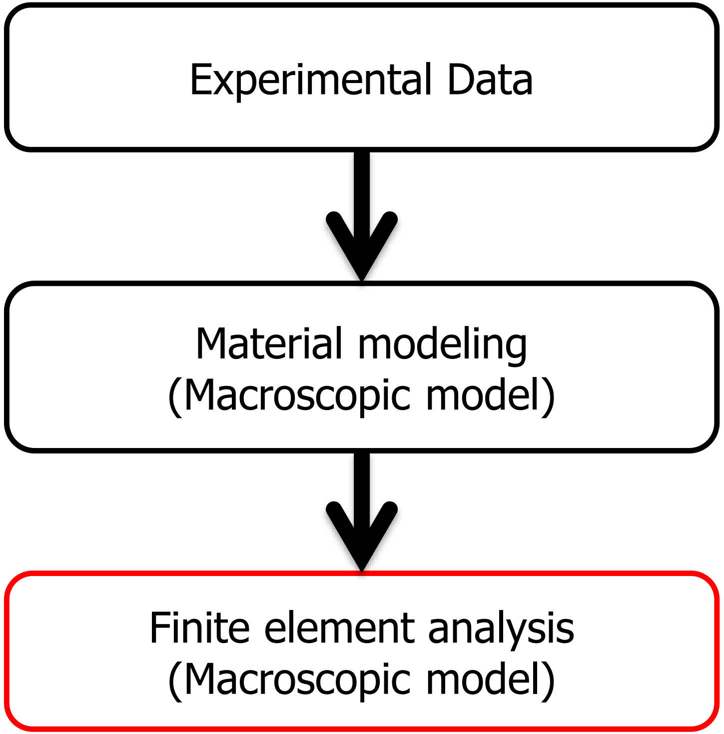
Problems

- ❑ Many models

Need

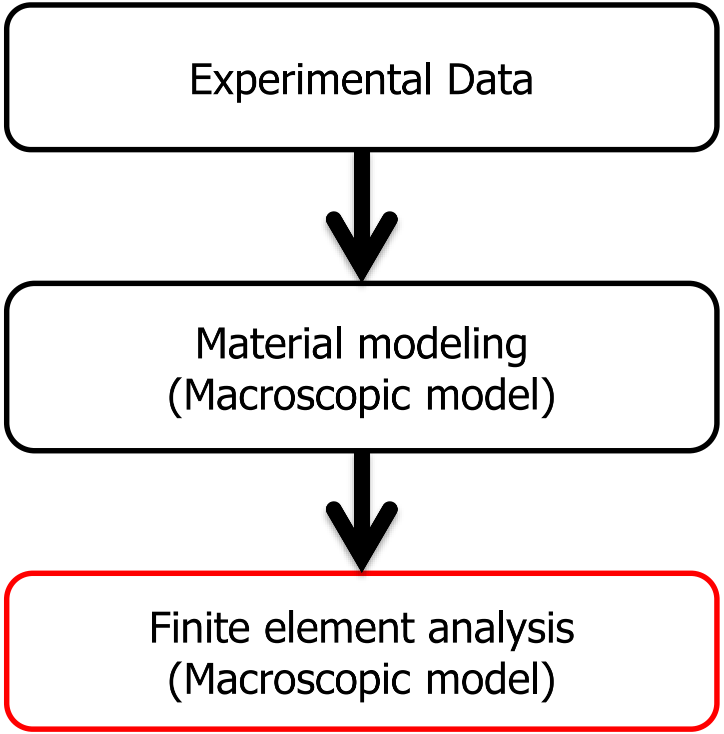
- ❑ *Selection strategy*
- ❑ Identification strategy

Problem statement



Problems	*Need*
❑ Many models	❑ *(Selection strategy)* ❑ Identification strategy
❑ Models not available in commercial FE code	❑ Implementation

