Thermoelastic Stress Analysis under signal undersampling conditions

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Abstract. This work examines how aliasing can be exploited in Thermoelastic Stress Analysis (TSA). By analyzing both oversampling and deliberate undersampling conditions, the study shows when aliased thermoelastic harmonics can be accurately recovered and when interference may degrade results. Findings demonstrate that carefully designed undersampling enables reliable amplitude extraction, supporting TSA with low-frame-rate IR cameras and high-frequency loading scenarios.

Introduction

Thermoelastic Stress Analysis (TSA) extracts harmonic temperature variations generated by cyclic elastic loading. These thermoelastic temperature modulations typically appear at frequencies *f* and 2*f*, where *f* is the primary loading frequency. Standard procedures for isolating these harmonics, such as the Discrete Fourier Transform (DFT) or Least-Squares fitting methods, require the signal to be sampled at sufficiently high frequencies. In common practice, at least three samples per cycle are needed (reducible to two when the harmonic phase is known [1]), meaning that a second harmonic requires $f_s \geq 3(2f)$, with f_s being the sampling rate. Therefore, identifying the second harmonic at 30 Hz would demand a minimum sampling rate of 90 Hz, or a first harmonic at 500 Hz an f_s of 1.5 kHz. Both scenarios become demanding and barely addressable for common cooled IR sensors, and definitely impossible for a slower microbolometer.

It is then useful to consider the possibility to capture high frequency thermoelastic harmonics by undersampling at specific rates able to capture aliased reproductions of the original waves of interest. The literature reports only a few applications or references to this practice [2,3], while no dedicated studies seem to have investigated the best aliasing sampling strategies for applying TSA. Such an approach could significantly widen TSA applicability in the following scenarios: 1) using either high-performance (30 fps) or low cost (< 10 fps) microbolometers [4]; 2) Use TSA at moderately high loading frequencies (200 Hz to 2 kHz, [3]); 3) extract thermoelastic harmonics during a long run fatigue test, where f_s must be much lower than f_L (load frequency) to handle extended monitoring datasets.

This work investigated the sampling conditions that allow aliased thermoelastic waves to be correctly identified.

Methodology

When sampling a harmonic signal of frequency f_L at sampling rate f_s, the aliasing frequency f_a can be expressed through a known relationship.

$$f_a = |f_L - m \cdot f_s| \quad \text{where } m \text{ is an integer ensuring that } f_a < f_s/2 = f_{ny} \tag{1}$$

The Nyquist (or folding) frequency $f_{ny}=f_s/2$ defines how real frequencies mirror into the discrete spectrum. This is also schematically reported in the Fourier spectrum of Fig. 1.

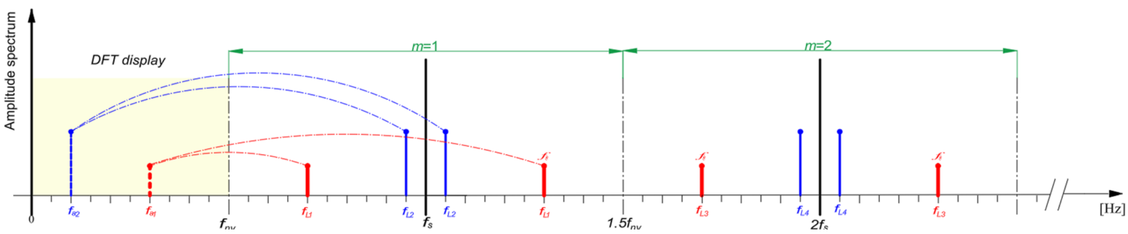

Figure 1: schematic representation of folding frequencies in a frequency domain spectrum for a given f_s.

If the frequencies associated with the thermoelastic harmonic are known beforehand, this relationship can be used to define a targeted subsampling scheme. Specifically, it becomes possible to predict where the aliased thermoelastic harmonics will appear in the DFT spectrum of the undersampled signal. To illustrate the concept, a simulated temperature signal is considered:

$$T(t) = 23 + 0.55 \cdot \sin(2\pi \cdot 15 \cdot t + \pi/2) + 0.05 \cdot \sin(2\pi \cdot 30 \cdot t + \pi/4) + 0.01 \sin(2\pi \cdot 45 \cdot t + \pi/4) + \underbrace{noise(t)}_{\text{gaussian noise}} \tag{2}$$

The harmonic components correspond to a first, second, and third predominant harmonics, over some white noise and a DC term. This synthetic signal is sampled and processed via DFT under two different sampling conditions: 1) oversampled at fs = 180 Hz, with N = 900 samples acquired over a 5 second window; 2) fs = 6.25 Hz, N = 25 samples, 5 sec window. The resulting amplitude spectra are shown in Fig. 2.

