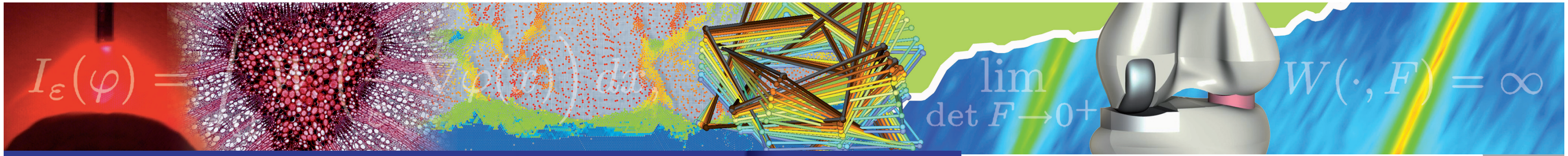


BSSM seminar
Celebrating 50 years of BSSM:

Showcase on leading edge experimental techniques

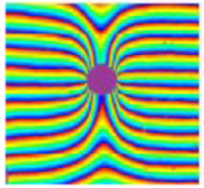
A special event to celebrate the BSSM fiftieth anniversary

3rd and 4th November 2014

Venue: National Physical Laboratory, UK.

TEDDINGTON

BSSM
BRITISH SOCIETY FOR STRAIN MEASUREMENT
50th Anniversary
1964-2014



GDR CNRS 2519
MESURES DE CHAMPS
ET IDENTIFICATION EN
MECANIQUE DES SOLIDES

Full-field measurements &
Identification in solid mechanics

Thermomechanical analysis of material behaviour
J. Barton & A. Chrysochoos

Part 2: Dissipation to characterise irreversible deformation mechanisms

A. Chrysochoos – LMGC – Montpellier University - France
GDR CNRS 2519

LMGC   UM2 UNIVERSITE MONTPELLIER 2   CNRS

LABORATOIRE DE MÉCANIQUE ET GÉNIE CIVIL - UM2/CNRS

Goals

Observing kinematic and energetic effects accompanying the material transformations

Constructing energy balances – consistency of behavioral models

Quantitative imaging techniques – full-field measurements

IRT : since 1983 – DIC : since 1991 – DIC & IRT : since 2000

DIC - Mechanics

- displacement fields
- strain and strain rate
- stress
- deformation energy rate

IRT - Thermodynamics

- absolute temperature
- heat source (via the heat equation)
- intrinsic dissipation
- coupling sources

Outline

Theoretical background
- dissipation, energy storage
- thermomechanical (thm) coupling effects

Experimental tools
- temperature fields
- displacement fields

Focusing on cyclic loading

Case study #1 : HCF of steel
- material vs. structure effects
- properties of intrinsic dissipation fields

Case study #2 : LCF of rubber
- intrinsic dissipation vs. strong coupling + thermal dissipation

Concluding comments

Theoretical background

Thm constitutive equations

Generalised standard material formalism

[Halphen & Nguyen, 75]

state variables	$\{T, \boldsymbol{\varepsilon}, \boldsymbol{\alpha}\}$
internal/free energy potential state equations	$e(s, \boldsymbol{\varepsilon}, \boldsymbol{\alpha})$ $\quad \psi(T, \boldsymbol{\varepsilon}, \boldsymbol{\alpha})$ $-s = \psi_{,T}$ $\quad \boldsymbol{\sigma}^r = \rho \psi_{,\boldsymbol{\varepsilon}}$ $\quad \mathbf{A}_\alpha = \rho \psi_{,\alpha}$
dissipation potential evolution equations	$\varphi(q, \dot{\varepsilon}, \dot{\boldsymbol{\alpha}}; T, \ldots)$ $-\frac{\nabla T}{T} = \varphi_{,q}$ $\quad \sigma^{ir} = \varphi_{,\dot{\varepsilon}}$ $\quad X_{\dot{\alpha}} = \varphi_{,\dot{\alpha}}$

Irreversibility

Material degradation $\quad d_1 = \sigma^{ir} : \dot{\varepsilon} + X_\alpha . \dot{\alpha}$

Heat diffusion $\quad d_2 = \varphi_{,q} . q = -\frac{\nabla T}{T} . q$

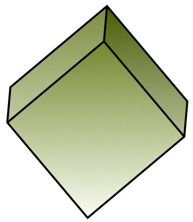
Energy balance (I)

$$w^{\bullet}_{def} = \sigma : \dot{\varepsilon} = \sigma^{r} : \dot{\varepsilon} + \sigma^{ir} : \dot{\varepsilon}$$

$$= \underbrace{\sigma^{r} : \dot{\varepsilon} + A_{\alpha} . \dot{\alpha}}_{w^{\bullet}_{e} + w^{\bullet}_{s}} + d_1$$

$w^{\bullet}_{e}$: rate of elastic energy

d_1 : intrinsic dissipation



$w^{\bullet}_{s}$: rate of strored energy

... incomplete balance !!

Energy balance (II)

- rate of internal energy

$$\rho\dot{e} = \rho C\dot{T} + (\sigma^{r} : \dot{\varepsilon} + A.\dot{\alpha}) - (T\sigma^{r}_{,T} : \dot{\varepsilon} + TA_{,T}.\dot{\alpha})$$

$$= \rho C\dot{T} + w^{\bullet}_{e} + w^{\bullet}_{s} - w^{\bullet}_{thc}$$

« thc » = thermomechanical couplings

- heat equation

$$\rho C\dot{T} + \mathrm{div}\,\mathbf{q} = \underbrace{\boldsymbol{\sigma}^{ir} : \dot{\boldsymbol{\varepsilon}} - \mathbf{A}.\dot{\boldsymbol{\alpha}}}_{d_1} + T\,\boldsymbol{\sigma}^{r}_{,T} : \dot{\boldsymbol{\varepsilon}} + T\,\mathbf{A}_{,T}.\dot{\boldsymbol{\alpha}} + r_e$$