Problem statement



Problems

- ❑ Many models
- ❑ Models not available in commercial FE code

Need

- ❑ *(Selection strategy)*
- ❑ Identification strategy
- ❑ Implementation

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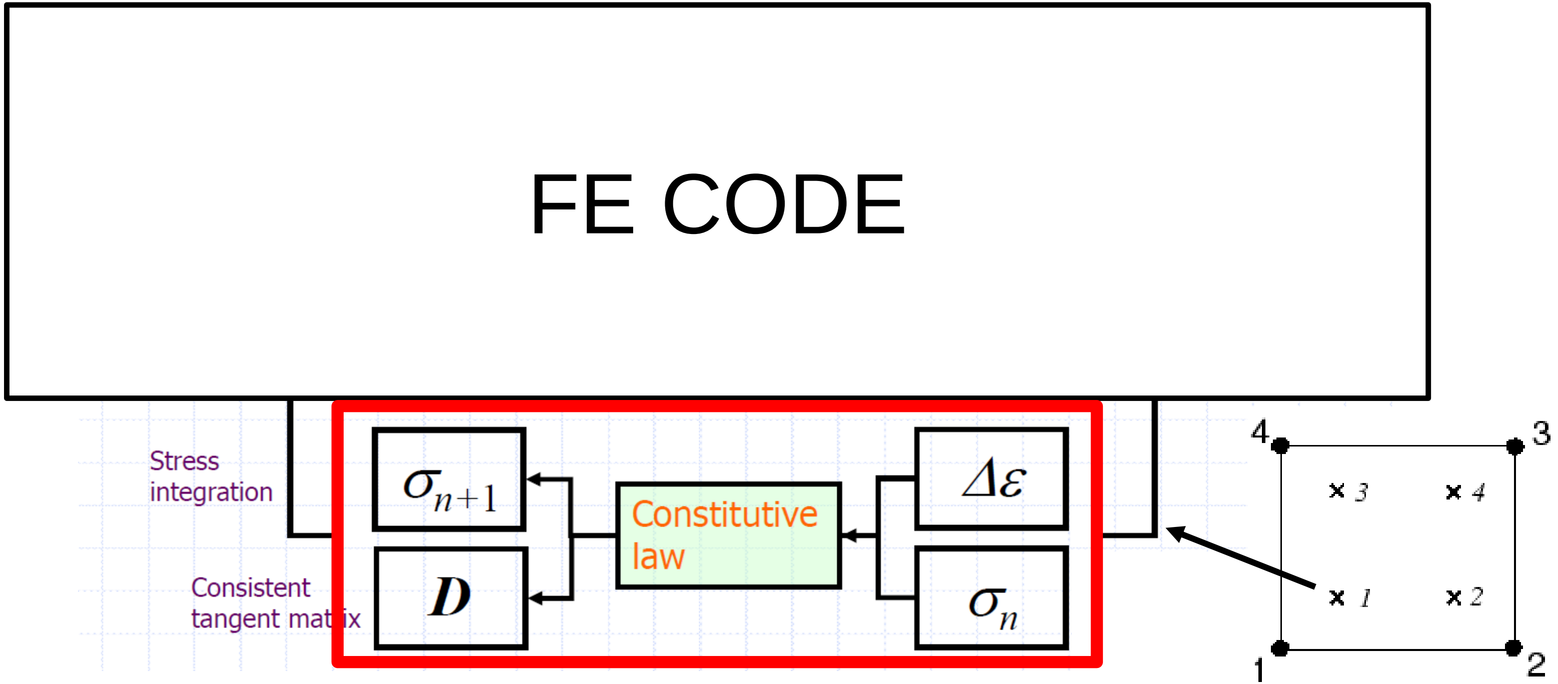
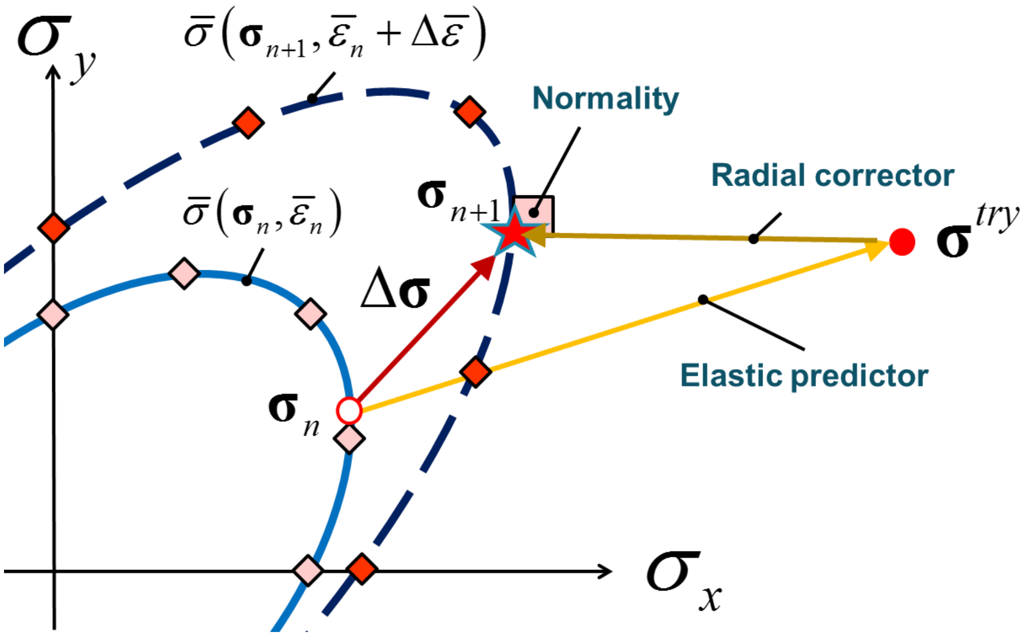
Implementation = stress reconstruction

