UNIVERSITY OF Southampton

Identification from full-field measurements – Short review and perspectives

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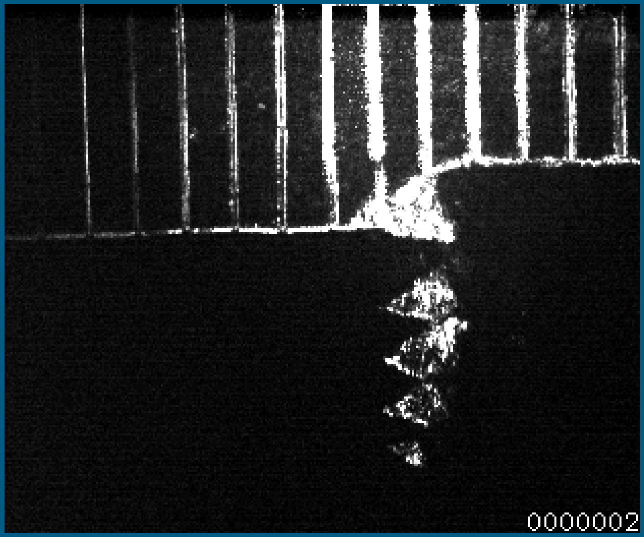
Introduction

- Great progress in computational mechanics
  - Simulation of machining
    - Large strains elasto-plasticity
    - Large strain rates
    - Localization
    - Friction/thermal behaviour



- Problem
  - Many material parameters required
  - How to obtain them?

Introduction

- Standard tests: tensile test on rectangular specimen
  - Uniform stress state
  - Uniaxial stress strain curve

  - Very poor information (very boring!)
  - Very restrictive assumptions (constraints)

Develop the experimental identification procedures of the future !

Introduction

- Step change: instrumentation
  - Standard tests rely on strain gauges / extensometer
  - Point or average/global measurements
    - Need for a priori stress distribution
- Technological breakthrough
  - Full-field strain measurements
  - Thousands or more simultaneous measurement points
  - Relieves usual constraints on testing configurations

Statement of the problem

- Basic equations

I Equilibrium equations (static)

$$\sigma_{ij,j} + f_i = 0$$ + boundary conditions — strong (local)

or

$$-\int_V \sigma_{ij}\varepsilon^*_{ij}dV + \int_{\partial V_f} T_i u^*_i dS + \int_V f_i u^*_i dV = 0$$ weak (global)

II Constitutive equations (elasticity)

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$$

III Kinematic equations (small strains/displacements)

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

Statement of the problem

	Known	Unknown
Direct problem	C_{ijkl} Geometry Boundary conditions	$\sigma_{ij}, \varepsilon_{ij}, u_i$

- Tools for solving this problem
  - Direct integration (closed-form solution)
  - Approximate solutions
  - Galerkin, Ritz
  - Finite elements, boundary elements…
  - etc…

Statement of the problem

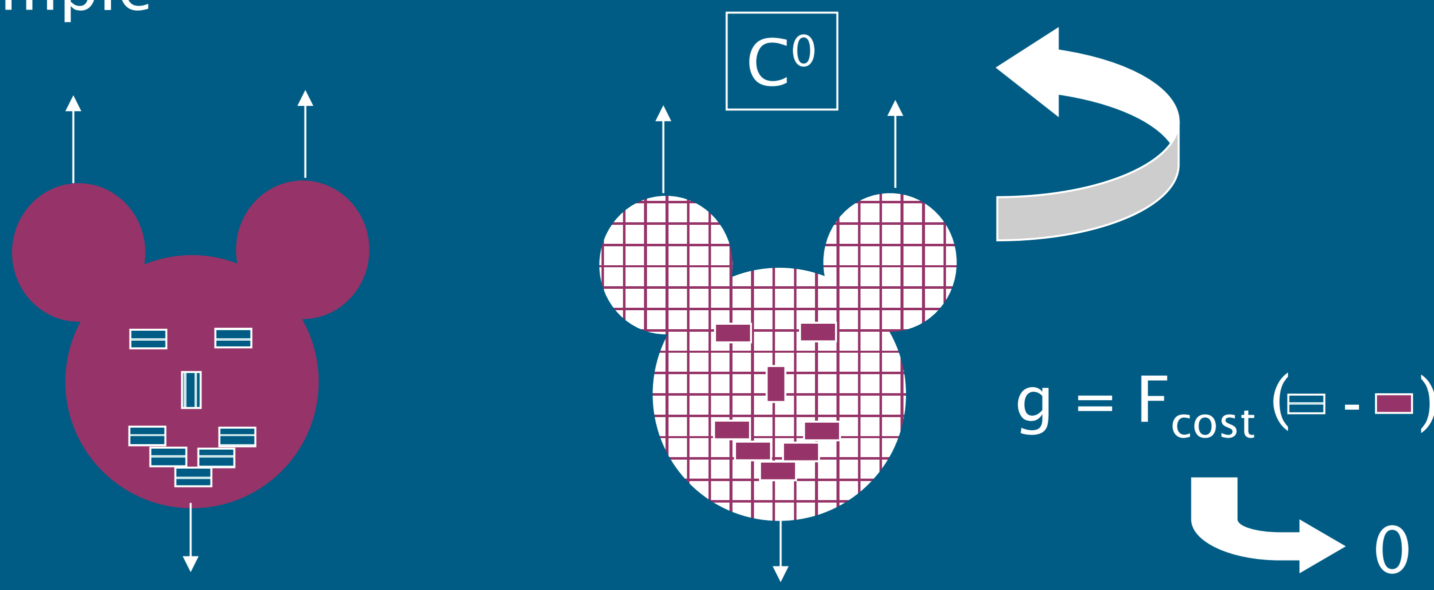
	Known	Unknown
Inverse problem	ε_{ij}, u_i (measured) Geometry Some information on the boundary conditions (load cell)	C_{ijkl} σ_{ij}

- Tools for solving this problem
  - Statically determinate tests:
    Closed form solution of Eq. I (uncoupled system)
    Force BC, simple geometry
    Ex.: tensile test, bending tests (on rect. beams) etc…

Resolution strategies

- Tools for solving this problem
  - Model updating
    Idea: iterative use of tool for direct problem (analytical or approximate)

Example



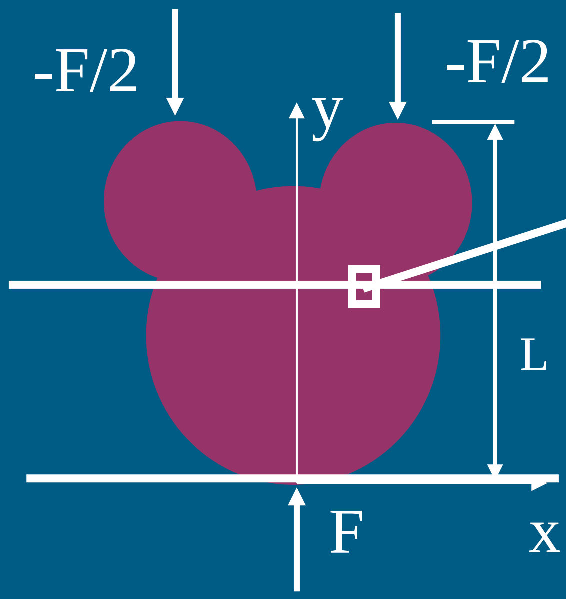
Resolution strategies

- Model updating
  - Advantages
    - General method (full-field measurements not compulsory)
    - Tools already developed
  - Shortcomings
    - Sensitive to boundary conditions (generally badly known)
    - CPU intensive (for numerical approximations and non-linear equations…)
    - Not fully dedicated to full-field measurements