- comments

C.1: C specific heat

C.2: $q = -k.gradT$

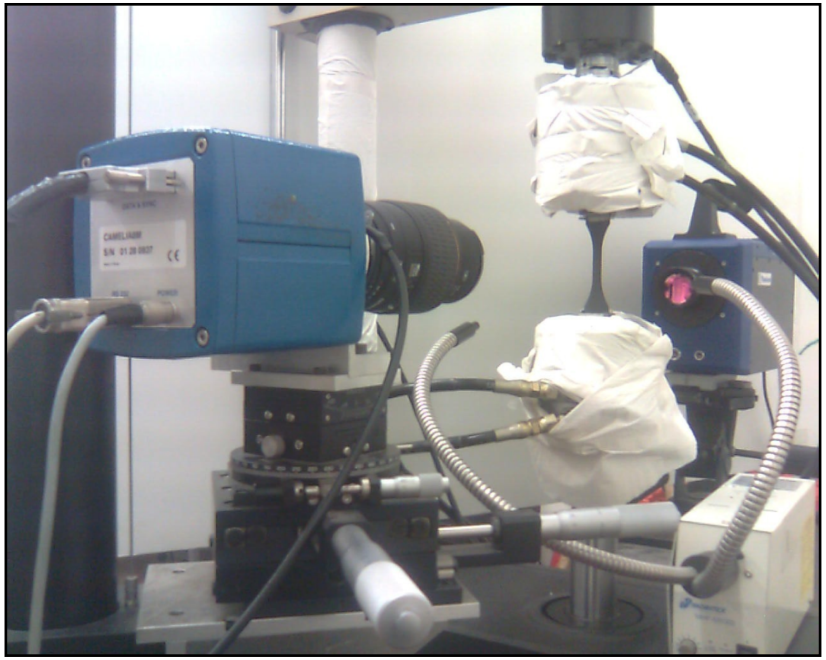
kinematics required

C.3: $\dot{T} = \dfrac{\partial T}{\partial t} + v.\nabla T$

Experimental tools

Quantitative imaging – full field measurement system

- **CCD camera**
- max frame rate 20 Hz
- 1280×1024 pixels
- $13\times13\ \mu m^2$
- 14 bits
- $\delta x \approx 0.1$ mm (min 15 µm)



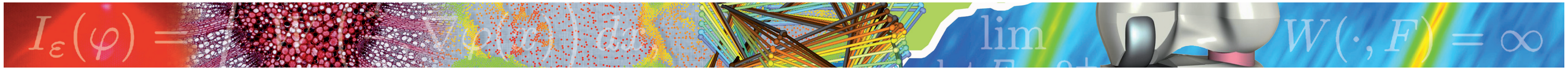
- **IRFPA camera**
- [3,5] mM
- max frame rate 250 Hz
- 640×320 pixels
- $18\times23\ \mu m^2$
- 14 bits
- $\delta x \approx 0.1$ mm (min 25 µm)
- $\delta T \approx 0.02$ °C

- **Hydraulic testing machine**
- load cell : ± 25 kN
- frame : ± 100 kN
- $\max(f_L)$= 50 Hz en R_s= -1

Refs

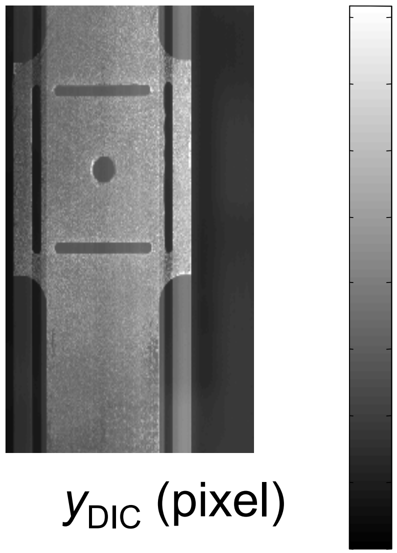
[Wattrisse et al., Exp. Mech, 2001]

[Chryso et al., IJES, 2000]



Combining DIC & IRT

Speckle image

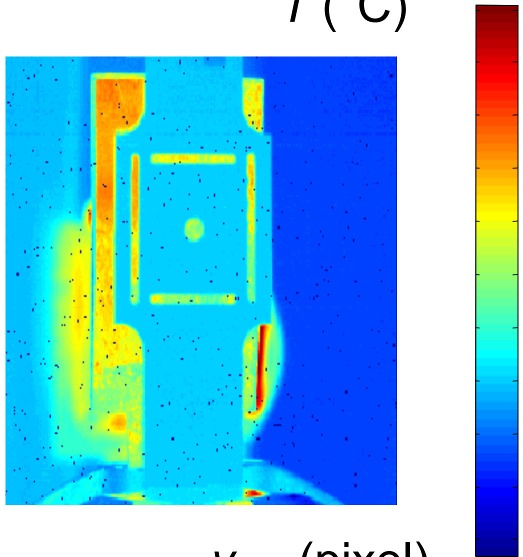


$U(t_{DIC}, x_{DIC}, y_{DIC})$

Map-to-map correspondence

$T(t_{IRT}, x_{IRT}, y_{IRT})$

IR image

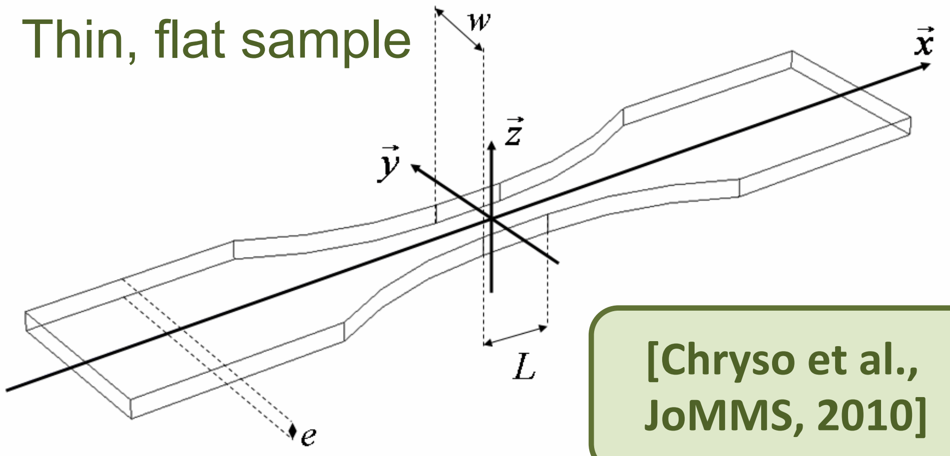


[Chryso et al., JoMMS, 2010]