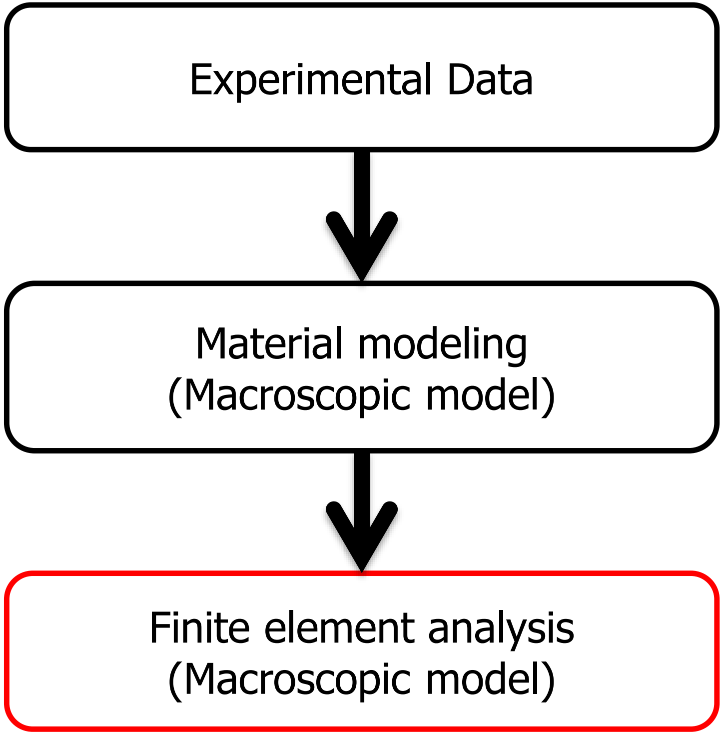


Stress integration – Stress reconstruction -Stress update

Stress update algorithm

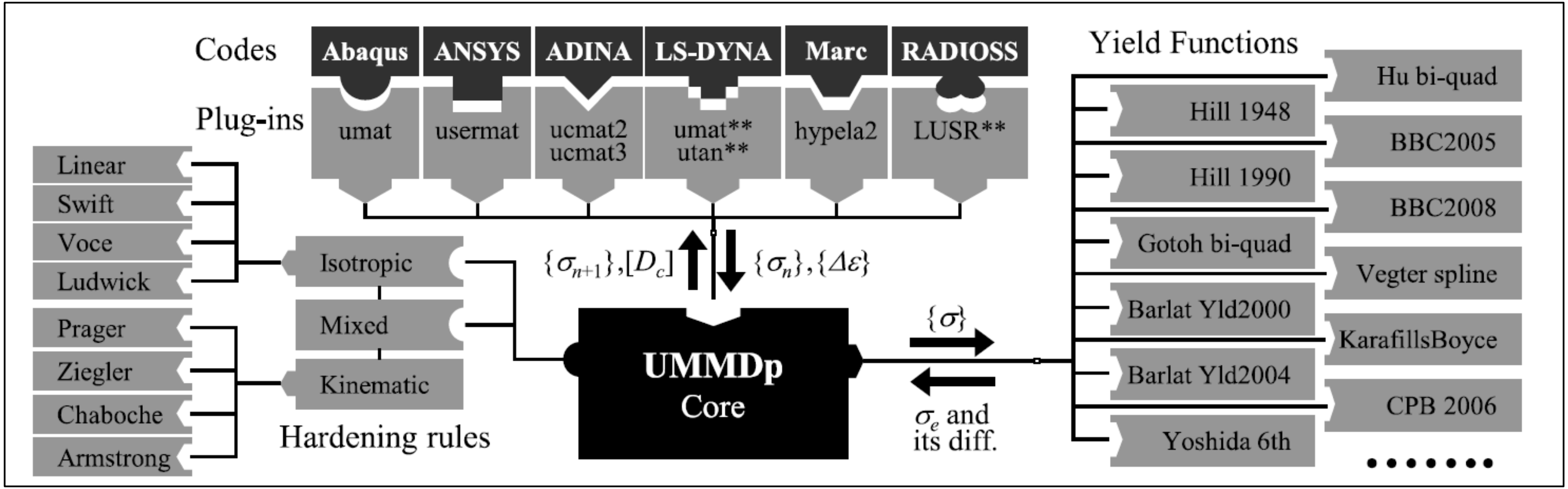


Problem statement



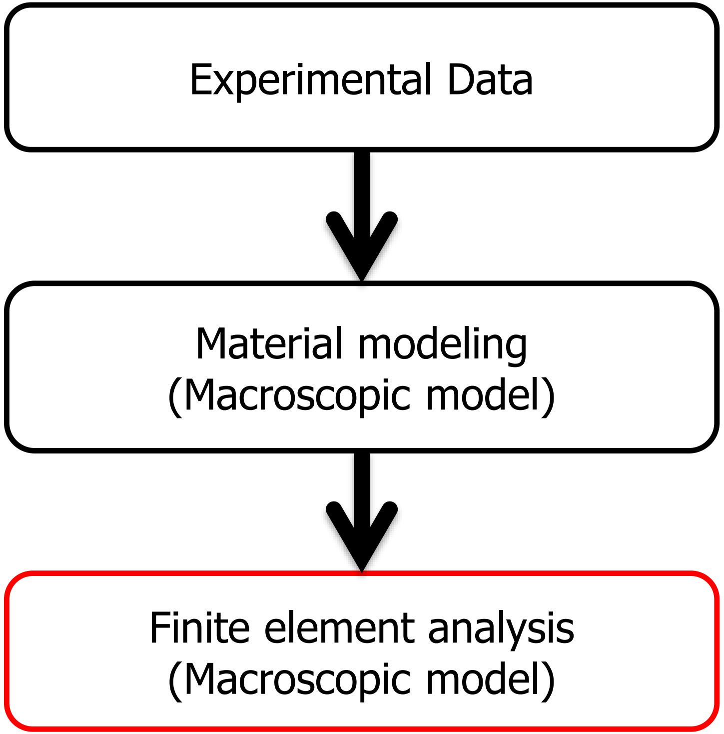
Problems	*Need*
❑ Many models	❑ *(Selection strategy)* ❑ Identification strategy
❑ Models not available in commercial FE code ❑ Many codes	❑ Implementation

Unified Material Model Driver for plasticity[1]



