Dynamic Constitutive Response of MAX Phase Ti_3SiC_2 Leveraging the Virtual Fields Method

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Abstract. Ti_3SiC_2 is a tough and machinable metal-ceramic MAX phase potentially well-suited to applications prone to impact loading. The bulk-scale deformation of Ti_3SiC_2 is hysteretic, enabling energy dissipation, and has been described across *quasi-static* strain rates by phenomenological models such as the Preisach-Mayergoyz (PM) model. However to the authors' knowledge, the dynamic response has yet to parametrically characterized. Possible reasons for this gap include the novelty of this material and challenges faced using conventional approaches, such as Kolsky (split-Hopkinson pressure) bar investigations. Kolsky studies often require many experiments to construct a complete constitutive response curve and may be affected by premature failure of brittle specimens. As a result, the more recent image-based inertial impact (IBII) test protocol is employed to reconstruct the response at high strain rates $(10^2 - 10^3\ s^{-1})$ using the stress gauge implementation of the virtual fields method (VFM). This work seeks to implement a hysteretic constitutive model (such as the PM model) alongside the stress gauge VFM to describe the dynamic deformation response of Ti_3SiC_2, including determination of rate dependence (if any).

Introduction and Background

Image-based Inertial Impact (IBII) Test. The IBII test is used for the large amount of data yielded by each experiment, whereby stress—strain curves may be reconstructed at any location on the specimen surface. In an IBII test, a thin plate specimen is impacted edge-on, and its surface deformation is recorded to extract full-field strain and acceleration maps. The maps are used with the VFM to reconstruct stress—strain curves and extract constitutive parameters pertaining to an assumed constitutive model [1]. IBII has been used to construct stress—strain curves for Ti_3SiC_2 [2], though the non-linear response was not characterised. This work builds on data obtained in [2] and leverages the idea that sections of the sample are being loaded and unloaded at various points in time, corresponding to internal wave propagations and interactions

Mechanical Hysteresis. MAX phase Ti_3SiC_2 exhibits desirable qualities of metals and ceramics, including machinability, high toughness (relative to metals), and a high brittle to plastic transition temperature. Its hysteretic bulk-scale stress—strain response affords high damage tolerance, where the area enclosed in the hysteresis loop is the energy dissipated. The response is history-dependent, and has been described under quasi-static conditions with the phenomenological Preisach-Mayergoyz (PM) model [3] and effective stress model [4].

Preisach-Mayergoyz Model. The PM model describes the branches (loading or unloading) of a hysteretic stress—strain response as respective systems of infinitesimal hysteresis operators connected in parallel. A hysteresis operator is defined as a two-position “relay” which is in the “up” position if the input (i.e. strain) is increasing and in the “down” position if the input is decreasing. Below a certain input, the relay is in the down position regardless of whether the input is increasing and decreasing. Likewise, above a certain input, the operator is in the up position regardless of whether the input is increasing or decreasing. The model describes the cumulative hysteresis response as a system of these infinitesimal operators. The model is not inherently mechanical and was originally formulated to describe magnetic hysteresis. The PM model is defined in [5].

Quasi-static mechanical hysteresis of bulk-scale Ti_3SiC_2 has been found to follow the necessary and sufficient conditions of the PM model, which was applied using data from loading-unloading cycles [3]. Each loading to unloading transition is a ‘reversal’ and corresponds to the stress/strain maxima of a unique hysteresis loop. If the loading is initiated from zero stress/strain, the respective reversal is considered first order. The highest first order reversal in the experimental data set corresponds to an ‘outermost’ or limiting loop of the hysteresis response space. Lesser reversals correspond to minor loops nested within the limiting loop. Hysteretic stress—strain behaviour is then fully predictable within the branches of the limiting loop. Symbolically, first order reversal stresses are denoted by α, and stress values of interest along the corresponding unloading branches are denoted by β. The strains corresponding to α and β are ε_α and $\varepsilon_{\alpha\beta}$, respectively (see Fig. 2a). A plot of α vs. β is generated and a strain summation, F(α, β), is calculated at each data point (see Fig. 2b). The summation is called the Everett function, and is given by:

$$F(\alpha, \beta) = \varepsilon_\alpha - \varepsilon_{\alpha\beta} \quad (1)$$

α vs. β vs. F(α, β) is plotted and fitted with a bicubic spline interpolation to generate a 3D surface, constituting the Preisach space (see Fig. 2b and 3b). Model predictions are then made using the space. This procedure is discussed further in the next section.