t_{DIC}	synchrocam	t_{IRT}	(50 µs)
(x_{DIC}, y_{DIC})	target	(x_{IRT}, y_{IRT})	(≈ 0.5 pixel)

Stress derivation

Thin, flat sample



[Chryso et al., JoMMS, 2010]

e: thickness, *w*: width, *L*: length

$$\sigma_{xx}(x,t)=\frac{F(t)}{S_0}\exp\left(\varepsilon_{xx}(x,t)\right)$$

$$\sigma_{xy}(x,y,t)=-\sigma_{xx}(x,t)\frac{\partial\varepsilon_{xx}(x,t)}{\partial x}y$$

$$\sigma_{yy}(x,y,t)=\frac{\partial}{\partial x}\left(\frac{\sigma_{xx}(x,t)}{2}\frac{\partial\varepsilon_{xx}(x,t)}{\partial x}\right)\cdot\left(\frac{w(x,t)^2}{4}-y^2\right)$$

① stress triaxiality neglected

plane stress

$$\begin{cases}\dfrac{\partial\sigma_{xx}(x,y,t)}{\partial x}+\dfrac{\partial\sigma_{xy}(x,y,t)}{\partial y}=0\\[2ex]\dfrac{\partial\sigma_{xy}(x,y,t)}{\partial x}+\dfrac{\partial\sigma_{yy}(x,y,t)}{\partial y}=0\end{cases}$$

② no volume variation

③ uniform distribution of tensile stress over a cross-section

④ no overall shear loading

⑤ no lateral stress

Heat rate

Heat equation averaged over the sample thickness

$$\rho C \left(\frac{\partial \bar{\theta}}{\partial t} + v_x \frac{\partial \bar{\theta}}{\partial x} + v_y \frac{\partial \bar{\theta}}{\partial y} + \frac{\bar{\theta}}{\tau_{th}} \right) - k \left(\frac{\partial^2 \bar{\theta}}{\partial x^2} + \frac{\partial^2 \bar{\theta}}{\partial y^2} \right) = \dot{\bar{w}}_h$$

Direct estimate of heat sources using noisy and discrete thermal data

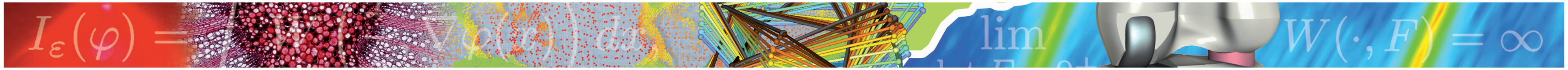
Thermal noise

- uniform power spectrum
- Gaussian probability distribution

Refs:
IJES : 2000
EXP-MECH : 2007
JoMMS : 2010
EXP-MECH : 2014

Estimate of the partial derivative operators

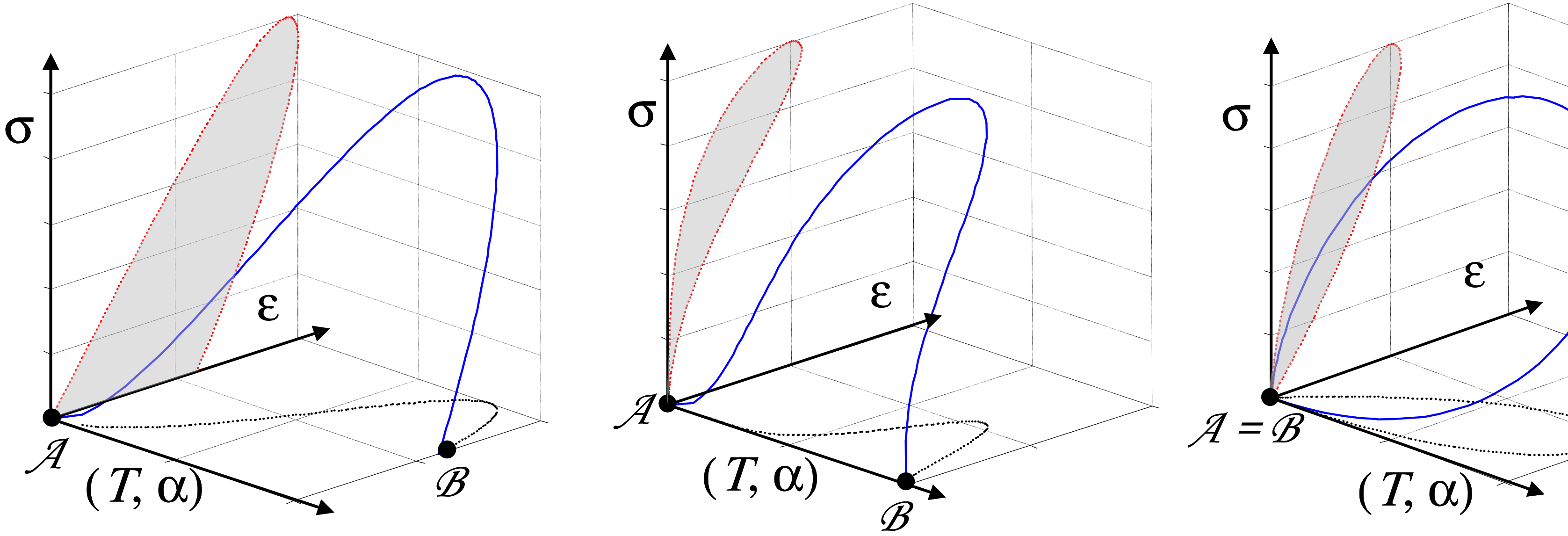
- thermal data projection onto spectral solutions (1995)
- periodic expansion and convolutive filtering by DFT (2000)
- local approximation of θ using l.sq. fitting (2004)
- POD: pre-filtering of thermal fields (2013)