[1]JANCAE, The Japan Association for Nonlinear CAE

Problem statement



Problems	*Need*
❑ Many models	❑ *(Selection strategy)* ❑ Identification strategy
❑ Models not available in commercial FE code ❑ Many codes	❑ Implementation

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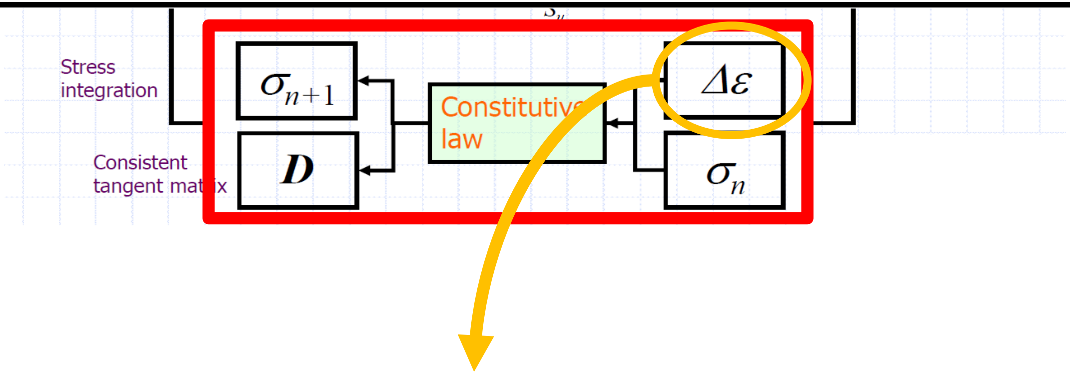
Inverse Material Identification



- ❑ Numerically converged value (e.g. FEMU)
- ❑ Experimentally measured value (e.g. VFM)

