Isochromatic fringe features for a crack under contact loading

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Abstract

Over the past decades, continuous welded rails (CWRs) have increasingly replaced jointed rail systems due to better performance and reduced maintenance. However, ambient temperature variation and high cyclical contact stress from the vehicular loads have a pronounced effect on CWRs due to restricted strains and stress build-up. In low temperatures, this can potentially lead to fracture of the rail and derailments. Therefore, an understanding of the stress state due to the combination of a crack and contact loading is needed. In order to study this effect closely, a set of equations for in-plane stresses is obtained by the linear superposition of contact stress field equations and crack-tip stress field equations. Using these along with the stress-optic law, theoretical isochromatics are plotted in order to get a qualitative appreciation of the influence of various parameters on the stress field. This finds its application in studying CWR fracture behaviour as well as other problems where such a load combination is prevalent.

Introduction

Studying the problem of combined contact and crack-tip stress field forms an important step towards the development of mathematical models which can be used for crack growth studies and failure prediction in CWRs and other mechanical components where such an adverse load combination is witnessed during service. In this context, photoelastic studies play an important role as the isochromatic fringes are very sensitive to changes in input parameters and some studies on this problem have been reported in the literature. Guagliano *et al* [1] have attempted the photoelastic determination of plane stress SIFs (K_I and K_{II}) of a surface crack on rails under the action of rolling contact loading for the case of a vertical surface crack. However, in order to capture the features of more complex stress fields arising from different crack orientations and lengths, a multi-parameter solution is required where different crack combinations can be analyzed in the presence of contact loading. There is a lacuna in the literature regarding a solution which can capture this combined phenomenon and aid such analyses. As a preliminary step in this direction, a combined set of equations is obtained by linear superposition and the theoretical equations of isochromatic fringes are obtained for the entire domain by invoking the stress-optics law. Finally, coloured fringes are plotted to see the influence of input fracture and contact stress parameters on the theoretical isochromatics with a focus on making critical observations for improvement.

Stress field equations

Within the linear elastic domain, the contact stress field equations relating normal and tangential loads by the friction law [2] and the corrected Atluri-Kobayashi multi-parameter stress field equations [3] for a planar condition are added in accordance with Fig. 1a as:

$$\begin{aligned}\sigma_x = &-\frac{a}{\pi\zeta}\left[\left(a^2+2x^2+2y^2\right)\frac{y\psi_1}{a}-\frac{2\pi y}{a}-3xy\psi_2+\mu\left\{\left(2x^2-2a^2-3y^2\right)\psi_2+\frac{2\pi x}{a}+2\left(a^2-x^2-y^2\right)\frac{x\psi_1}{a}\right\}\right]\\ &+\sum_{n=1}^{\infty}\frac{n}{2}A_{In}r^{\frac{n-2}{2}}\left\{\left\{2+(-1)^n+\frac{n}{2}\right\}\cos\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\right\}\\ &-\sum_{n=1}^{\infty}\frac{n}{2}A_{IIn}r^{\frac{n-2}{2}}\left\{\left\{2-(-1)^n+\frac{n}{2}\right\}\sin\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-3\right)\theta\right\}\end{aligned} \quad (1)$$

$$\begin{aligned}\sigma_y = &-\frac{ay}{\pi\zeta}\left[a\psi_1-x\psi_2+\mu y\psi_2\right]+\sum_{n=1}^{\infty}\frac{n}{2}A_{In}r^{\frac{n-2}{2}}\left\{\left\{2-(-1)^n-\frac{n}{2}\right\}\cos\left(\frac{n}{2}-1\right)\theta+\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\right\}\\ &-\sum_{n=1}^{\infty}\frac{n}{2}A_{IIn}r^{\frac{n-2}{2}}\left\{\left\{2+(-1)^n-\frac{n}{2}\right\}\sin\left(\frac{n}{2}-1\right)\theta+\left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-3\right)\theta\right\}\end{aligned} \quad (2)$$