Focusing on cyclic tests

Monochromatic uniaxial
loading

Focus on a load-unload cycle



(i) $\mathcal{A} \neq \mathcal{B}$ *(ii)* $\varepsilon_{\mathcal{A}} = \varepsilon_{\mathcal{B}}$ *(iii)* $\mathcal{A} = \mathcal{B}$

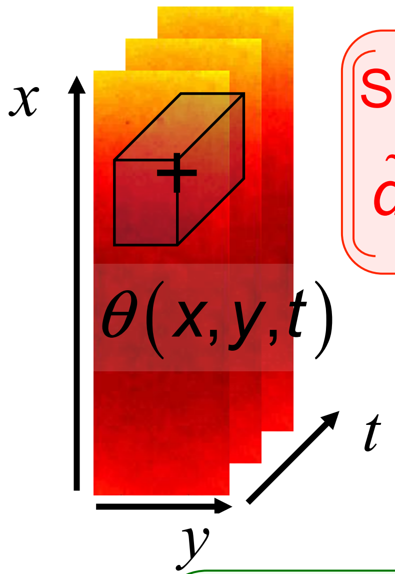
(i) $$w_{\text{def}} = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} \sigma : \dot{\varepsilon}\,\mathrm{d}t = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} d_1\,\mathrm{d}t + \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} (\rho\dot{e} - \rho C\dot{T} + w^{\bullet}_{\text{thc}})\,\mathrm{d}t$$

(ii) Hysteresis loop : $w_{\text{def}} = A_{\text{h}}$ (for uniaxial loading)

(iii) Load-unload cycle = thermodynamic cycle $w_{\text{def}} = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} d_1\,\mathrm{d}t + \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} w^{\bullet}_{\text{thc}}\,\mathrm{d}t$

IR image processing

$$\rho C\left(\frac{\partial\bar{\theta}}{\partial t}+\frac{\bar{\theta}}{\tau_{th}}-\frac{k}{\rho C}\left(\frac{\partial^2\bar{\theta}}{\partial x^2}+\frac{\partial^2\bar{\theta}}{\partial y^2}\right)\right)=\bar{d}_1+\bar{s}_{the}$$



Slow evolution of mean dissipation

$$\tilde{d}_1=\int_{cycle} f_L\,\bar{d}_1\,d\tau$$

In phase with loading

$$\tilde{w}_{the}=\int_{cycle} f_L\,\bar{s}_{the}\,d\tau$$

Linear PDE + Linear BC

$$\bar{\theta}=\bar{\theta}_d+\bar{\theta}_{the}$$

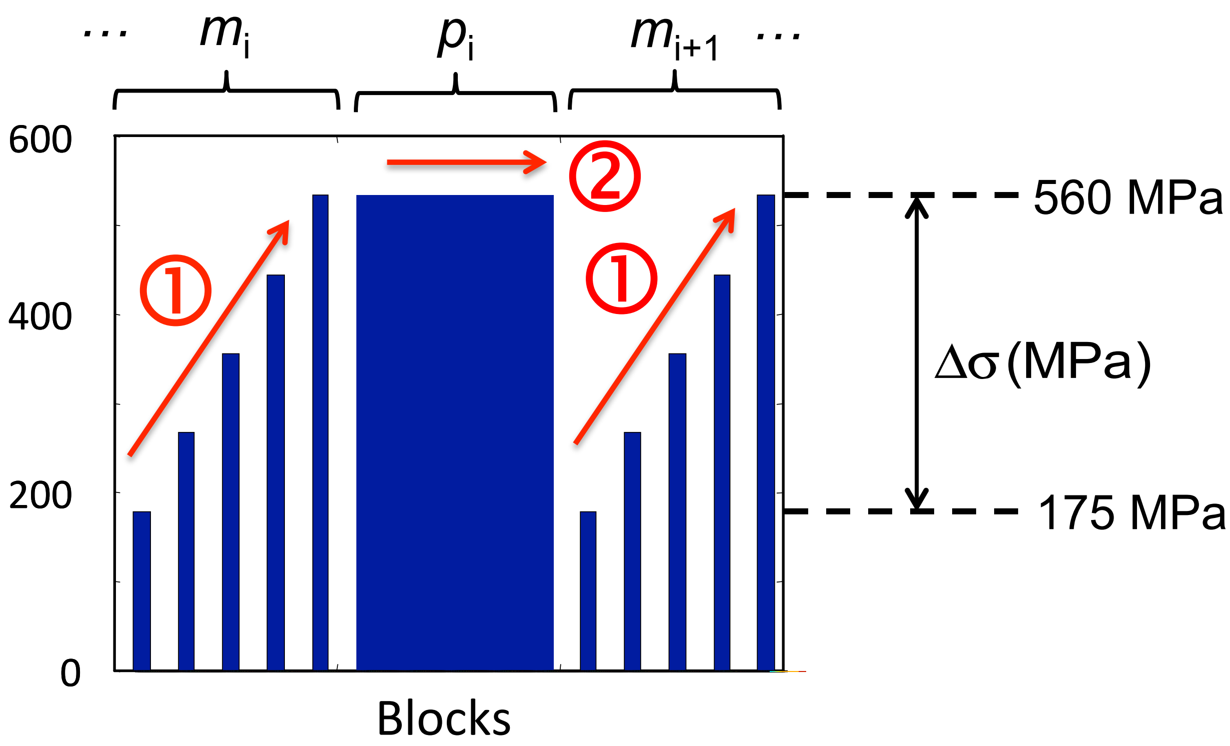
Local approximation function

$$\theta_{app}(x,y,t)=\underbrace{p_1(x,y)\cos(2\pi f_L t)+p_2(x,y)\sin(2\pi f_L t)}_{\text{periodic response}}+\underbrace{p_3(x,y)\,t+p_4(x,y)}_{\text{drift}}$$

