Meshfree Digital Image Correlation for Accurate Deformation Measurement

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Abstract. A meshfree digital image correlation (MF-DIC) is proposed for more accurate deformation measurement. The classical meshfree method, element free Galerkin method (EFGM), which has been widely used in numerical studies, is used to construct the shape function in DIC. The principle of the proposed MF-DIC is different from the classical global DIC and subset-based local DIC. It does not resort to the subset or element for shape function construction. Instead, the shape function is constructed from a set of scattered nodes in a small support domain surrounding each point of interest. The shape function in MF-DIC is C^1-continuous, which guarantees a direct strain measurement from displacement maps. Benchmark tests on DIC challenge 2.0 shows that the proposed MF-DIC has an excellent balance between spatial resolution and measurement resolution for displacement and strain measurements.

Introduction

Digital image correlation (DIC) [1] is an advanced experimental mechanics method for measuring displacement and strain fields. It measures the displacement fields by comparing the image series of a speckled sample surface using image matching technique. The DIC algorithms, according to principal difference, can generally be classified to local DIC and global DIC [2]. Local DIC assumes continuous deformation in a small subset surrounding each calculation point. Each point is matched independently between reference image and deformed image. Local DIC is very efficient and has a good accuracy. It is currently the most popular DIC algorithm and has been adopted in the majority of open-source and commercial DIC software. The global DIC inspired by the finite element concept in numerical simulation, constructs the shape function from a mesh. The shape function is continuous in the whole region of interest instead of only in a small subset. All points are then matched simultaneously. The synergy of the shape function in DIC and numerical simulation offers global DIC the powerful ability to bridge experiments and simulation. However, in global DIC, the shape function still is only C^0 continuous at the nodes, and a high-quality mesh is required.

We propose to directly integrate the classical numerical method, meshfree method, into DIC algorithm for more accurate deformation measurement, which is termed as meshfree DIC (MF-DIC). In MF-DIC, a set of scattered nodes is first generated on the reference image. The displacement of every point on the reference image can be determined from the scattered nodes in a small domain surrounding this point. Similar to the finite-element based global DIC, the displacements at all nodes are then optimized simultaneously using the high-efficiency inverse-compositional Gaussian-Newton (IC-GN) algorithm. The strain field can then be analytically calculated from the displacement field.

Meshfree DIC

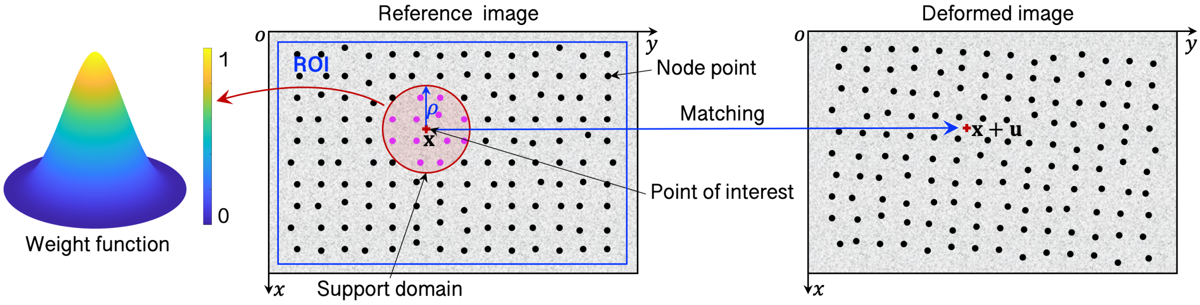

Fig. 1 Principal diagram of MF-DIC

Similar to all DIC techniques, in MF-DIC, the displacement maps are estimated by minimizing a correlation criterion. For simplicity, the sum of squared differences (SSD) cost function is used:

$$C_{SSD} = \int \left(f(\mathbf{x}) - g\big(\mathbf{x} + \mathbf{u}(\mathbf{x})\big)\right)^2 d\mathbf{x} \tag{1}$$

where f and g are the image intensity on the reference image and deformed image, respectively. The displacement $\mathbf{u}(\mathbf{x})$ at point $\mathbf{x}$ can be approximated using the displacement of the nodes in its support domain, see the circle in Fig. 1, which is expressed as

$$u(\mathbf{x}) \approx u^h(\mathbf{x}) = \sum_{i=1}^{M} \varphi_i(\mathbf{x}) u_i \tag{2}$$

where $u^h(\mathbf{x})$ is the approximated displacement at $\mathbf{x}$. M denotes the number of nodes in the support domain of $\mathbf{x}$, u_i the vertical displacement components of i-th node in this support domain, $\varphi_i(\mathbf{x})$ the shape function of *i*-th node, According to meshfree method, the displacement can then be approximated as

$$u^h(\mathbf{x}) = \mathbf{p}^{\mathrm{T}}(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{U}_{\mathrm{S}} \tag{3}$$

where $\mathbf{U}_{\mathrm{S}} = (u_1, u_2, \ldots, u_M)^T$ is the displacement vector of the nodes in the support domain. where

$$\mathbf{A}(\mathbf{x}) = \sum_{i=1}^{M} W(\mathbf{x} - \mathbf{x}_i)\mathbf{p}(\mathbf{x}_i)\mathbf{p}^{\mathrm{T}}(\mathbf{x}_i) \tag{4}$$

$$\mathbf{B}(\mathbf{x}) = [W(\mathbf{x} - \mathbf{x}_1)\mathbf{p}(\mathbf{x}_1) \quad W(\mathbf{x} - \mathbf{x}_1)\mathbf{p}(\mathbf{x}_1) \quad \ldots \quad W(\mathbf{x} - \mathbf{x}_M)\mathbf{p}(\mathbf{x}_M)] \tag{5}$$

The quadric spline weight function (see left inset in Fig. 1) is adopted:

$$W(\mathbf{x} - \mathbf{x}_i) = \begin{cases} 1 - 6r_i^2 + 8r_i^3 - 3r_i^4, & 0 \le r_i < 1 \\ 0, & r_i \ge 1 \end{cases} \tag{6}$$

with $r_i = \|\mathbf{x} - \mathbf{x}_i\|_2/\rho$ and ρ the radius of the support domain. $\mathbf{p}(\mathbf{x}) = (1, x, y, x^2, y^2, xy, \ldots,)^{\mathrm{T}}$ is the polynomial basis. Minimizing the cost function in Eq. (1), we can get the displacement at all nodes simultaneously with good accuracy. The displacement in Eq, (2) is C^1 continuous at $\mathbf{x}$. Consequently, the strain can be analytically calculated from the displacement directly. More details and results could be found in Ref. [3].

Conclusions

The performance of the proposed MF-DIC was evaluated on DIC challenge 2.0 [4]. According to the retrieved Metrological Efficiency Indicator [5], the proposed MF-DIC was proven to hold excellent balance between spatial resolution and measurement resolution compared with 26 state-of-the-art DIC algorithms. MF-DIC as a sister method to meshfree methods in computational mechanics, is also promising to be integrated with meshfree methods for constitutive parameters identification.

Reference

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