⇒ Alternative tool: the Virtual Fields Method

Resolution strategies

- The Virtual Fields Method
  - Idea: use global equations (and not local)



$$\varepsilon_{yy}(x_0, y_o) \quad \sigma_{yy}(x_0, y_o)?$$

$$\int_S \sigma_{yy} dxdz = -F$$

Integrate over y

$$\int_V \sigma_{yy} dxdydz = -FL$$

Resolution strategies

- Constitutive behaviour

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{xy} & Q_{xx} & 0 \\ 0 & 0 & \dfrac{Q_{xx} - Q_{xy}}{2} \end{bmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{pmatrix}$$

In-plane linear elastic isotropy

$$\int_V \sigma_{yy} dxdydz = -FL \Rightarrow \int_V (Q_{xy}\varepsilon_{xx} + Q_{xx}\varepsilon_{yy}) dxdydz = -FL$$

Material is homogeneous

$$Q_{xx} \int_V \varepsilon_{yy} dxdydz + Q_{xy} \int_V \varepsilon_{xx} dxdydz = -FL$$

Resolution strategies

- Surface measurements only

Constant strains through the thickness

$$Q_{xx}\int_S \underbrace{\varepsilon_{yy}}_{?}dxdy + Q_{xy}\int_S \varepsilon_{xx}dxdy = -\frac{FL}{t}$$

$$\int_S \varepsilon_{yy}dxdy \approx \sum_{i=1}^{n} \varepsilon_{yy}^{i} s^{i}$$

$\mathbf{s^i}$ is the surface of each pixel

n is the number of strain data points

If all pixels have the same size s (usually the case for CCD/CMOS based measurements)

$$\sum_{i=1}^{n} \varepsilon_{yy}^{i} s^{i} = s\sum_{i=1}^{n} \varepsilon_{yy}^{i} = \frac{S_d}{n}\sum_{i=1}^{n} \varepsilon_{yy}^{i} = S_d\,\overline{\varepsilon_{yy}} \qquad \overline{\varepsilon_{yy}} = \frac{1}{n}\sum_{i=1}^{n} \varepsilon_{yy}^{i}$$

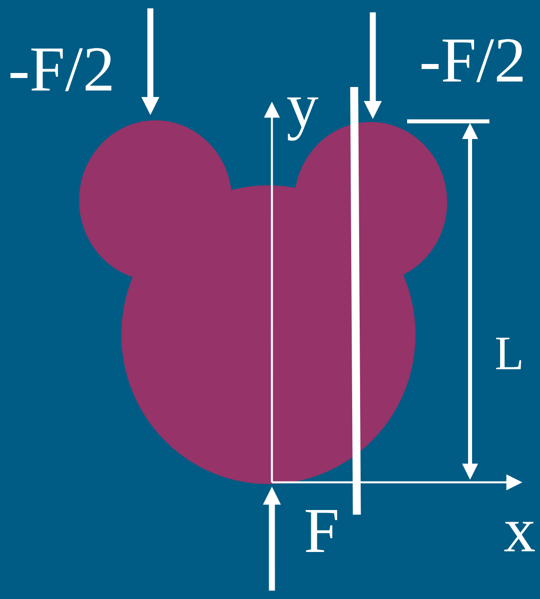
$\mathbf{S_d}$ is the surface of the disc

Resolution strategies

- Finally

$$Q_{xx}\bar{\varepsilon}_{yy} + Q_{xy}\bar{\varepsilon}_{xx} = \frac{-FL}{tS_d}$$

$$Q_{xx}\bar{\varepsilon}_{xx} + Q_{xy}\bar{\varepsilon}_{yy} = 0$$



$$\int_S \sigma_{xx}\,dxdz = 0$$

Integrate over x

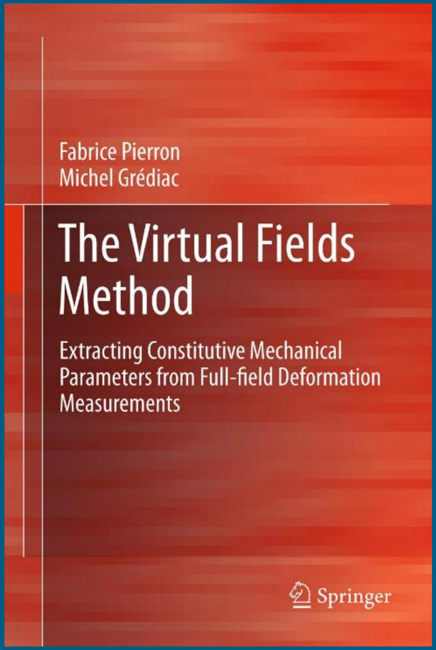
$$\int_V \sigma_{xx}\,dxdydz = 0$$

$$\begin{bmatrix} \bar{\varepsilon}_{xx} & \bar{\varepsilon}_{yy} \\ \bar{\varepsilon}_{yy} & \bar{\varepsilon}_{xx} \end{bmatrix} \begin{pmatrix} Q_{xx} \\ Q_{xy} \end{pmatrix} = \begin{pmatrix} 0 \\ \dfrac{-FL}{tS_d} \end{pmatrix}$$

$$Q_{xx} = \frac{-FL\bar{\varepsilon}_{yy}}{tS_d(\bar{\varepsilon}_{yy}^2 - \bar{\varepsilon}_{xx}^2)} \qquad Q_{xy} = \frac{FL\bar{\varepsilon}_{xx}}{tS_d(\bar{\varepsilon}_{yy}^2 - \bar{\varepsilon}_{xx}^2)}$$

Resolution strategies

- More details in



- Other strategies
  - Avril S., Bonnet M., Bretelle A.-S., Grédiac M., Hild F., Ienny P., Latourte F., Lemosse D., Pagano S., Pagnacco E., Pierron, F. **(2008)**. Overview of identification methods of mechanical parameters based on full-field measurements. *Experimental Mechanics, 48(4), 381-402.*

The pioneers

- Prof. Michel Grédiac - 1989
  - Ecole des Mines de St-Etienne, France, now University of Clermont-Ferrand
  - Motivation: reduced number of tests from composite identification
  - Bending test on anisotropic plate, full-field slope measurements
  - Virtual Fields Method (though term coined in 2000)

Prof. Alain Vautrin

Grédiac M., & Vautrin, A. (1990). A new method for determination of bending rigidities of thin anisotropic plates. Journal of Applied Mechanics-Transactions of the ASME, 57(4), 964-968.