$p_i(x,y)$, $i=1,..,4$, are 2nd order polynomials of x and y

Fatigue loading

[Boulanger, PhD 2004]
[Berthel, PhD 2008]
[Blanche, PhD 2012]



① m_i: series of "mini" cycle blocks (3000 cycles) at different stress ranges:
energy balance at "constant fatigue state"

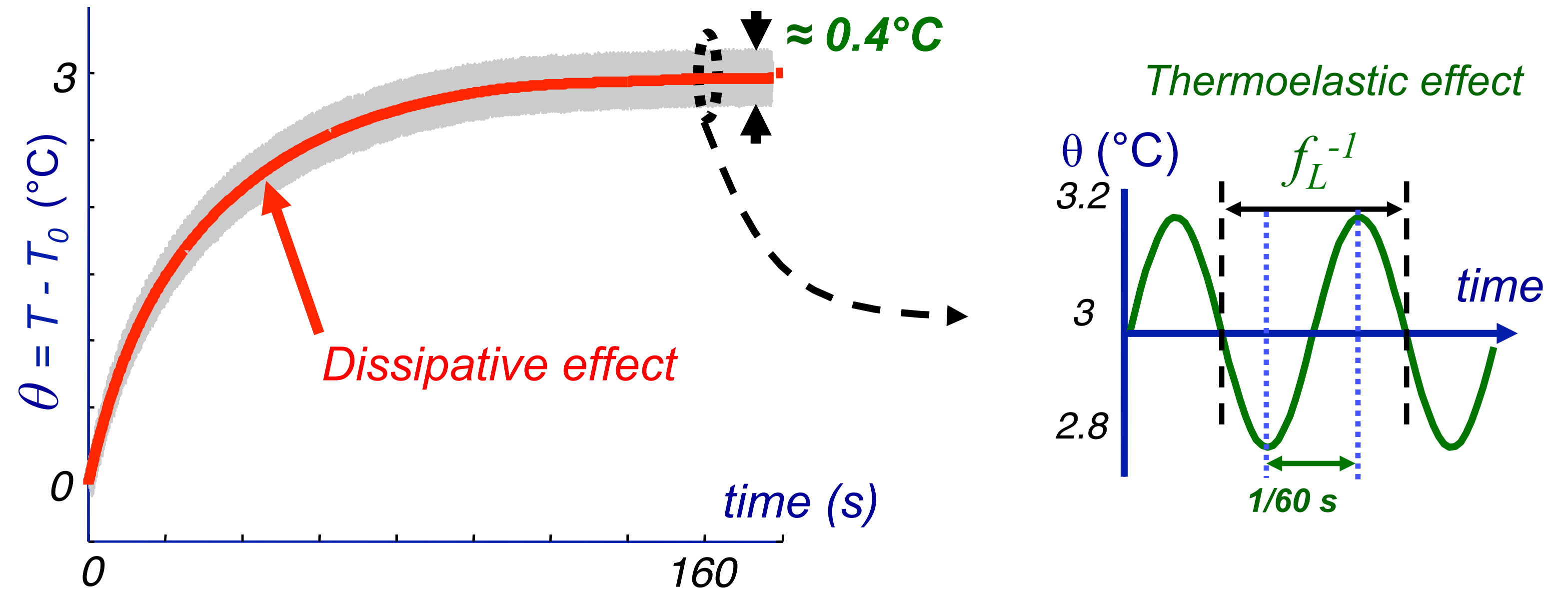
② p_i: large blocks (100 000 cycles) at constant stress range:
energy balance evolution induced by fatigue mechanisms

highest stress range ≈ fatigue limit

Thermal and calorific effects

HCF test on DP 600 steel

$\Delta\sigma = 500$ MPa, $R_\sigma = \sigma_{min}/\sigma_{max} = -1$ et $f_L = 30$ Hz