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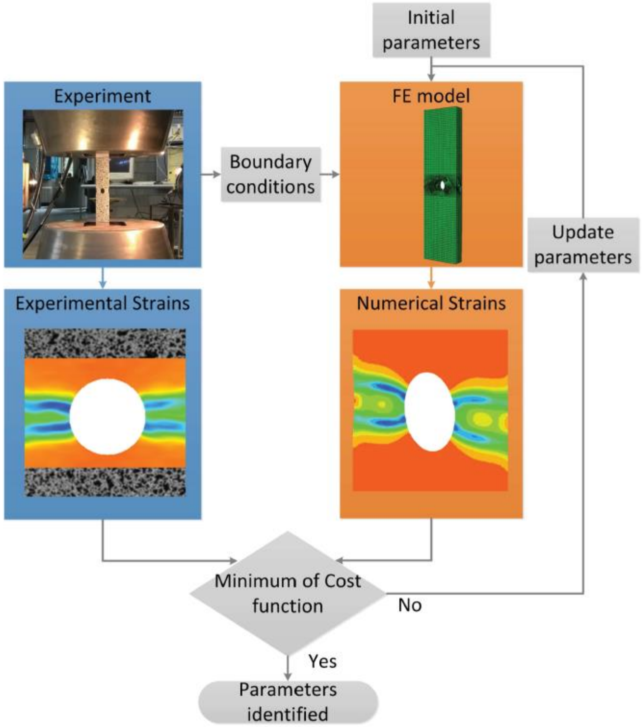
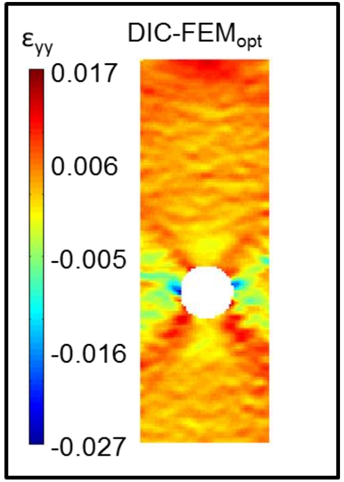
Finite Element Model Updating (FEMU)



Finite Element Model Updating (FEMU)

$$C(\boldsymbol{p}) = \sum_{i=1}^{m}\sum_{j=1}^{n_i}\left[\left(\frac{\varepsilon_{xx,ij}^{exp} - \varepsilon_{xx,ij}^{num}}{\varepsilon_{xx,RMS,i}^{exp}}\right)^2 + \left(\frac{\varepsilon_{yy,ij}^{exp} - \varepsilon_{yy,ij}^{num}}{\varepsilon_{yy,RMS,i}^{exp}}\right)^2 + \left(\frac{\varepsilon_{xy,ij}^{exp} - \varepsilon_{xy,ij}^{num}}{\varepsilon_{xy,RMS,i}^{exp}}\right)^2\right]$$