Grédiac, M. (1989). Principle of virtual work and identification. *Comptes Rendus de L'Académie des Sciences, Serie II, 309(1), 1-5.*

The pioneers

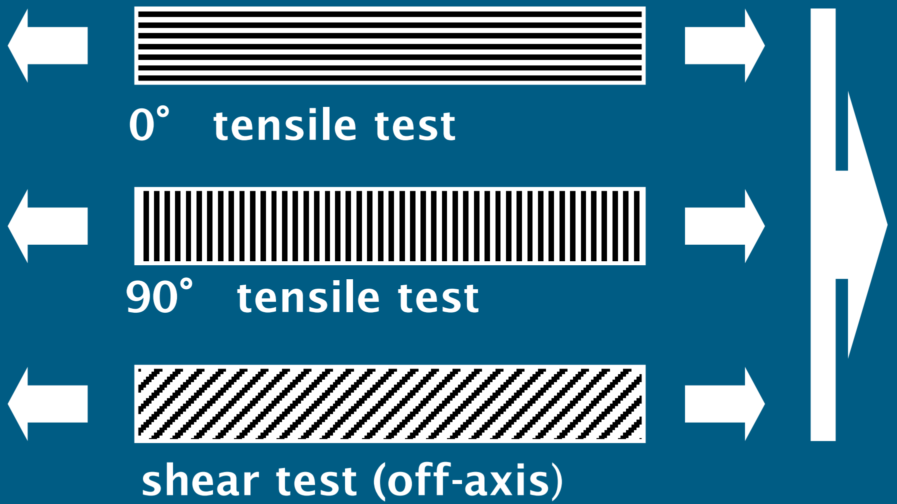
- Prof. Cees Oomens - 1991
  - Technical University of Eindhoven
  - Motivation: biological materials
  - Extended to elasto-plasticity later on (1998)
  - Measurements by image correlation

Meuwissen, M. H. H., Oomens, C. W. J., Baaijens, F. P. T., Petterson, R., & Janssen, J. D. (1998). *Journal of Materials Processing Technology*

Oomens, C.W.J., Ratingen v, M.R., Janssen, J.D., Kok, J.J., & Hendriks, M.A.N. (1993). *Journal of Biomechanics.*