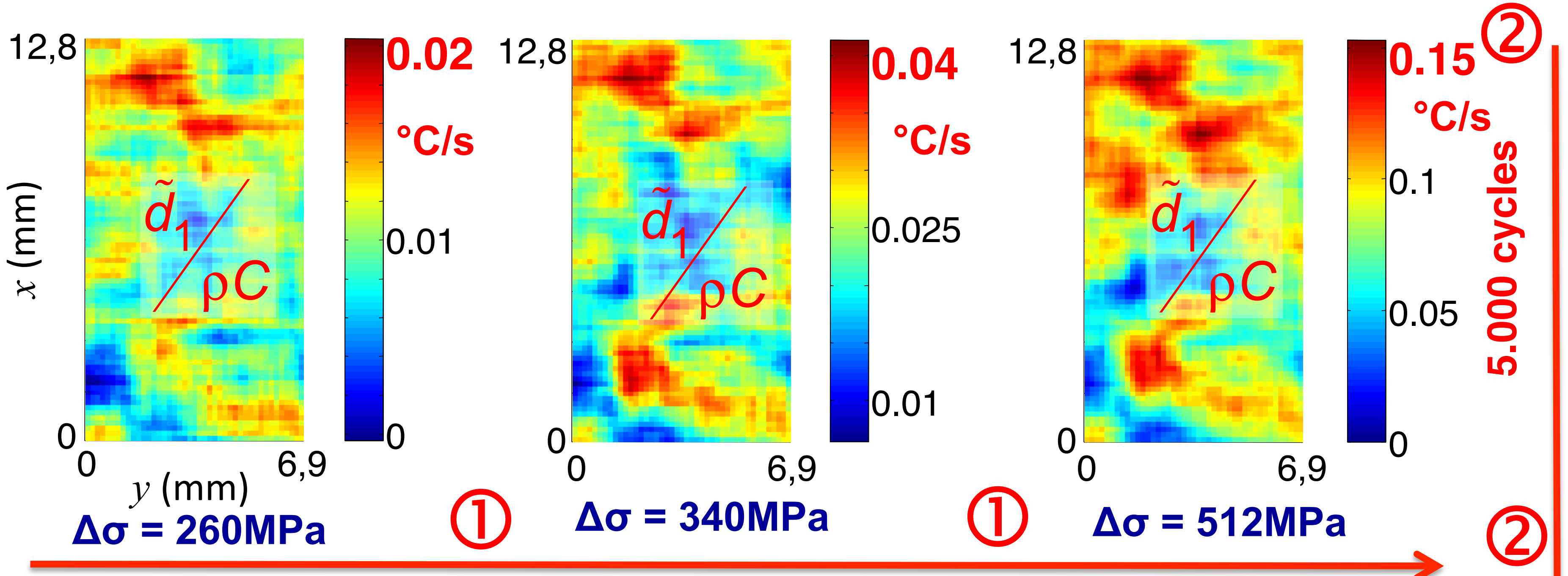


0.4°C every 1/60 s → $\dfrac{\Delta s_{the}}{\rho C} \approx 75\ °C.s^{-1}$

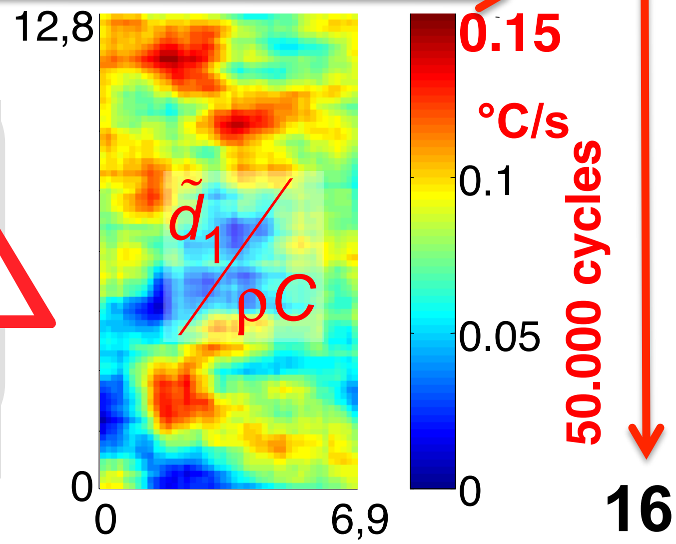
3°C within 160 s → $\dfrac{\tilde{d}_1}{\rho C} \approx 0.1°C.s^{-1}$

Dissipation properties (I)

f_L = 30Hz and R_σ = -1



- *heterogeneous distribution*
- *similar distribution, whatever Δσ*
- $\tilde{d}_1(\Delta\sigma)$ *non linear,* $\tilde{d}_1 \propto f_L$
- *slow time evolution of dissipation*
- *plastic shakedown ? (TQ ratio ≈ 50 % !)*

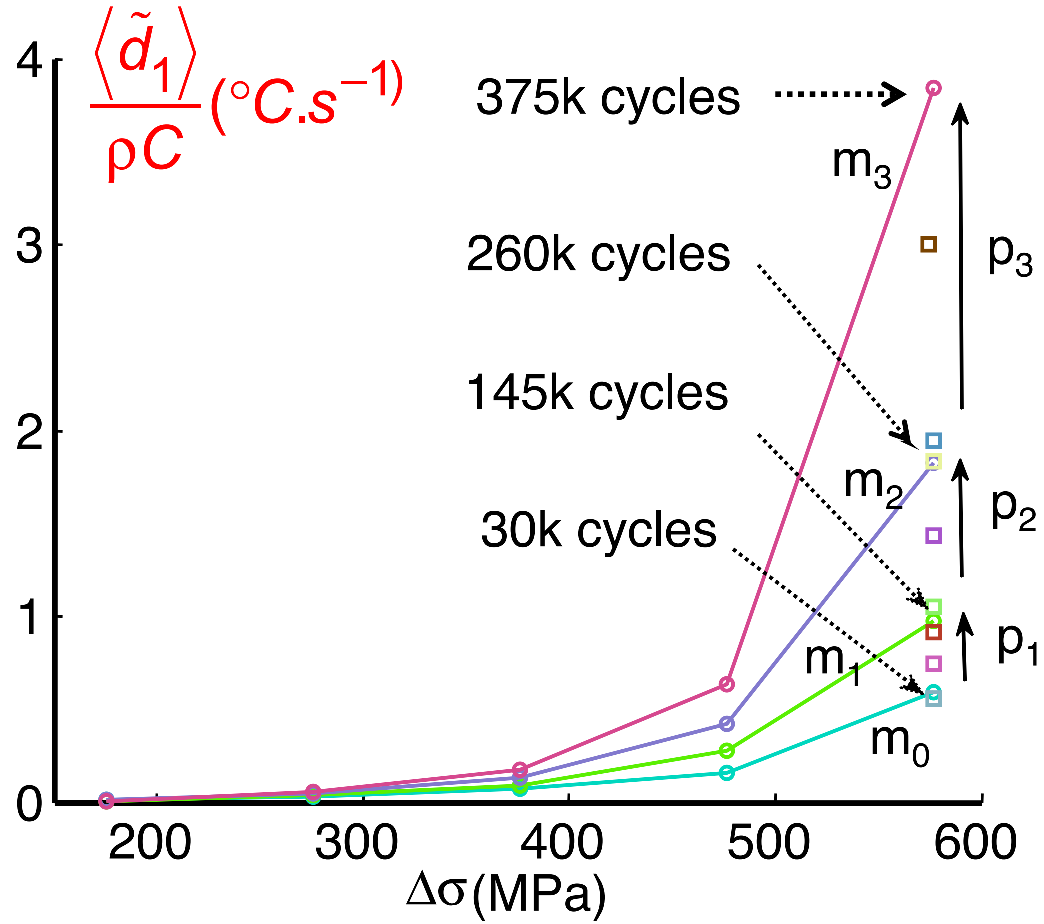


Dissipation properties (II)

