A framework for non-deterministic model validation using full-field measurements

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Summary. A technique to quantitatively assess the predictive capability of non-deterministic models against full-field measurements is demonstrated. The technique, based on the orthogonal decomposition of the measured and predicted field responses and the subsequent statistical pooling of their coefficients, allows the aggregation of multiple full-field response data in a universal probabilistic space. This enables, the incorporation of high-volume field data into an area-based comparison metric and allows for simple and efficient assessment of a model's predictive capability.

Introduction

Computational models are used for the design and optimisation of structures. It is important for engineers to possess confidence that these numerical surrogates can accurately represent the actual structure well enough that they can be exploited to draw meaningful inferences. It is also valuable to be able to convey that confidence in terms of a quantitative statement to decision makers and certification bodies alike. This is usually done through a process known as validation where the degree of the model's proximity to the real world is determined.

In order to perform a validation, comparisons between model results and experimental data must be made. Traditionally, this has been performed using point measurements (e.g. strain-gauges, accelerometers) thus leaving the engineers ignorant about the state of the largest part of the structure. Nonetheless, the advent of full-field measurements offers the potential of more comprehensive validation. However, this abundance of data is accompanied by numerous technical issues, including comparing data from sources with different coordinate systems, data pitch, and data types, many of which can be alleviated using techniques such as orthogonal decomposition [1]. This technique has been proven to be efficient by transforming original incomparable field results to equal sized feature vectors thus allowing for quantitative comparison.

In addition to the aforementioned, the advances in scientific computing along with the need for comprehensive risk assessment of critical structures has shifted the focus from deterministic models to Monte Carlo simulations, where multiple runs of the same model with varying inputs are performed. The need to evaluate these multiple predictions creates new challenges, which this paper addresses. The outcome is to collate the different available pieces of information from both the model and the experiments in a way that allows for efficient and quantitative validation.

Background

It is normal practice for experimentalists to obtain multiple measurements (when possible) that allow them to draw inferences about a quantity of interest. Then, using some descriptive statistic or confidence intervals, the population parameters from which these values are assumed to arise from can be estimated. One way to present multiple observations is to use Empirical Distribution Functions (EDF). These consist of multiple steps that correspond to the cumulative relative frequency of the observations. Thus, they monotonically increase and their range lies between [0,1] (the probability space). Traditionally, EDFs are used to identify whether the empirical data are adequately represented by the selected family of probability distributions (e.g. normal, uniform, etc.) or to quantify the difference between various datasets (e.g. Model Predictions vs Experimental Measurements). Of course, with more experimental data, the steps become less abrupt and the function looks smoother. Examples of various empirical distributions are given in Fig 1.
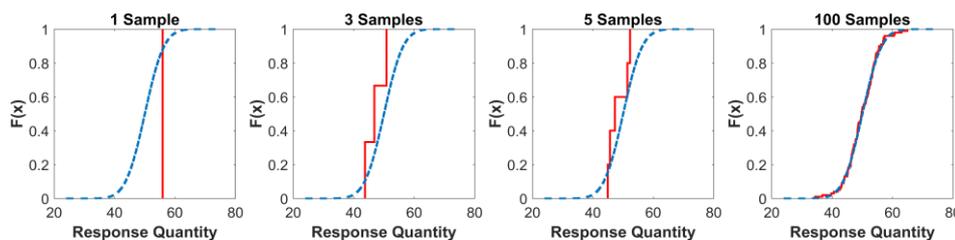

Fig 1. The empirical distribution functions (continuous-lines) for different sample sizes have been plotted against the cumulative distribution function (dashed-lines) of their normally distributed population ($\mu=50,\sigma=5$).

Independently, measurements from repetitions of the experiment and predictions from Monte Carlo simulations can be accumulated to build analogous EDFs which are then plotted against each other.

