

# Contour Method Residual Stress Uncertainty Evaluation and Sensitivity Analysis: a Friction Stir Welded Plate Case-Study

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**Abstract.** Residual stress in welded components can be evaluated at the macro scale using the Contour Method experimental technique. Given that several sources of uncertainty affect each step of the Contour Method procedure and very little is found in the literature about this aspect, we proposed a comprehensive approach to tackle it. Our method was applied on a Friction Stir Welded plate and eventually we performed a sensitivity analysis to recognise which is the most significant source of uncertainty that affects the application of the Contour Method.

**Keywords** Residual Stress · Friction Stir Welding · Contour Method · Uncertainty · Monte Carlo Method

## Introduction

The Contour Method (CM) is a destructive technique addressed to evaluate the residual stress at the macro scale. This method was developed by Prime in the early 2001 [1]. A Wire Electrical Discharge Machine (WEDM) cuts in half a plate containing residual stress, and due to the elastic stress relaxation, the cut surfaces deform. A Coordinate Measuring Machine (CMM) measures the normal component of the displacement,  $Z(x, y)$ , over each obtained surface following a raster-scan pattern. Following, using the set of samples  $Z(x, y)$  on the surface grid points,  $(x, y)$ , an interpolation process provides the interpolated function  $S(x, y)$ . Finally, a Finite Element (FE) simulation was employed to prescribe the interpolated displacement  $S(x, y)$ , changed in sign, over the cut surface of a 3D model of the half-plate. The FE simulation allowed then to assess the residual stress component, normal to the cut surface,  $\sigma_{zz}(x, y)$ .

Both the CMM displacement measurements and its interpolation are affected by uncertainties, which propagate throughout the CM procedure. To the best of our knowledge, the sources of uncertainty have been taken into account only by Olson et al. [2], so far. These authors develop an effective methodology, although they seem not to neatly separate the uncertainties arising from the measurement process and from the interpolation procedure.

We proposed an approach that separates the contributions of these uncertainties. Besides, we defined an additional source of uncertainty given by the WEDM repeatability. An application of the developed method was then applied to a Friction Stir Welded (FSW) plate. Eventually we performed a sensitivity analysis aimed at identifying the influence of each uncertainty source on the evaluated residual stress.

## Materials and Methods

We chose an AA6082-T6 FSW plate of dimensions width x thickness x length: 170 x 4x 770 mm. The CMM we used measures each sample  $Z(x, y)$ , with  $\sim 1\mu\text{m}$  of uncertainty. The data interpolation was performed by Python [3] routines. We adopted Gmsh [4] for designing the FE models and for data post-processing, and code\_aster [5] as a FE solver. In our approach we *separately* applied the Contour Method for each half-plates and we considered for each analysis three *independent* sources of uncertainty, listed below.

**CMM measurement uncertainty.** We estimated the effect of this source of uncertainty with the Monte Carlo method. For each Monte Carlo trial, we added a zero-mean random noise to  $Z(x, y)$ , having standard deviation equal to the uncertainty of the CMM, and thus we applied the CM. As a result we obtained a set of  $\sigma_{zz}(x, y)$ , whose standard deviation is  $U_M(x, y)$ .

**Interpolation uncertainty.** We fixed a third-order bivariate spline to interpolate  $Z(x, y)$ , thus giving the function  $S(x, y)$ . We added a zero-mean random noise to  $S(x, y)$ , having standard deviation equal to the standard deviation of  $Z(x, y) - S(x, y)$ . Then we calculated the interpolation uncertainty  $U_I(x, y)$  with the Monte Carlo method as done for  $U_M(x, y)$ .

**WEDM repeatability uncertainty.** Let us denote the k-th half-plate with the superscript <sup>(k)</sup>. We applied the CM without any data perturbation. As a consequence we obtained  $\sigma_{zz}^{(1)}(x, y)$  and  $\sigma_{zz}^{(2)}(x, y)$  for each half

plate, in our particular case of a single cut. The WEDM repeatability uncertainty  $U_w(x, y)$  is computed as the standard deviation of the set  $(\sigma_{zz}^{(1)}(x, y), \sigma_{zz}^{(2)}(x, y))$ .