Interpretation of curves

m_i = dissipation induced by activated “micro-defects” at constant fatigue state for different stress ranges

p_i = dissipation drift at constant stress range, reflecting a slow evolution of the fatigue state



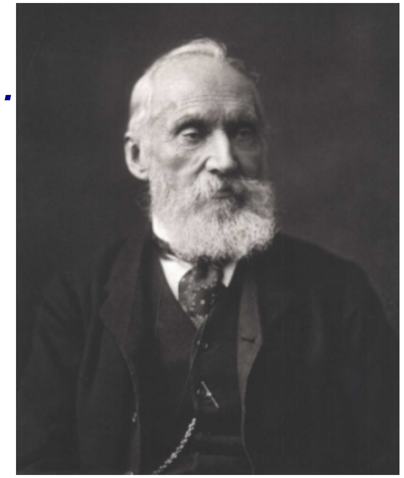
➢ *energy safeguard: kinetics of fatigue progress*

Case study #2 : back to thm couplings

The most simplistic non-adiabatic thermoelastic (the) model…

$$\begin{cases} \varepsilon = \dfrac{\sigma}{E} + \lambda_{th}(T - T_0) \\ \dot{T} + \dfrac{T - T_0}{\tau_{th}} = -\dfrac{E\lambda_{th} T \dot{\varepsilon}}{\rho C} \end{cases}$$

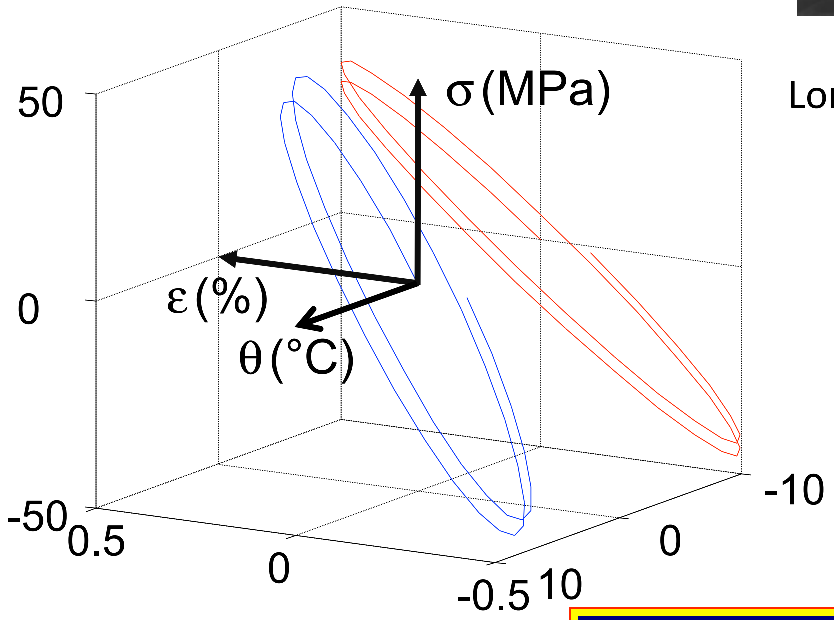
↔ « 0D » approach; linear heat losses



William Thomson
Lord Kelvin (1824-1907)

E = 1000 MPa
ρ = 1000 kg.m^{-3}
C = 1000 J.kg^{-1}.K^{-1}

λ_{th} = 50 10^{-5} K^{-1}× **100**
τ_{th} = 30 s
T_0 = 294 K



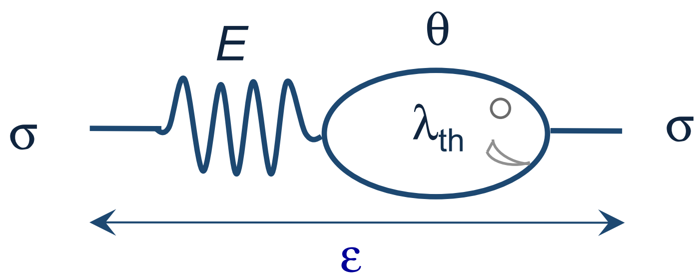
Toward a stabilised cycle…

Thm couplings + thermal dissipation

$$\tilde{w}_{def} = A_h = \tilde{w}_{the}$$

Thm couplings vs. viscosity

Thermoelastic coupling (i.e. d_1=0)
anisothermal framework



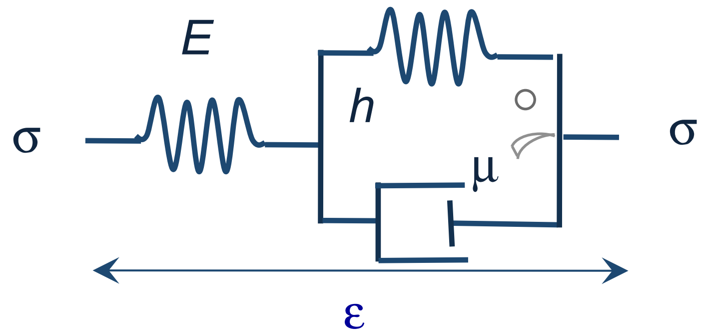
state variables (θ, ε)

$$\begin{cases} \varepsilon = \dfrac{\sigma}{E} + \lambda_{th}\theta \\ \dot{\theta} + \dfrac{\theta}{\tau_{th}} = -\dfrac{E\lambda_{th}(T_0+\theta)\dot{\varepsilon}}{\rho_0 C_0} \end{cases}$$

rheological equation

$$\sigma + \tau_{th}\,\dot{\sigma} \approx E\varepsilon + E\tau_{th}\left(1+\chi\right)\dot{\varepsilon}$$