Figure 3, instead, compares the thermoelastic signal maps obtained in oversampling and undersampling conditions from a cracked single-edge notched tension sample under fatigue sinusoidal loading.

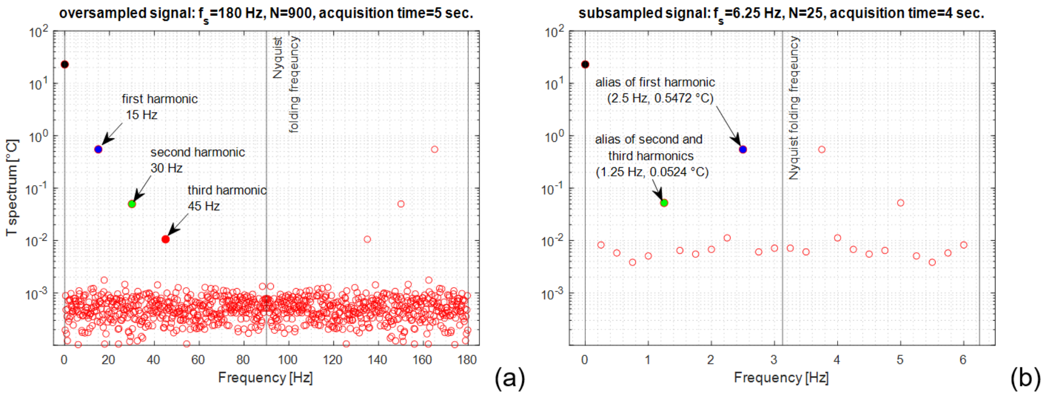

Figure 2: DFT spectrums of the signal of Eq. (2): a) oversampling case; b) subsampling.

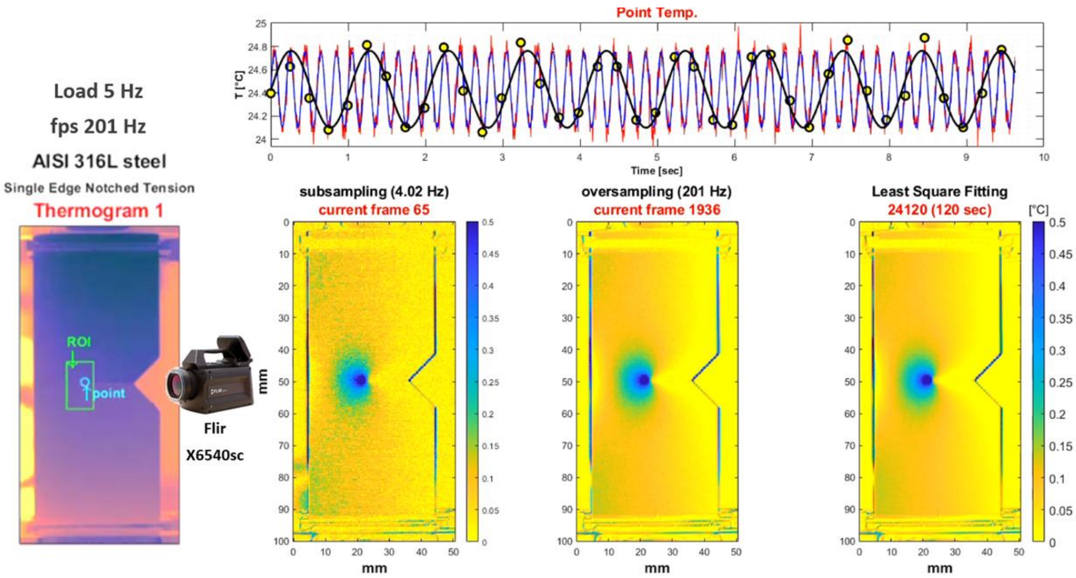

Figure 3: Cracked SENT specimen under sinusoidal loading and comparison of thermoelastic signal full-field maps in oversampling and subsampling conditions [4].

Results and conclusions

Figure 2 shows that using the aliasing relationship in Eq. (1), the first harmonic at 15 Hz produces an alias at 2.5 Hz in the subsampled dataset when m=2. The DFT estimates its amplitude as 0.547, differing from the true value by 0.54%, where the error would further reduce if the aliased wave is sampled for a longer time. Figure 2 further demonstrates the ability of subsampling to reproduce full-field thermoelastic maps, although at the cost of increased noise when using the same sampling time window.

The study demonstrates that a purposely designed undersampling strategy can effectively capture thermoelastic harmonics. While it requires careful selection of the undersampling frequencies and number of acquired samples, it clarifies how to avoid aliasing interference and opens the door for high-frequency TSA applications using affordable equipment.

References

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