Uncertainty of constitutive model parameters identified from heterogeneous DIC full-field measurements

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Abstract. Full-field strain measurement techniques are a powerful tool for material characterization. To evaluate reliability of the full-field identification procedure, it is crucial to investigate the influence of measurement uncertainty on the identified material parameters. In this paper, we present a novel method for evaluating uncertainty of strains measured with DIC, which propagates to the identified material parameters. We demonstrate the effectiveness of this approach by applying it to an open-hole sample as an example.

Introduction

Digital image correlation (DIC) allows full-field strain measurements, enabling us to perform experiments where the samples exhibit heterogeneous stress/strain response. Such an experiment can replace several series of simple experiments, as we can identify multiple material parameters from a single heterogeneous experiment. To assess the reliability of the identification, we need to calculate how the uncertainty in the measured strains propagates through the identification procedure to the identified values of the material parameters.

Methodology

Our approach to evaluating the confidence intervals of inversely identified constitutive model parameters from full-field measurements involves two parts. Firstly, we evaluate the uncertainty of DIC measurements using a collection of our previous measurements. Secondly, we examine the influence of measurement uncertainty on the identified parameters.

Evaluation of error distributions from past measurements. To account for uncertainties that could impact the overall measurement uncertainty of the DIC strain, we evaluated our existing collection of measurements. Our measurements were taken using the same DIC setup and a variety of planar samples [1-3], and we followed a similar approach to Badaloni [4], taking still images at the beginning of each measurement when the specimen was not loaded, allowing us to assume that the measured surface strains should be zero. However, we found that the strain values in reality closely follow a normal distribution $\boldsymbol{\mathcal{N}}(\boldsymbol{\mu},\boldsymbol{\sigma})$, indicating measurement errors. Since the strains should be zero, we considered the measured values as measurement errors $\boldsymbol{e}$. To avoid generalizing errors from a single measurement to the measurement uncertainty of a complete DIC system, which can vary slightly between setups [5], we evaluated 850 individual measurements made over the last years with the same DIC measurement setup of Dantec Dynamics Q-400 using two 2 MP cameras with 35 mm focal length lenses. We employed a two-step procedure, as illustrated in Fig. 1, to include the errors of the individual measurements in the total measurement uncertainty. First, we determined the mean $\boldsymbol{\mu_i}$ and standard deviation $\boldsymbol{\sigma_i}$ of the measured strain values (errors) at the beginning of each measurement separately for each strain component.

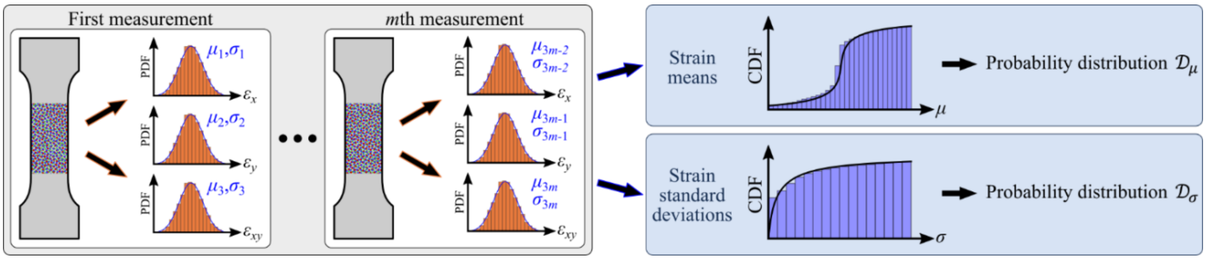

Fig. 1: The evaluation of strain error distributions from past measurements. First, the errors of the individual strain fields of the measurements are modelled using normal distributions (left). Second, since the normal distributions vary between measurements, we observe the probability distribution of the means and standard deviations of all measurements (right).

Next, we collected the means and standard deviations of the measurements and analysed their probability distribution. We discovered that the measurement errors were biased, meaning that their means were not equal to zero, as seen in Fig. 1. To account for the effects of the bias and the distribution of standard deviations in the next step of error propagation, we modelled the calculated distributions using the probability laws $\boldsymbol{\mathcal{D}_\mu}$ and $\boldsymbol{\mathcal{D}_\sigma}$. The mean values of the errors $\boldsymbol{\mathcal{D}_\mu}$ follow a generalized hyperbolic (GH) distribution with parameters $\{\boldsymbol{\lambda},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\delta},\boldsymbol{\mu}\}=\{\mathbf{0.0425978},\mathbf{2498.15},\mathbf{0},\mathbf{4.58485}\times\mathbf{10^{-6}},\mathbf{4.74097}\times\mathbf{10^{-6}}\}$. The standard deviations of the errors $\boldsymbol{\mathcal{D}_\sigma}$ follow a log-normal distribution with multiplicative parameters, where $\boldsymbol{\mu}^*=\mathbf{0.00184}$ and $\boldsymbol{\sigma}^*=\mathbf{6.05}$ (corresponding to a $\mathbf{MOE}(\boldsymbol{\varepsilon})_{\mathbf{68.3\%}}=\mathbf{0.005}$ strain). Importantly, the obtained probability distributions account for not only random noise, but also other systematic errors related to the experimental setup, which are often overlooked by researchers [6, 7].