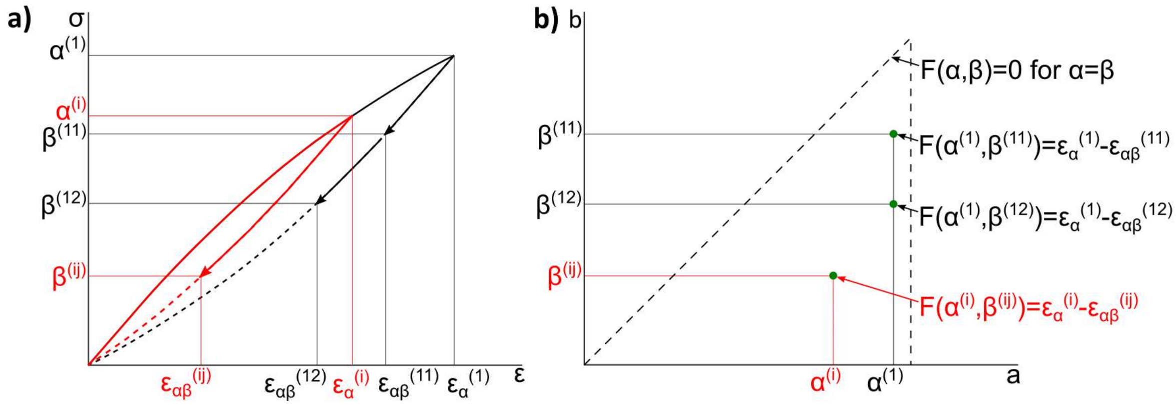

Fig. 1. a) Reversal stresses and strains, b) constructing the corresponding Preisach space

Method and Preliminary Results

Ultra-high-speed images obtained in [2] for six Ti_3SiC_2 specimens were re-processed with the stress gauge implementation of the VFM, where the stress gauge equation (which employs rigid body virtual fields) was used to construct 24 first-order reversal curves at specimen cross-sections spaced 1.5 mm apart. As observed in [2], the reversal stresses of respective cross-sections decrease with distance from the impact edge, due to wave attenuation. Also, the hysteresis loops do not fully close on unloading. A few loops showing these behaviours are overlayed in Fig. 3a (in the legend, the lowest 'x' corresponds to the cross-section farthest from the impact edge). A Preisach space was generated for each specimen using their respective 24 reversals and the Everett Function. The space corresponding to Fig. 3a is shown in Fig. 3b. Fit curves for the loading branches of each specimen response were generated using respective Preisach spaces. The fit curves consisted of plotting α vs. F(α, β) for β = 0. The residual strains of original stress strain hysteresis loops caused the resulting curves to fit sub-optimally. The reason for these residual strains will be further investigated.

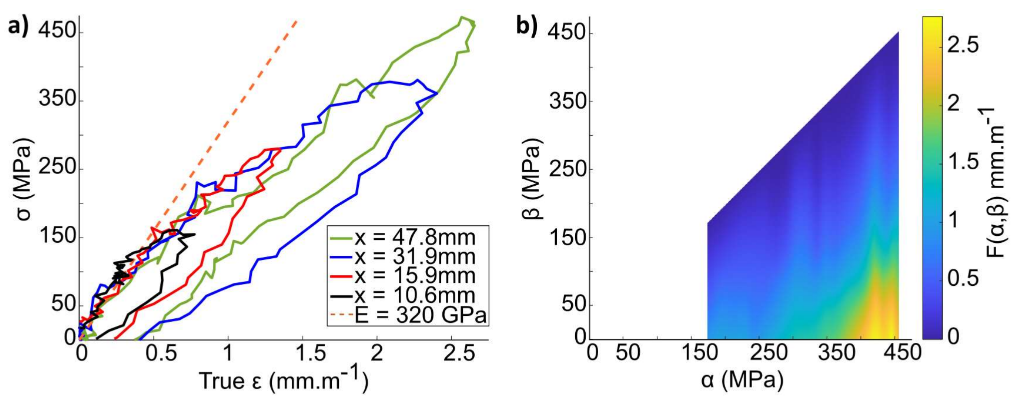

Fig. 3. a) Experimental reversals based on imaging from [2], b) corresponding Preisach space

Conclusion

Whether the IBII data meets the necessary and sufficient conditions of the PM model is yet to be determined; however, this is a first order attempt at describing the dynamic constitutive response of these hysteretic materials. Preisach spaces were obtained, though the residual strain of the hysteresis loops adversely affects the fit of the model predictions. The reason for open loops is to be determined but could result from material behaviour not included in the model (e.g. plasticity or damage). If the PM model is ultimately realized to be insufficient, a modified Preisach or different hysteresis model such as the effective stress model will be investigated utilizing the VFM.

References

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