**Total uncertainty.** Finally, we combined the above uncertainties taking their quadrature:

$$U_T^{(k)}(x, y) = ((U_M^{(k)}(x, y))^2 + (U_I^{(k)}(x, y))^2 + (U_w(x, y))^2)^{1/2}; \quad k=1,2 \rightarrow U_T(x, y) = ((U_T^{(1)}(x, y))^2 + (U_T^{(2)}(x, y))^2)^{1/2} \quad (1)$$

**Sensitivity analysis.** We numerically amplified by a factor 2, 5 and 10 the standard deviation of the noise used in the evaluation of  $U_M^{(k)}(x, y)$  and  $U_I^{(k)}(x, y)$ , whereas  $U_w(x, y)$  is fixed. Accordingly, we computed the corresponding  $\sigma_{zz}^{(k)}(x, y)$  along with  $U_T^{(k)}(x, y)$ , and we assessed the influence of each source of uncertainty on  $\sigma_{zz}^{(k)}(x, y)$  and critically discussed it.

### Preliminary results

Fig. 1(a) shows the contour plot of  $\sigma_{zz}^{(1)}(x, y)$  while Fig. 1(b) shows the uncertainty-affected residual stress  $\sigma_{zz}^{(1)}(x, y)$  along the path depicted in Fig. 1(a). The error bars represent the uncertainty  $U_T^{(1)}(x, y)$  according to the Eq. (1), where  $U_M^{(1)}(x, y)$  and  $U_I^{(1)}(x, y)$  were evaluated running a 10-trial Monte Carlo analysis. Despite the limited number of trials, our method gave larger error bars at the plate borders.

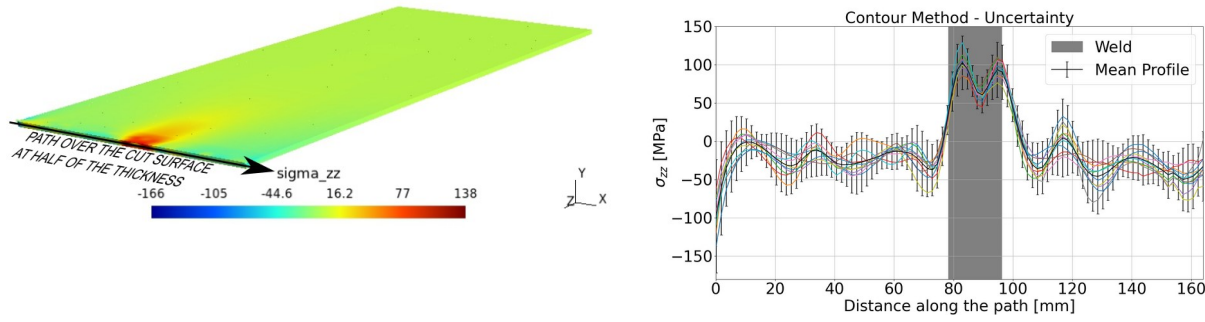


Figure 1: Preliminary results. (a) Contour plot of  $\sigma_{zz}^{(1)}$ . (b) Uncertainty evaluation of  $\sigma_{zz}^{(1)}$  along the path

### Conclusions

We have presented an extension of the current state of the art methodology for evaluating the uncertainty of the CM, and we applied our methodology on a FSW plate. The proposed route takes directly into account the uncertainty of the CMM along with those given by the data interpolation and by WEDM repeatability. By combining these factors, we estimated the total residual stress uncertainty carried by the CM. Finally we performed a sensitivity analysis to determine which sources of uncertainty show the greatest influence on the evaluated residual stress. We hope our approach will give a better insight on the uncertainties affecting residual stress evaluation and therefore find new ways to improve the accuracy of the CM.

### Acknowledgments

The authors acknowledge L.U.P. di Bailotti (Udine, Italy) for the WEDM cut and the Laboratory for Advanced Mechatronics (University of Udine) for the CMM measurements.

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