Influence of DIC uncertainty on the identified parameters. To assess how the bias and random errors of the system propagate through the identification chain, we conducted Monte Carlo simulations (MC) to estimate the uncertainties of the material parameters. In each MC run, we generated new errors $\boldsymbol{\varepsilon}$ using the probability distributions $\boldsymbol{\mathcal{D}}_{\mu}$ and $\boldsymbol{\mathcal{D}}_{\sigma}$ that we approximated earlier. We randomly drew the mean $\boldsymbol{\mu}$ and standard deviation $\boldsymbol{\sigma}$ from these distributions, and used them to generate strain errors $\boldsymbol{\varepsilon}$ for the entire field, since our DIC full-field strain measurements closely follow $\boldsymbol{\mathcal{N}}(\boldsymbol{\mu}, \boldsymbol{\sigma})$. Next, we examined how these randomly generated errors $\boldsymbol{\varepsilon}$ propagate through the Finite Element Model Updating (FEMU) identification method, leading to an evaluation of the confidence interval. To investigate the impact of both random error and system bias on the identified parameters $\widetilde{\boldsymbol{\theta}}$, we analyzed how small perturbations in the data affected the model parameters. We followed a similar approach to Mathieu et al. [8], obtaining the following relation: $\boldsymbol{\Delta\theta} = ((\mathbf{J}^{\boldsymbol{T}}.\mathbf{J})^{-\mathbf{1}}.\mathbf{J}^{\boldsymbol{T}}).\boldsymbol{\varepsilon} = \mathbf{K}.\boldsymbol{\varepsilon}$, where $\mathbf{J}$ is the sensitivity matrix derived by varying the parameters $\widetilde{\boldsymbol{\theta}}$.

Results

We demonstrate the proposed method using an open-hole specimen [1] that is 25.4 mm wide and 0.8 mm thick, with a 4.2 mm diameter hole in the centre. The specimen is made of a unidirectional carbon fibre-epoxy composite with fibre orientation parallel to the longitudinal axis. The necessary material parameters for numerical computation of the Jacobian matrix were obtained from measurements and are equal to: $\widetilde{\boldsymbol{\theta}} = \{E_{xx}, E_{yy}, G, \nu\} = \{125\ \mathrm{GPa}, 5.6\ \mathrm{GPa}, 7.9\ \mathrm{GPa}, 0.23\}$. Applying our method, we generated a set of parameter variations $\boldsymbol{\Delta\theta}$, the probability distributions of which are shown in Fig. 2.

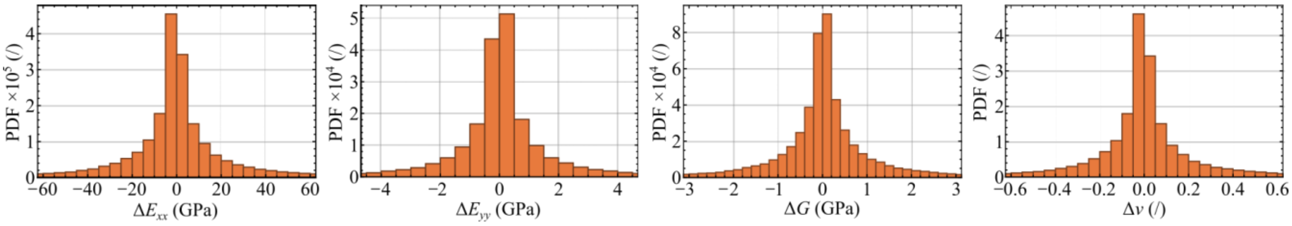

Fig. 2: Probability density distributions of the variations of the material parameters, determined by the proposed method.

The results indicate that the probability distributions of the variations in material parameters do not follow a normal distribution, as measurement errors were not modelled as such. By evaluating the Margin Of Error (MOE) of the distributions with 68.3% confidence, we obtain relative MOE values of $\mathrm{MOE}(\widetilde{\boldsymbol{\theta}}) = \{13.7, 21.4, 11.0, 73.7\}\%$. In contrast, if we were to model the errors using a normal distribution, as in the classical method [9], without taking into account system bias, we would obtain unrealistically small MOE values of $\mathrm{MOE}(\widetilde{\boldsymbol{\theta}}) = \{0.64, 2.02, 1.26, 2.99\}\%$.

Conclusion

A novel method for evaluating DIC measurement uncertainty is presented. In contrast to conventional methods, we considered the systematic error (bias) of the measured strain and the random error separately. The obtained error probability distributions are used to evaluate the uncertainties of inversely identified material parameters from full-field strain measurements. The proposed procedure is demonstrated on an open-hole sample. Our results demonstrate that the probability distributions of the identified orthotropic elastic parameters exhibit a non-normal shape, providing a more realistic measure of MOE compared to the literature.

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