$$\tau_{xy} = -\frac{a}{\pi\zeta}\left[y^2\psi_2 + \mu\left\{\left(a^2+2x^2+2y^2\right)\frac{y\psi_1}{a} - \frac{2\pi y}{a} - 3xy\psi_2\right\}\right] + \sum_{n=1}^{\infty}\frac{n}{2}A_{In}r^{\frac{n-2}{2}}\left\{\begin{array}{l} -\left\{(-1)^n+\frac{n}{2}\right\}\sin\left(\frac{n}{2}-1\right)\theta \\ +\left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-3\right)\theta \end{array}\right\} \quad (3)$$

$$-\sum_{n=1}^{\infty}\frac{n}{2}A_{IIn}r^{\frac{n-2}{2}}\left\{-\left\{(-1)^n-\frac{n}{2}\right\}\cos\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\right\}$$

where, $\zeta = \frac{1}{\frac{1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}\left[\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right]; A_{I1} = \frac{K_I}{\sqrt{2\pi}}; A_{II1} = \frac{-K_{II}}{\sqrt{2\pi}}$

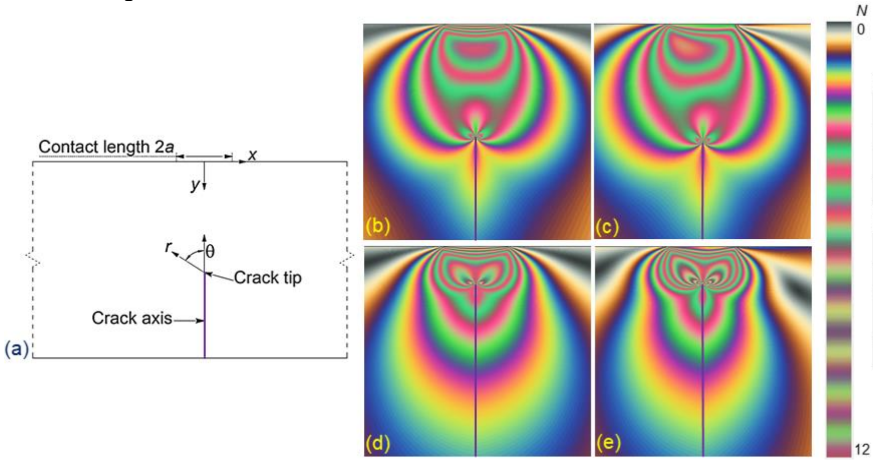
From the stress-optics law, the principal stress difference is given as:

$$\sigma_1 - \sigma_2 = \frac{NF_\sigma}{h} \quad (4)$$

where, N, F_σ and h represent the fringe order, material stress fringe value and model thickness, respectively. Using stress transformation to obtain principal stresses from Eq. 1-3 and substituting in Eq. 4, the whole field theoretical fringe order equations are obtained and plotted using a suitable colour code (Fig. 1).

Results and discussion

The theoretical coloured isochromatics are plotted from some cases by modifying the input parameters as presented in Fig. 1b-e.



Contact parameters

Semi-contact length (a)	0.4 mm
Friction coefficient (μ)	0.1
Young's Moduli [$E_{1,2}$]	2.2 GPa
Poisson's ratio (ν)	0.34

Fracture parameters

A_{I1}	1.0 MPa$\sqrt{\text{mm}}$
$A_{In\,(n\neq1)}$, A_{IIn}	0.0
Material stress fringe value (F_σ)	12 N/mm/fr.
Thickness	6 mm

Fig. 1 Theoretical isochromatics for (a) General schematic and inputs; Vertical crack (b) without friction (c) with friction; Advancing vertical crack (d) without friction (e) with friction

Frontal loops ahead of the crack are observed with a single parameter A_{I1} solution in the case of advancing crack in contact stress field which requires multiple parameters to reconstruct for a general crack problem. This suggests interaction between the crack tip and contact stress field due to crack extension. Also, the case of advancing crack with friction leads to stress amplification. Further, fringe orders greater than zero are observed along the crack axis which is a free surface. This observation suggests a need for a closer look into the localised nature of contact as well as crack tip stress field which needs to be accommodated into the equations by further conducting numerical and photoelastic experimental studies.

Conclusion

This preliminary study is a step towards a parametric analysis of the case of contact loading in the presence of a crack. There is need for mathematical improvements to better capture the localized effects which can be achieved by numerical and experimental studies. This refinement in the solution can help obtain fracture and contact parameters reliably for complex stress fields problems such as CWR fracture which forms the next step for this study.

References

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