Gaussian Process Latent Force Models for Point Load Estimation using Distributed Strain Measurements

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Abstract. In structural health monitoring frameworks, accurately estimating a structure's loading history with quantifiable uncertainty is crucial for fatigue life assessment. A linear Gaussian Process Latent Force Model (GPLFM) is used to recover a simulated point load on a 0.8m composite sandwich blade using distributed strain measurements, with a view towards experimental validation. The development of a reduced-order strain modal GPLFM with joint input-state estimation utilising the Kalman filter are outlined and factors impacting its performance in the recovery of an experimental shaker load discussed.

Introduction

Accurate estimation of dynamic loading is crucial for predicting a structure's remaining useful life. Strain measurements from fibre Bragg gratings (FBGs) are being increasingly considered in offshore monitoring due to their ability to be embedded in composite structures, resistance to electromagnetic interference, and immunity to lightning issues [1]. This work employs strain measurements within a Bayesian framework to estimate point loads applied to a composite blade structure using a Gaussian Process Latent Force Model (GPLFM) [2], utilising a linear modal superposition (MSUP) model updated via experimental modal analysis.

Gaussian Process Latent Force Models

Linear dynamic structural systems can be represented as a state space model, characterised by a transition model that describes how states (i.e., displacement, velocity) evolve over time, and a measurement function that relates these states to observed variables. From a Bayesian probabilistic perspective, these deterministic functions simply become densities. If the transition is a first order Markov process, and the noise model is assumed to be a Wiener process in continuous time, the following state space model can be obtained:

$$\dot{\boldsymbol{z}}(t) = F_s\boldsymbol{z}(t) + G_s\boldsymbol{u}(t) + \boldsymbol{q}(t) \qquad \boldsymbol{q}(t)\sim\mathcal{N}(0, Q) \tag{1}$$

$$\boldsymbol{y}_t = H\boldsymbol{z}(t) + D\boldsymbol{u}(t) + \boldsymbol{v}(t) \qquad \boldsymbol{v}(t)\sim\mathcal{N}(0, R) \tag{2}$$

Where F_s is the transition matrix, G_s the input matrix, H is the observation matrix and D is the feed-through matrix, which may be zero depending on the parameter being observed. Following discretisation, Equations 1 and 2 can be solved for exactly in a Bayesian framework using the Kalman filter [3] and Rauch-Tung-Striebel (RTS) smoother algorithms [4].

As continuous dynamic structural systems theoretically have an infinite number of degrees of freedom, it is often more suitable to transform from a physical perspective to the modal domain, where behaviour is characterised by fundamental modes of vibration. For a probabilistic state space system with strain measurements as the observation, this gives rise to the following model:

$$\dot{\boldsymbol{z}} = \begin{bmatrix} \boldsymbol{0} & \mathbb{I} \\ -\Omega^2 & -2Z\Omega \end{bmatrix}\boldsymbol{z} + \begin{bmatrix} \boldsymbol{0} \\ \widetilde{\Phi} \end{bmatrix}\boldsymbol{u} \tag{3}$$

$$\boldsymbol{\varepsilon} = [\widetilde{\Phi}_s \quad 0]\boldsymbol{z} \tag{4}$$

Where $\boldsymbol{z} = [z_1 \dots z_{n_m}, \quad \dot{z}_1 \dots \dot{z}_{n_m}]^{\boldsymbol{T}}$ is the modal state vector, and n_m is the number of modes considered. Note that the first half of the state vector (modal displacements) is related to the measured strain values by the mass-normalised strain mode shape matrix, $\widetilde{\Phi}_s$.