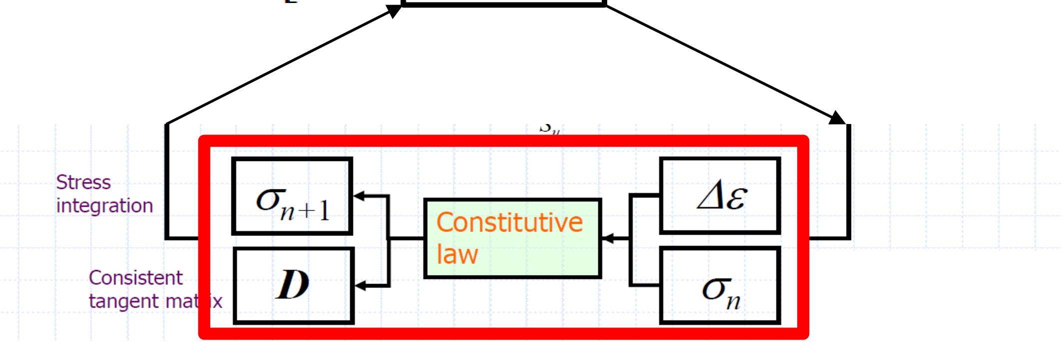


Virtual Fields Method

Global Equilibrium

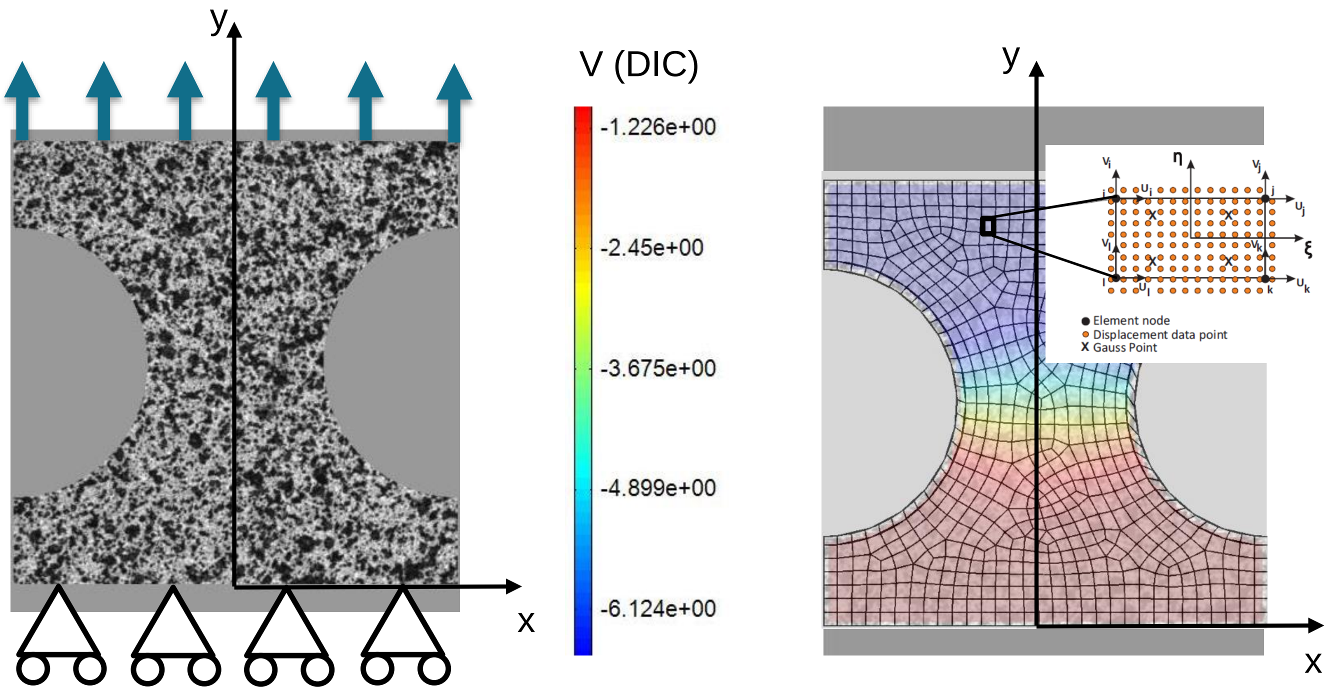
$$-\int_V \boldsymbol{\sigma}(\boldsymbol{p},\boldsymbol{\varepsilon}) : \boldsymbol{\varepsilon}^* \, dV + \int_{\partial V} \boldsymbol{T} \cdot \boldsymbol{u}^* \, dS = 0$$

$$C(\boldsymbol{p}) = \sum_{i=1}^{k} \sum_{j=1}^{m} \left[-\int_V \boldsymbol{\sigma}_j(\boldsymbol{p},\boldsymbol{\varepsilon}_j) : \boldsymbol{\varepsilon}^{*i} \, dV + \int_{\partial V} \boldsymbol{T}_j \boldsymbol{u}^{*i} \, dS = 0 \right]^2$$