Motivation

- Extract more information from 1 test



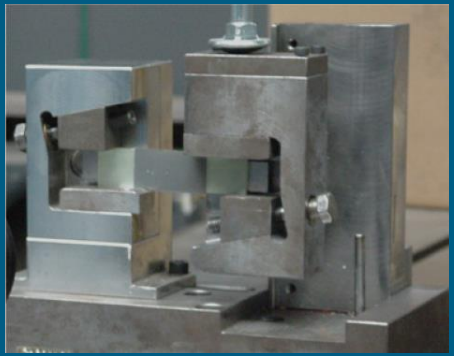
E_{xx}

E_{yy}

ν_{xy}

G_{xy}

3 tests
4 parameters



E_{xx}

E_{yy}

ν_{xy}

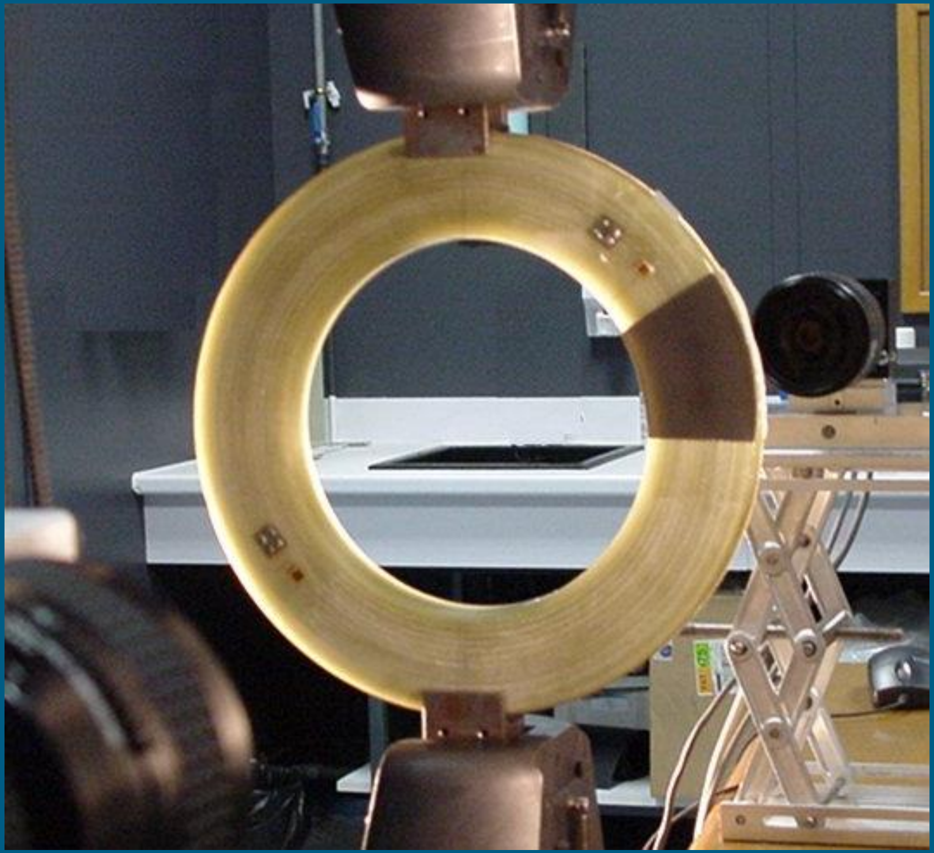
G_{xy}

1 test

Chalal, H., Avril, S., Pierron, F., & Meraghni, F. (2006). Composites Part A

Motivation

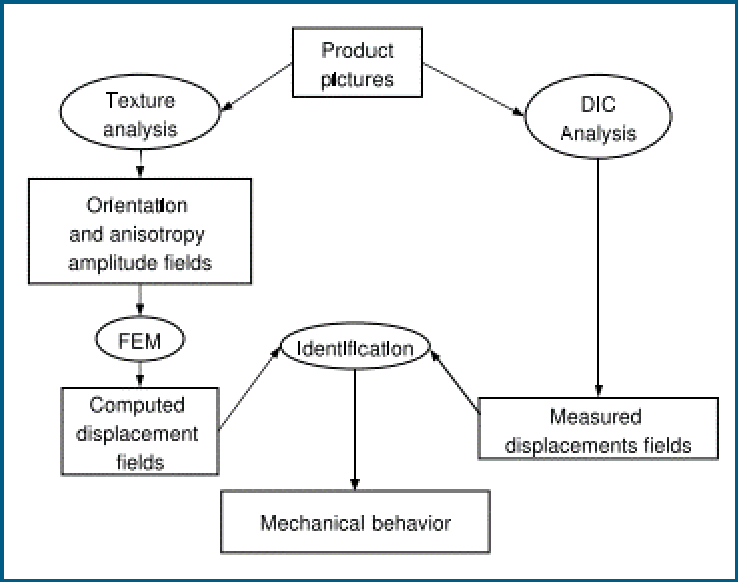
- Complex test geometry

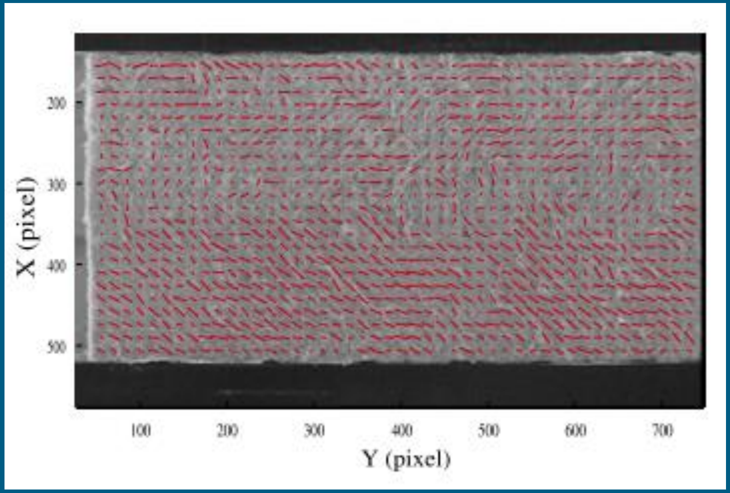


Moulart, R., Avril, S., & Pierron, F. (2006). Composites Part A.

Motivation

- Complex material behaviour
  - Crimped mineral wools
  - Spatially varying material directions
  - DIC and FEMU

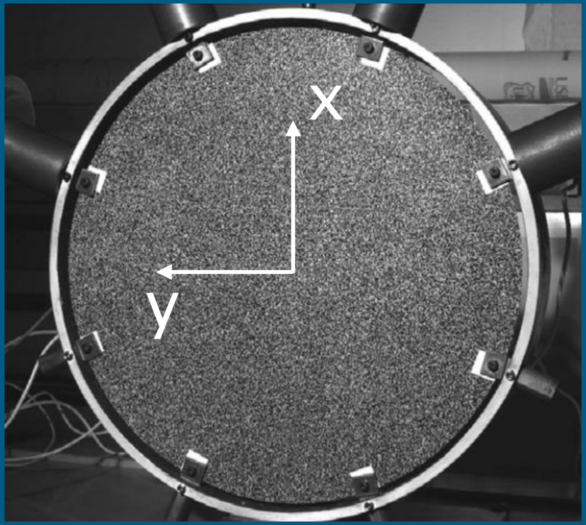




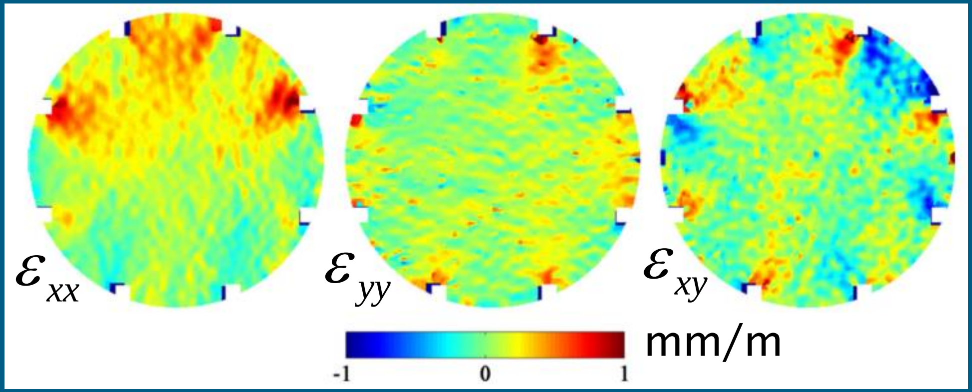
Witz, J.-F., Roux, S., Hild, F., & Rieunier, J.-B. (2008). Journal of Engineering Materials and Technology.

Motivation

- Complex material behaviour
  - Orthotropic paper webs
  - DIC with drumhead test
  - VFM: Stiffness and orthotropy axes
  - Next step: heterogeneity



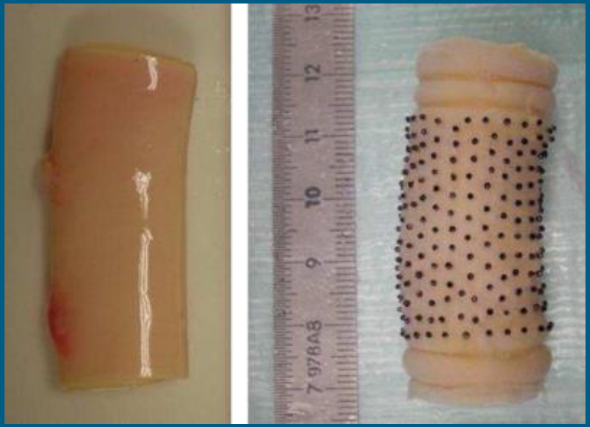
Drumhead specimen



Considine, J. M., Pierron, F., Turner, K. T., & Vahey, D. W. (2014). Experimental Mechanics

Motivation

- Complex material behaviour
  - Biological materials
  - Arterial segments
  - Inflation tests
  - Marker tracking
  - VFM in large deformation
  - Hyperalastic model



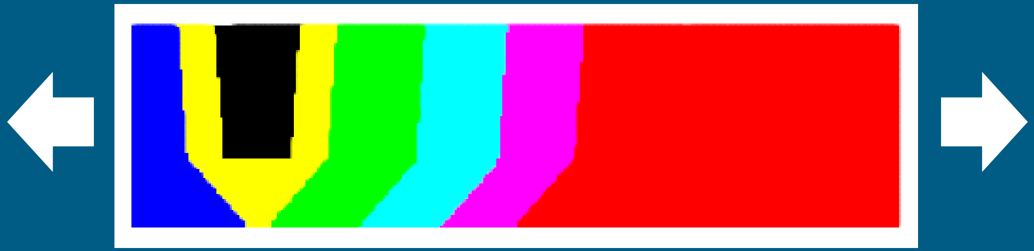
Arterial segments

Avril, S., Badel, P., & Duprey, A. (2010). Journal of Biomechanics.