Viscous dissipation
Isothermal framework

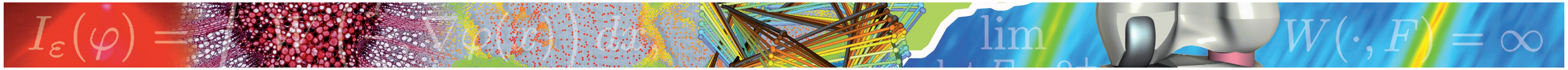


state variables(ε, ε_v)

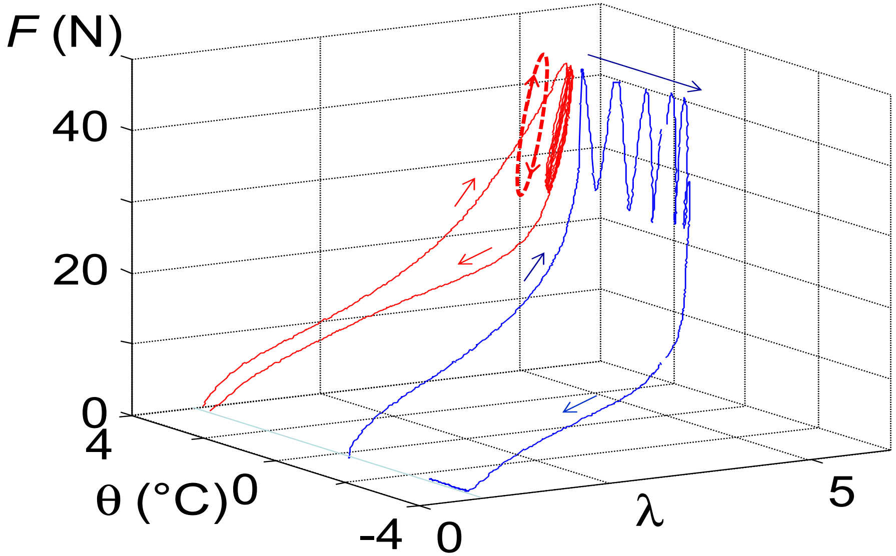
$$\begin{cases} \sigma = E(\varepsilon - \varepsilon_v) \\ \sigma = h\varepsilon_v + \mu\,\dot{\varepsilon}_v \end{cases}$$

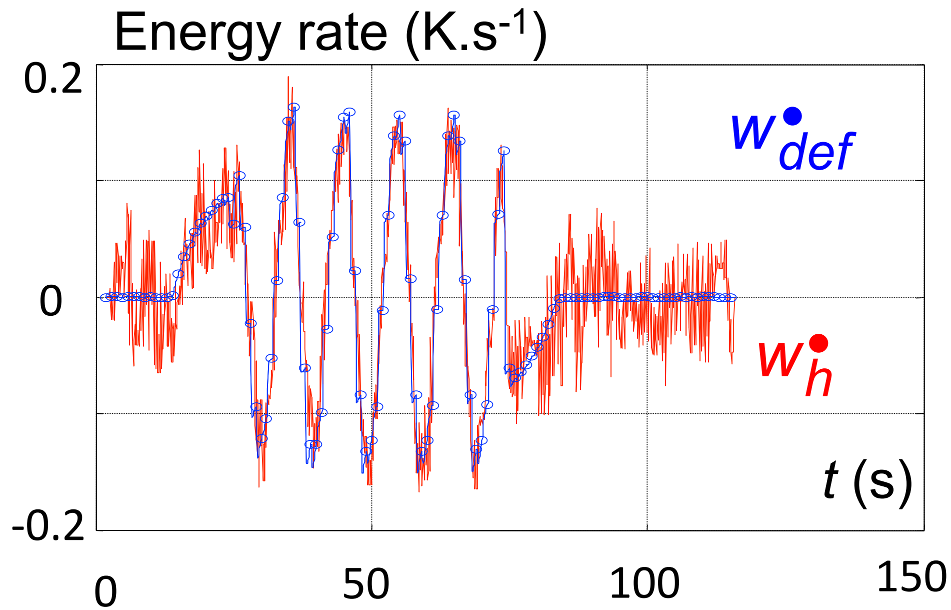
rheological equation

$$\sigma + \frac{\mu}{E+h}\dot{\sigma} = \frac{Eh}{E+h}\varepsilon + \frac{E\mu}{E+h}\dot{\varepsilon}$$



Entropic elasticity of natural rubber





Gough (1805) – Joule (1857) – Treloar (1960)

$e(s,\varepsilon) = e_{nr}(T)$ perfect gas analogy

$\psi_{nr}(T,\varepsilon) = TK_1(\varepsilon) + K_2(T)$

$$w^{\bullet}_{def} = w^{\bullet}_{h}$$

[Saurel, PhD 99]

[Honorat, PhD 06]

[Caborgan, PhD 11]

Concluding comments

- Full-field measurements
  Material vs. Structure effects

- Temperature, the 1st state variable …
  thermal effects vs. calorimetric effects
  not totally intrinsic (heat diffusion)

- Heat sources of different nature
  Thm coupling sources : thermo-sensitivity
  dissipation sources : irreversibility of material deformation

- Energy balance and constitutive equations
  stored energy / state laws
  dissipated energy / evolution law

- Rate-dependent behaviour
  d_1 (viscosity) vs. [thm coupling + d_2 (heat diffusion)]