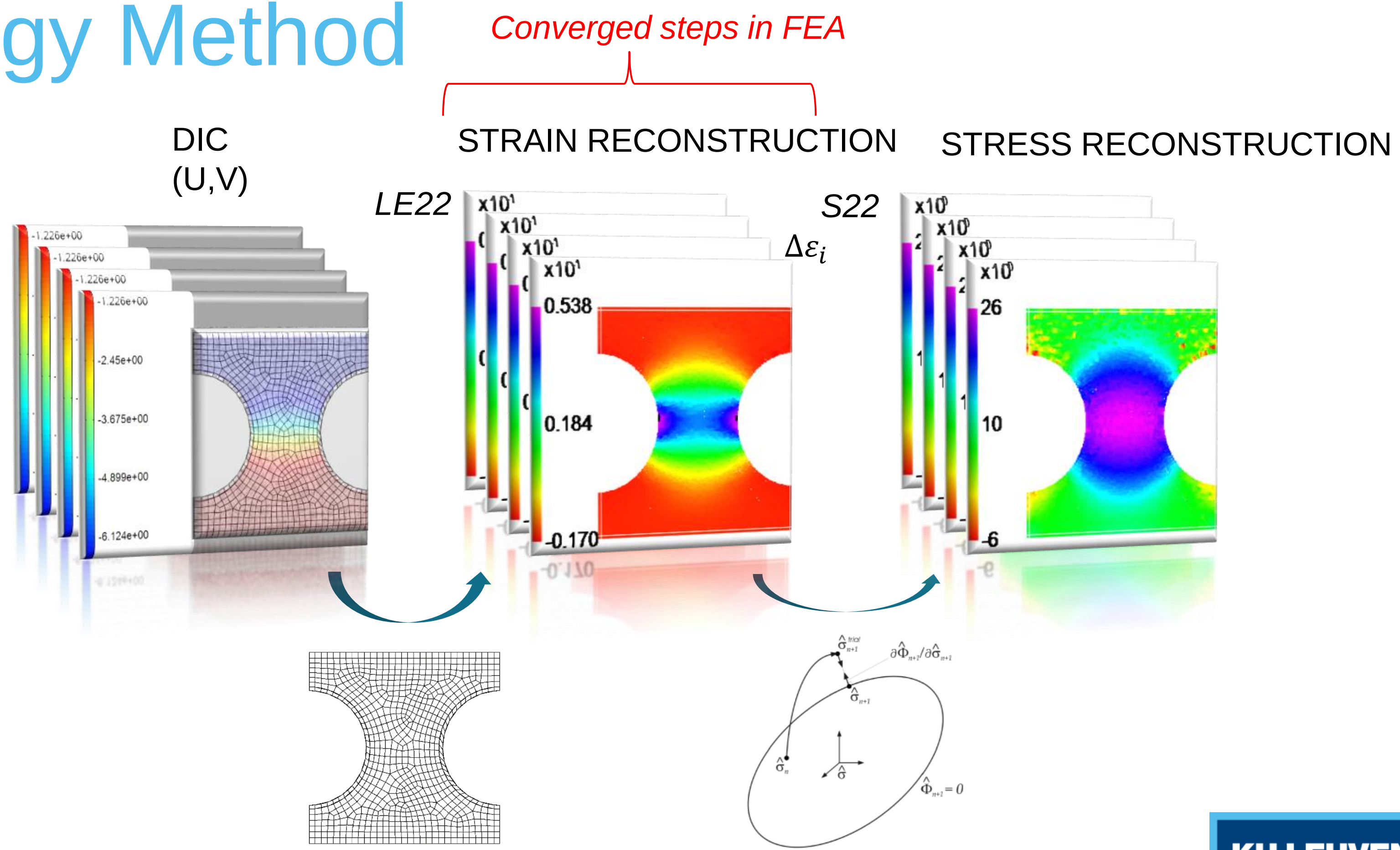


Energy Method

$$C(\mathbf{p}) = \frac{1}{2}\sum_j^m\left[W_{int,j}(\mathbf{p}) - W_{ext,j}\right]^2$$