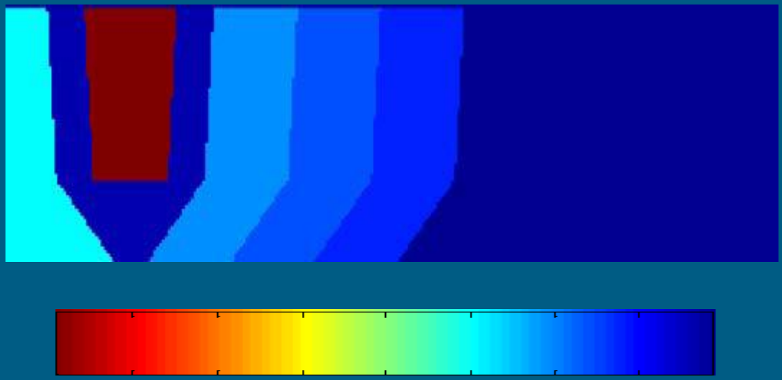
Motivation

- Heterogeneous materials
  - Welds

Seven zones
14 parameters

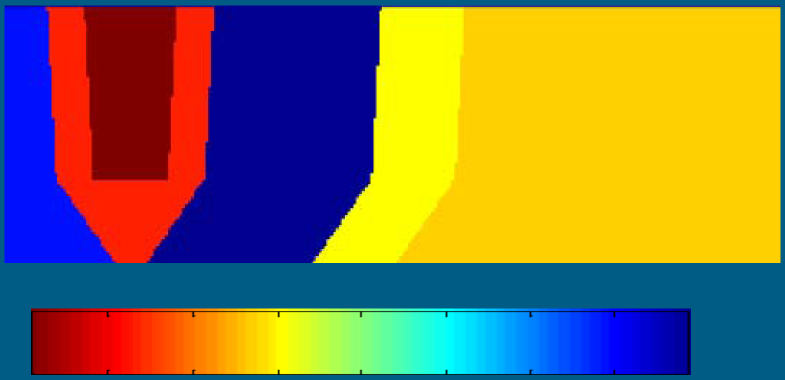


Yield stress (MPa)



694 682

Hardening modulus (MPa)

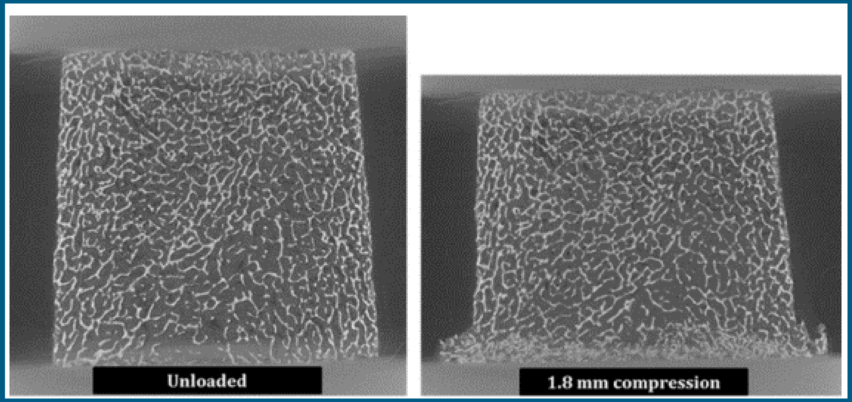


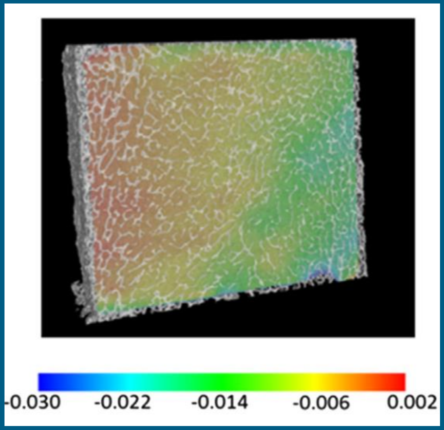
3200 2000

Sutton, M. A., Yan, J. H., Avril, S., Pierron, F., & Adeeb, S. M. (2008). Experimental Mechanics.

Hot topics

- Identification from volume strain data
  - X-ray CT in-situ compression of bone
  - Digital Volume Correlation (DaVis package)
  - VFM to identify Poisson's ratio (non-uniform strain distribution)



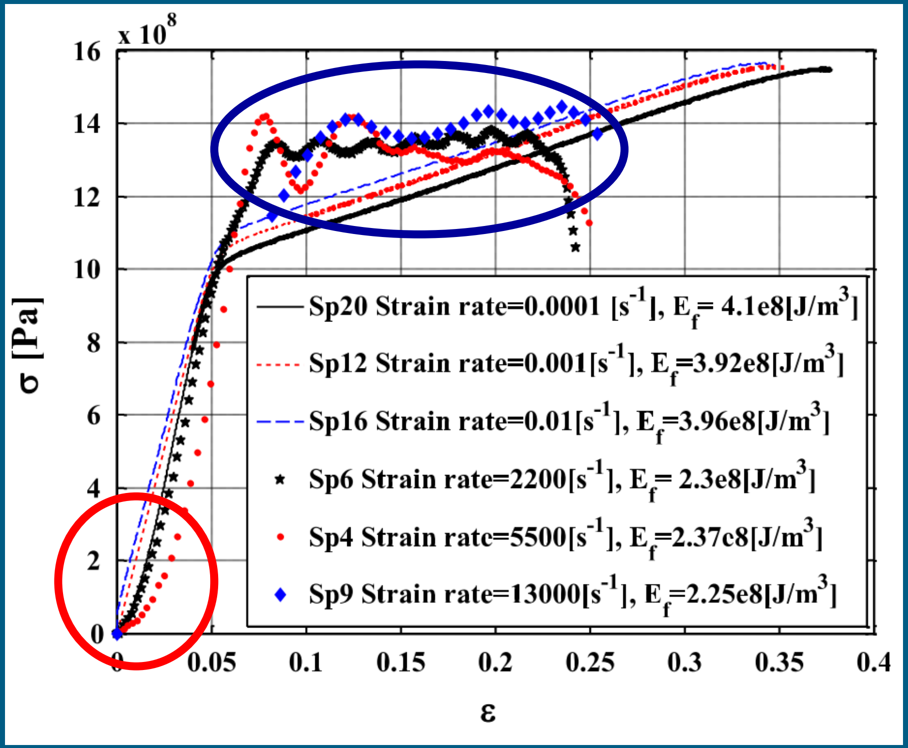


Axial strain

Gillard, F., Boardman, R., Mavrogordato, M., Hollis, D., Sinclair, I., Pierron, F., & Browne, M. (2014). Journal of the Mechanical Behavior of Biomedical Materials.

Hot topics

- High strain rate behaviour
  - Early days for FFM at high rates
  - Potential for step change in test data quality



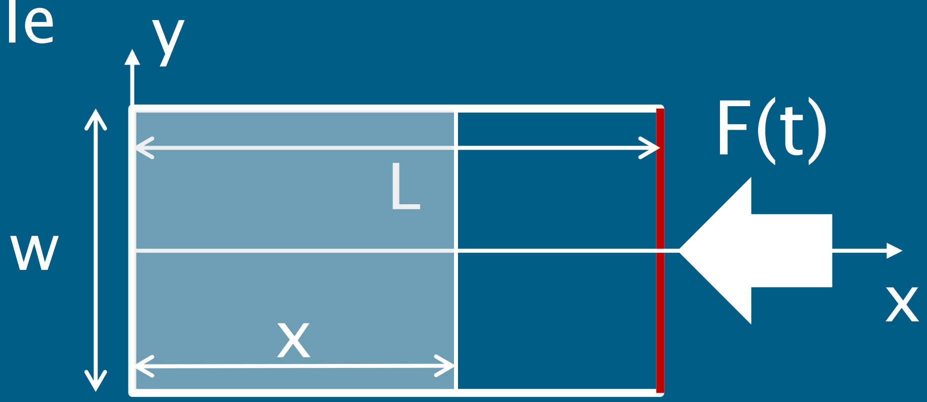
Ringing (dispersion)

Inertia effects

Osovski S. et al., Scripta Materialia, 2012. 67(7-8): p. 693-695.