However, it is not uncommon for stress analysts to take single measurements at a number of point locations, for instance using strain gauges. In this scenario, an EDF can be constructed for the measurements. An example is shown in Fig 2. where the dashed curve represents predictions from a Monte Carlo simulation and the vertical line represents a single observation from an experiment. The different plots in figure 2. could represent measurements taken at different locations in a structure. If it is assumed that the measurements follow the distribution described by the predictions; then, following the u-pooling method suggested in [2], one can assign a u_i value to each observation that corresponds to the value of the predicted EDF at the intersection point (horizontal line in Fig 2.). According to the probability integral transform theorem, these u_i should be uniformly distributed. The EDF of the u_i-values is then plotted against the uniform Cumulative Distribution Function (CDF) and the difference to the uniform CDF expressed in the form of an integral defined as: $A = \int_{-\infty}^{+\infty} |U(x) - G(x)| dx$, which corresponds to the grey-filled area on the right side of Fig 2.
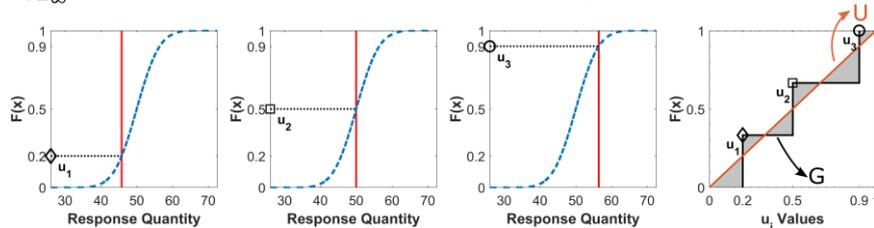

Fig 2. The empirical distribution functions for a single measurement (vertical continuous line) and for Monte Carlo predictions (dashed line) at three measurement locations (left three graphs) together with the corresponding three u_i values plotted against the uniform CDF (right graph).

Test Case

A case study was selected, in order to demonstrate the use of this technique to validate a computational model using full-field measurements. The object of interest was a composite T-joint representative of those found on stiffened aircraft panels. Information about the computational model and experimental testing can be found in [3]. Orthogonal decomposition was used to reduce the matrix of field data to a vector of kernel coefficients for both the measurements and predictions. The resultant coefficients for two repetitions of the Monte Carlo simulation and one set of measurements obtained from digital image correlation are shown in figure 3. These coefficients were used as inputs to the u-pooling method as described above to yield the quantitative comparison shown in Fig 3.
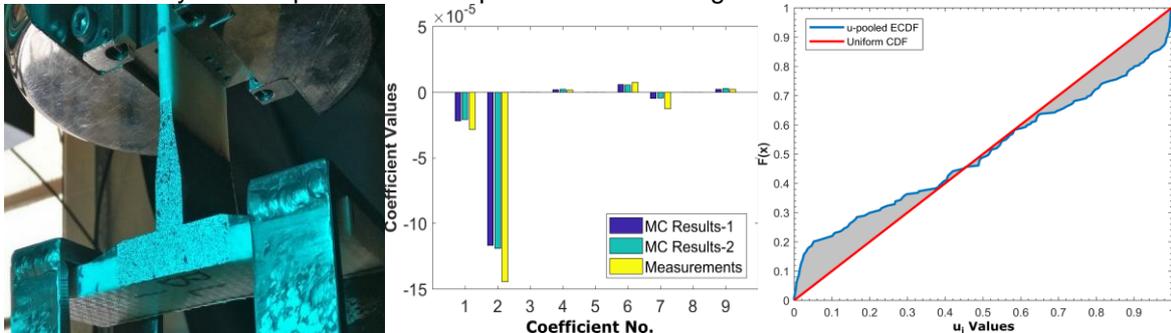

Fig 3. The first nine kernel coefficients for a region of interest from two Monte Carlo simulations and one digital image correlation measurement (left) and a comparison of the corresponding u-values with a uniform CDF (right).

Conclusion

A new method that allows the aggregation and evaluation of multiple field datasets has been demonstrated. This allows engineers to provide a quantitative statement about a model's predictive capability while incorporating all the available pieces of information in a compressed quantitative measure. It should be stated that it is possible to combine multiple regions of interest and different scales in the u-pooling thus allowing for efficient full-field validation.

References
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