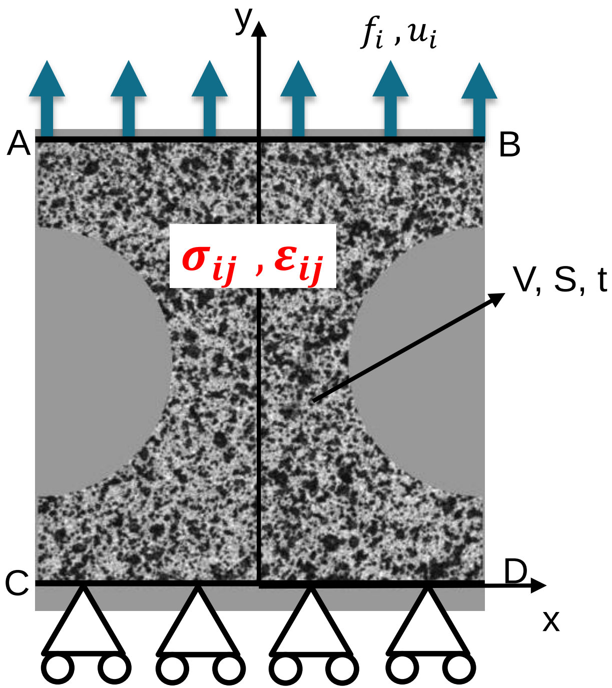


Energy Method



Energy Method

During a quasi-static test the internal work equals the external work



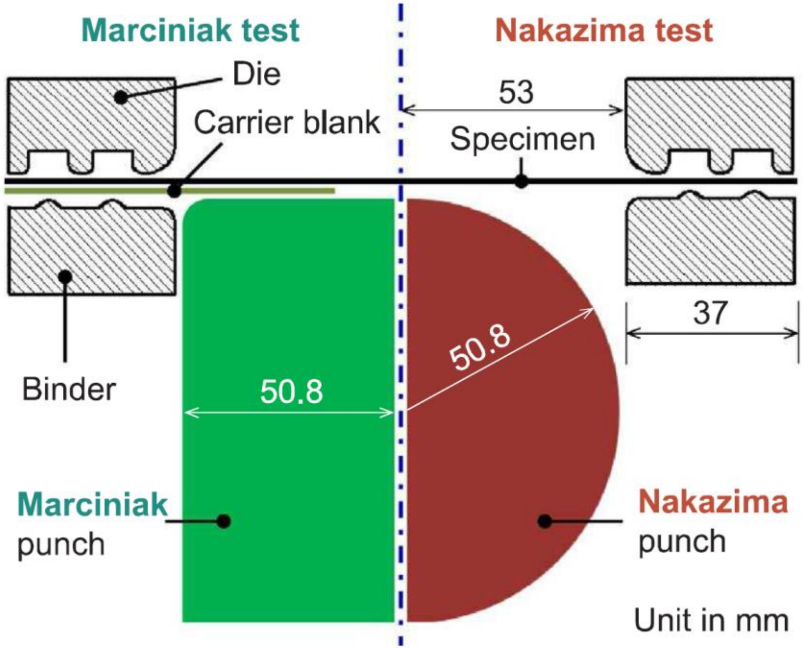
- ✓ Lines A-B and C-D remain straight
- ✓ The Volume V is constant
- ✓ The stresses and strains are constant through the thickness of the sheet

$$W_{ext} = \int_S \left(\int_{u_i} f_i du_i \right) dS = \int_u F\, du$$

$$W_{int} = \int_V \left(\int_{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij} \right) dV = \int_S t \left(\int_{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij} \right) dS$$

$$C(\mathbf{p}) = \frac{1}{2} \sum_{i=0}^{N} ((W_{ext})_i - (W_{int}(\mathbf{p}))_i)^2$$

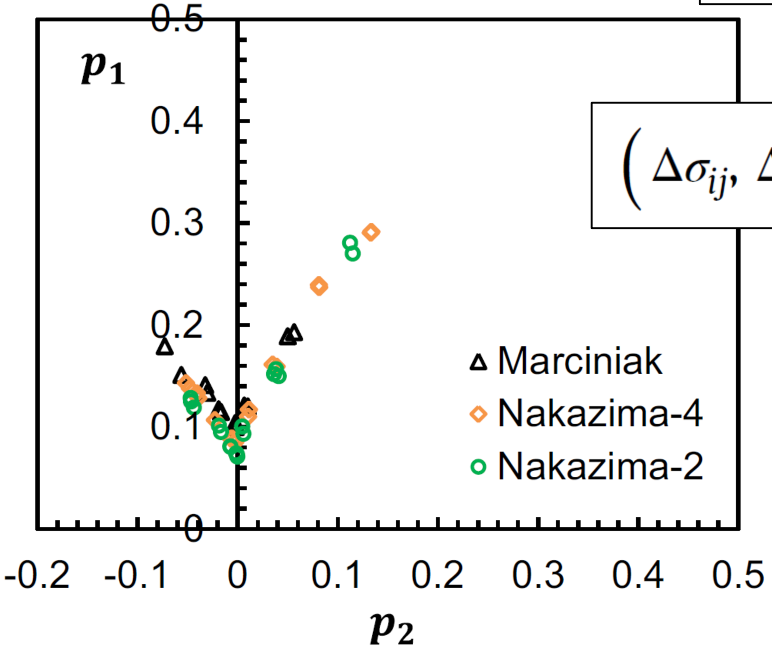
Forming Limit Curve



Forming Limit Curve*

Compensate for nonlinear strain path

$$\left(\Delta\sigma_{ij},\ \Delta p_{ij},\ \Delta e_{ij}, \Delta\bar{\varepsilon}_p\right) = \mathrm{UMAT}\left(P_i,\ \bar{\varepsilon}_p,\ \sigma_{ij},\ \Delta\varepsilon_{ij}\right)$$



* Junying Min et al., *Compensation for process-dependent effects in the determination of localized necking limits*, International Journal of Mechanical Sciences117(2016)115–134

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