High strain rate testing

- Use inertia as a load cell
  - Example



thickness: h

- Equilibrium of the structure

$$F(t) = \int_V \rho a_x(t)\,dV$$

$$\overline{\sigma_1}^{\text{red line}}(t) = \rho L \overline{a_1}^{S}(t)$$

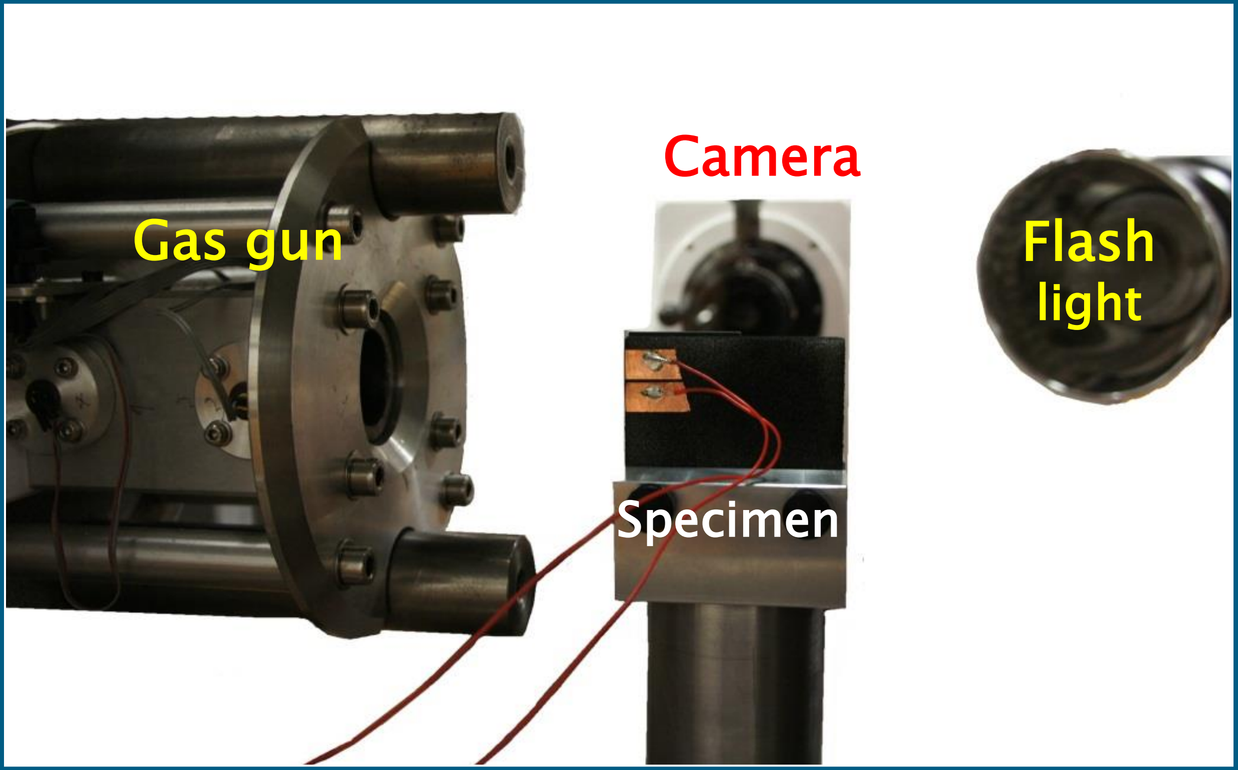
$$\boxed{\overline{\sigma_1}^{y}(x,t) = \rho L \overline{a_1}^{S}(x,t)}$$

$$\int_V \rho a_x(t)\,dV = \rho V \overline{a_1}^{\text{surface}}$$

$$F = hw\overline{\sigma_1}^{\text{red line}}$$

High strain rate testing

- Experimental set-up



Projectile: steel, 30mm diameter, 40mm long, 30 $m.s^{-1}$

Pierron, F., Zhu, H., & Siviour, C. (2014). Beyond Hopkinson's bar. Philosophical Transactions of the Royal Society A, 372(2023).

High strain rate testing

- Camera

  SHIMADZU HPV-X
  Inter-frame time: 0.2 μs
  Spatial resolution: 400 by 250
  Recorded images: 128

- Grid method
  - Grid pitch : 0.6 mm
  - 5 sampling pixels per period

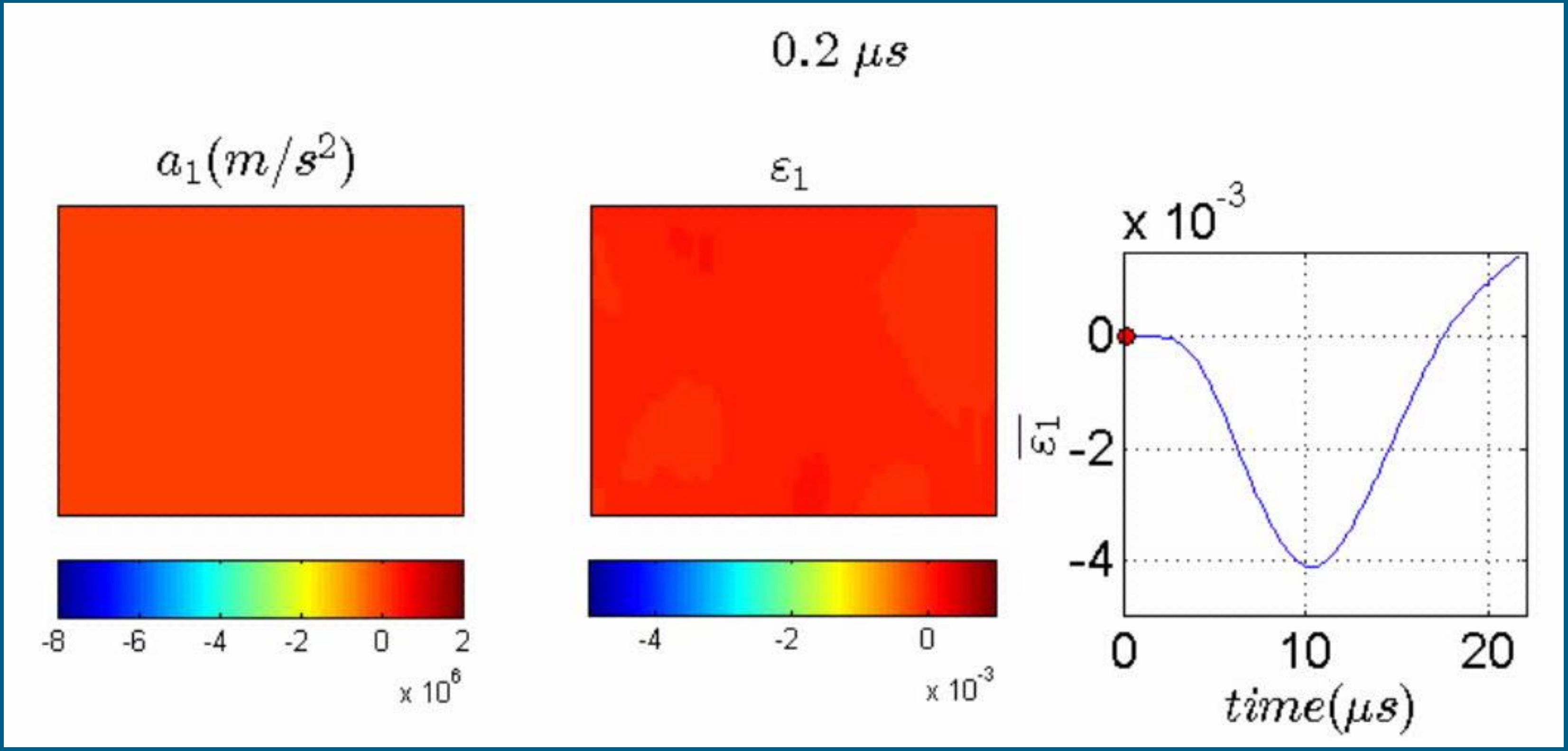
- Material
  - Carbon/epoxy QI
    (no strain rate sensitivity)