In the case where the input load to the system is also unknown, the state vector can be augmented such that both the states and input are filtered simultaneously. In the latent force formulation, the input forces are treated as Gaussian processes, defined by a mean and a covariance function such that $u_t \sim GP\big(0, k(t,t')\big)$. Hartikainen and Särkkä [5] demonstrated that this Gaussian process can be given equivalently in state space form as a stochastic differential equation driven by white noise. The resulting augmented state vector and state transition model is given by:

$$\boldsymbol{q} = [z_1, \dots, z_{n_m}, \quad \dot{z}_1, \dots, \dot{z}_{n_m}, u_1, \dots, u_{n_m}, \quad \dot{u}_1, \dots, \dot{u}_{n_m}]^{\boldsymbol{T}} \tag{5}$$

$$\dot{\boldsymbol{q}} = \begin{bmatrix} F_s & G_s \\ \boldsymbol{0} & F_k \end{bmatrix}\boldsymbol{q} + L\boldsymbol{w} \tag{6}$$

Where F_s is the system transition matrix, G_s determines how inputs enter the system, and F_k is the transition matrix for the input stochastic differential equation, which is dependent on the chosen covariance function. L is a selection matrix for input derivatives and $\boldsymbol{w}$ is a white noise matrix with spectral density $S(w)$, also dependent on the chosen covariance function. In this work, the Matérn covariance function with a smoothness parameter v = 3/2 was used, giving rise to two hyperparameters $\theta = [l, \sigma^2]$ that can be used to encode forcing properties. Optimisation for these parameters is done through minimising the energy function $\phi(\theta)$ of the Kalman filter, which is proportional to the negative marginal log likelihood of the observed data conditioned on the parameters being optimised:

$$\phi(\theta) \propto -p(y_{1:T} | \theta) \quad (7)$$

Model Derivation

The above model was tested in the recovery of a random shaker excitation, on a 0.8m CRFP blade structure, utilising an accelerometer in the flapwise bending direction and four strain gauge measurements connected via a quarter bridge configuration aligned along the blade span axis. The applied load was validated using a compressive loadcell measurement.

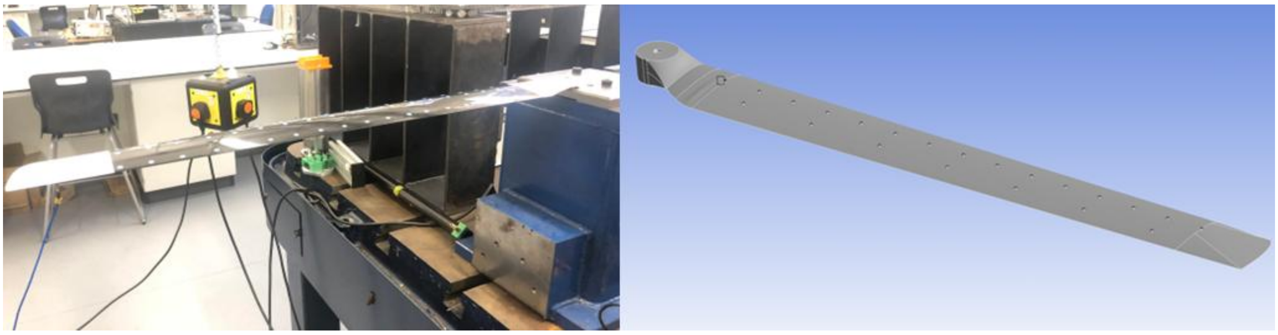

Fig. 1: (left) Composite blade impact modal analysis, (right) finite element model of blade structure with strain mode shape evaluation points

A finite element model was updated using impact modal analysis results using a Bayesian least squares parameter estimator. The resulting model was used to acquire modal parameters for the reduced order formulation described above. More information on model updating methods can be found in [6].

Discussion

With a satisfactory reduced-order model, we can assess whether incorporating this linear superposition of modes into the GPLFM framework accurately recovers a known shaker load. It will be seen that the introduction of measurements based on sparse strain sensors decreases the signal to noise ratio, which must be considered when learning the GPLFM. The process is further complicated when one considers whether the representation of the dynamics in the anisotropic composite structure can be adequately captured in the linear superposition setting.

A final point of care which must be taken is in tuning of the small number of hyperparameters associated with the method. The GPLFM requires learning of an overall scaling of the force magnitude and of the characteristic length scale of the signal, this tuning is also impacted by the presence of noise. Further work should be carried out to extend the approach shown here to cases where multiple and distributed loads are impacting the structure of interest as this will require extension of the underlying GP models.

References

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