$[0/\pm 45/90]_s$

E = 47.5 GPa, ν = 0.3

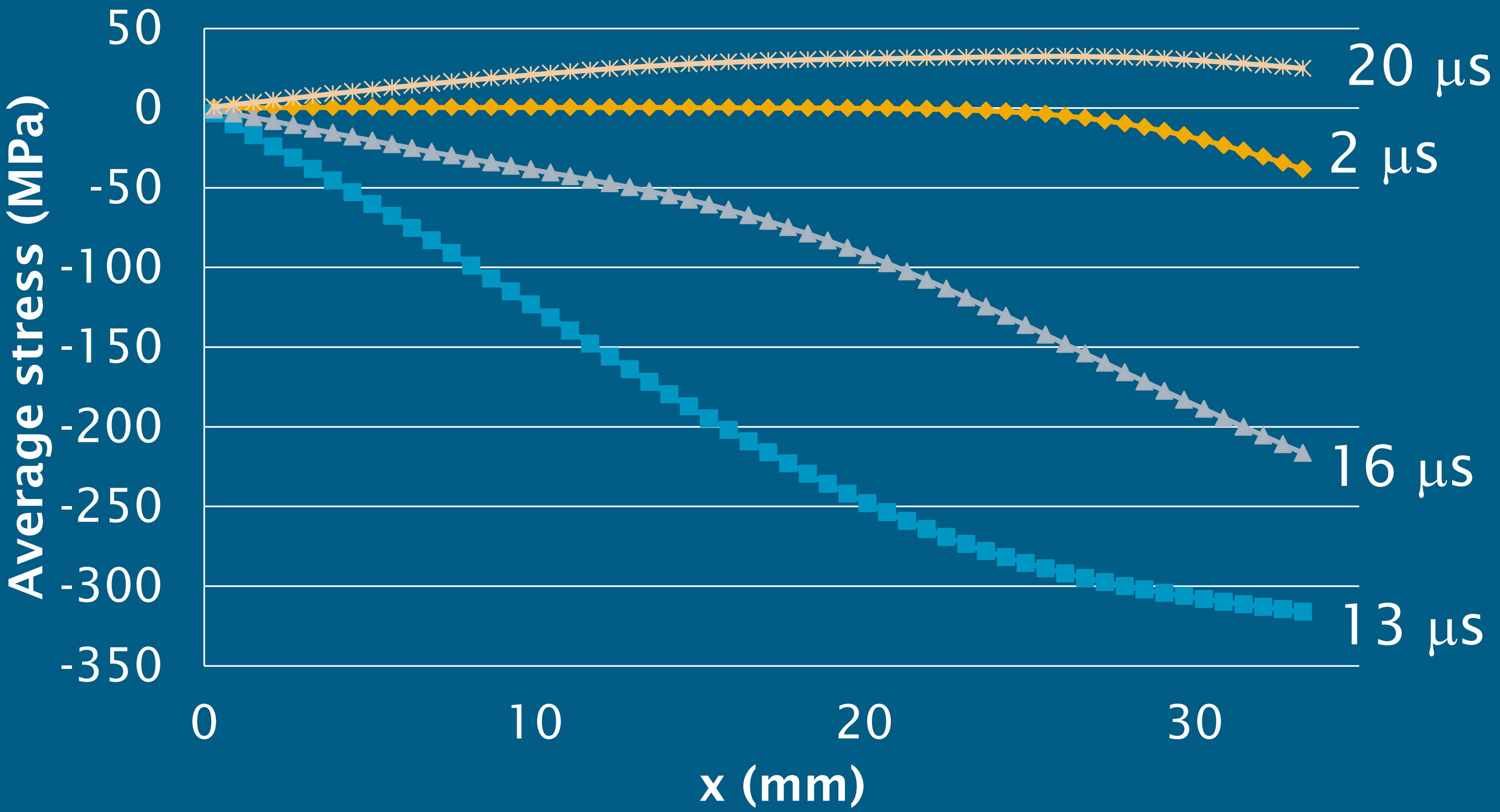
40 x 30 x 3.6 mm

High strain rate testing



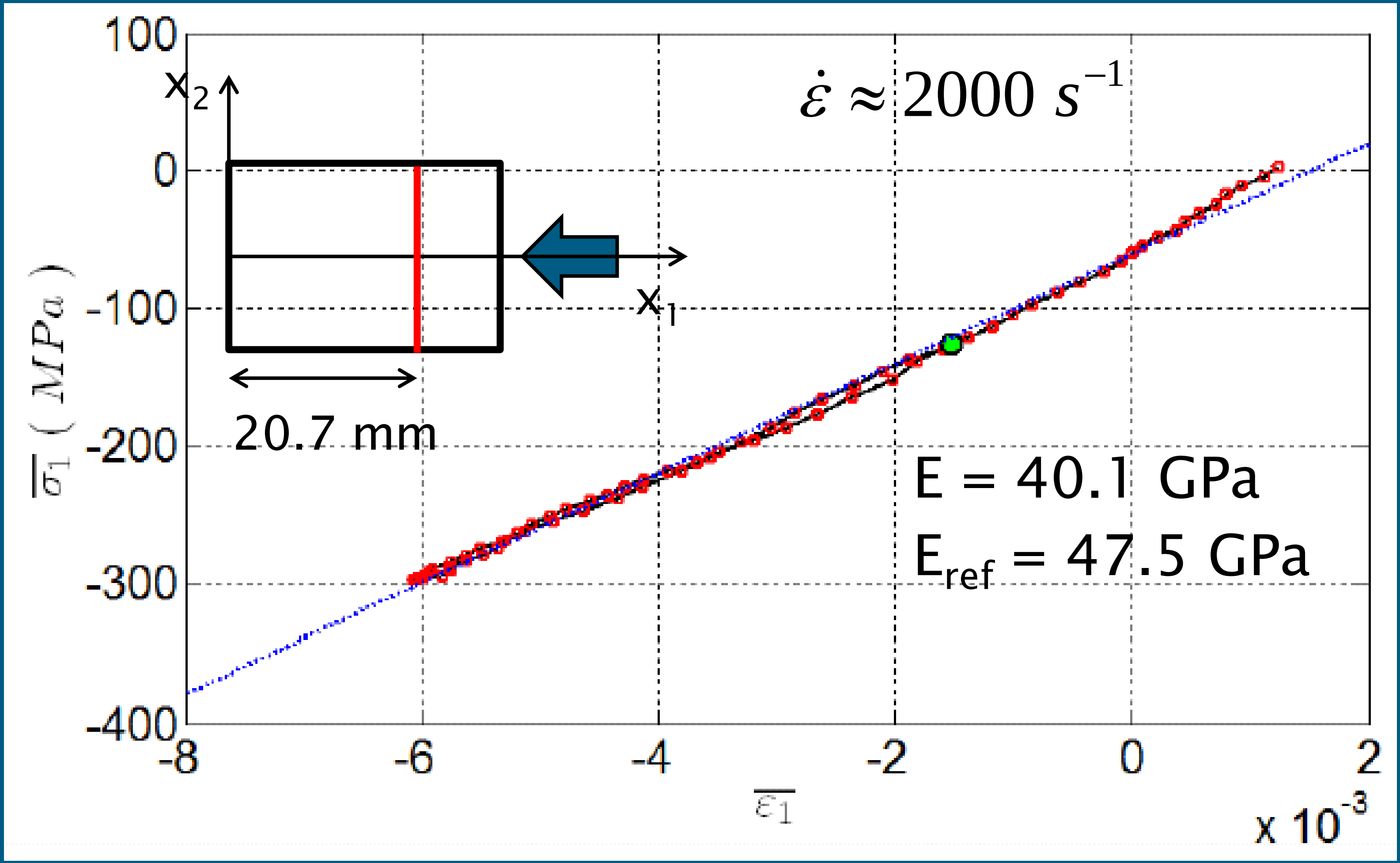
High strain rate testing

- Stress reconstruction



High strain rate testing

- Stress-strain curve

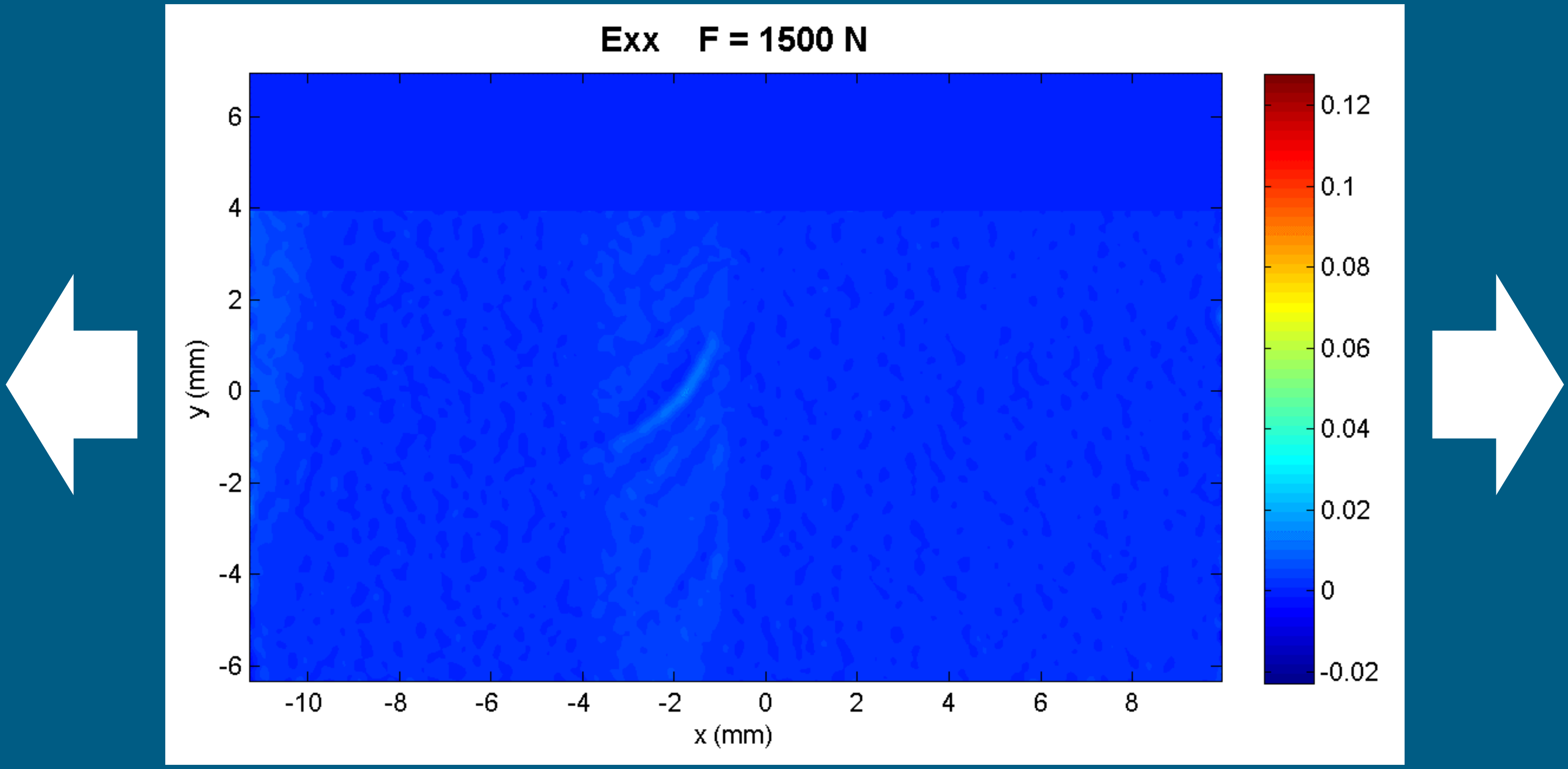


Future directions

- Design of new ‘standard tests’
  - Application specific
    - Welds
    - Composites
    - Metal forming
- Error propagation, uncertainty quantification
  - Simulator (see Pascal Lava’s presentation)
- Full integration with measurements
  - MatchID, see Pascal Lava’s presentation
  - Release of operational tools for the community

Thank you for your attention

- Tensile test on a magnesium friction stir weld



Welded zone | Base material

Tensile strain