Practical assessment of the accuracy of volumetric digital image correlation measurements for the analysis of geomaterials

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1) Introduction : microCT in-situ tests on geomaterials
2) Short review of DIC and DIC error sources
3) Quantification of discrete-DIC errors
4) Quantification of systematic errors

Micromechanics of Materials

Macro

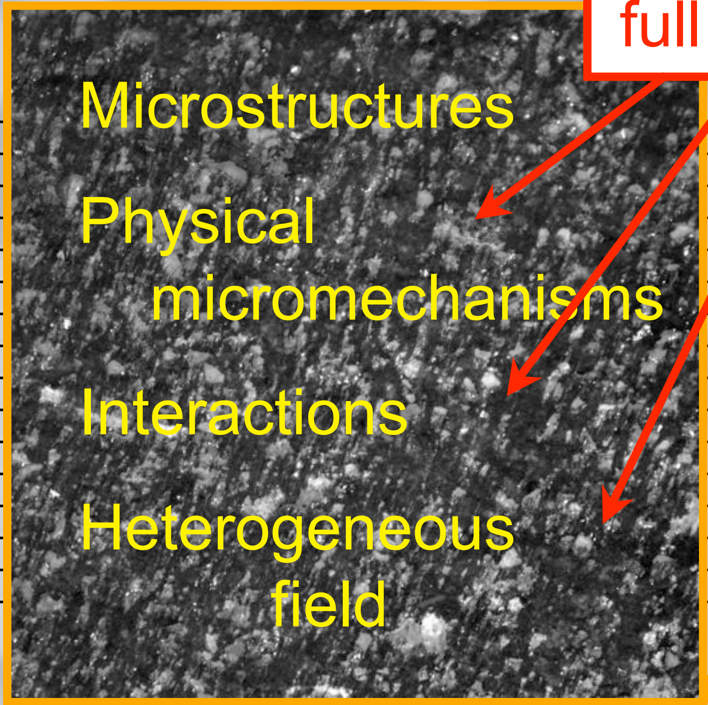
$$\Sigma(t) \longrightarrow E(t)$$

Scale transition

2D and 3D in situ tests
+ multiscale
full field measurement

Micro

$$\sigma(x) \qquad \varepsilon(x)$$



Microstructures

Physical micromechanisms

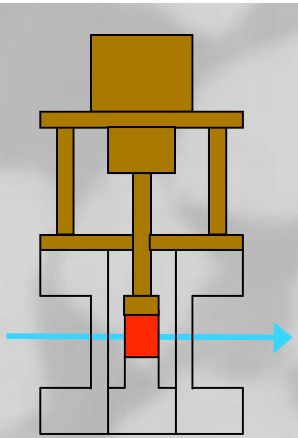
Interactions

Heterogeneous field

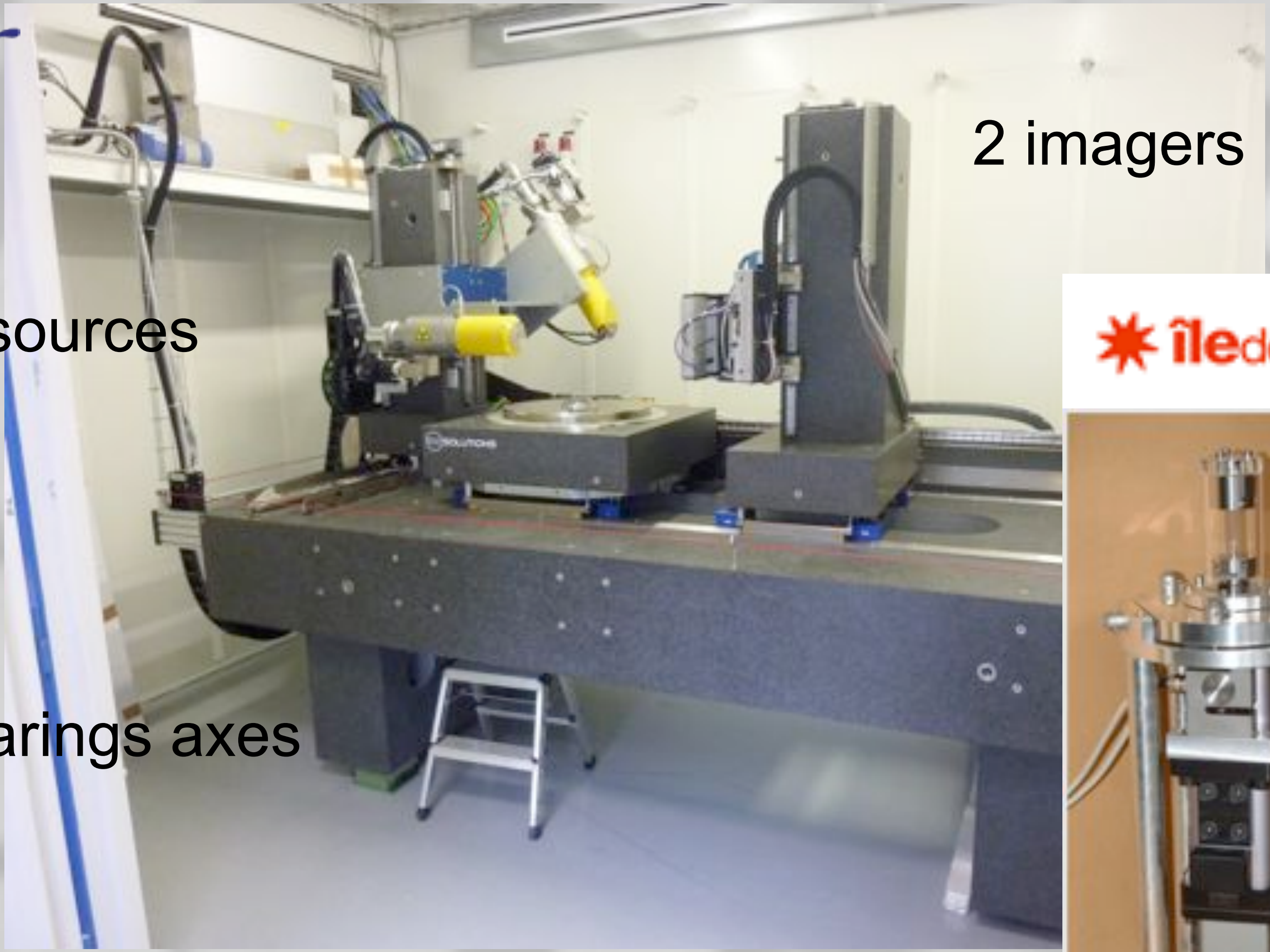
In situ tests in microCT

(cf E. Maire)

ESRF ID15,
& L3SR



Laboratory microCT setup at Navier



2 imagers

2 sources

Air bearings axes

100kg rotation stage

Manufacturer: RX Solutions, 2010-2012

7 in situ testing devices

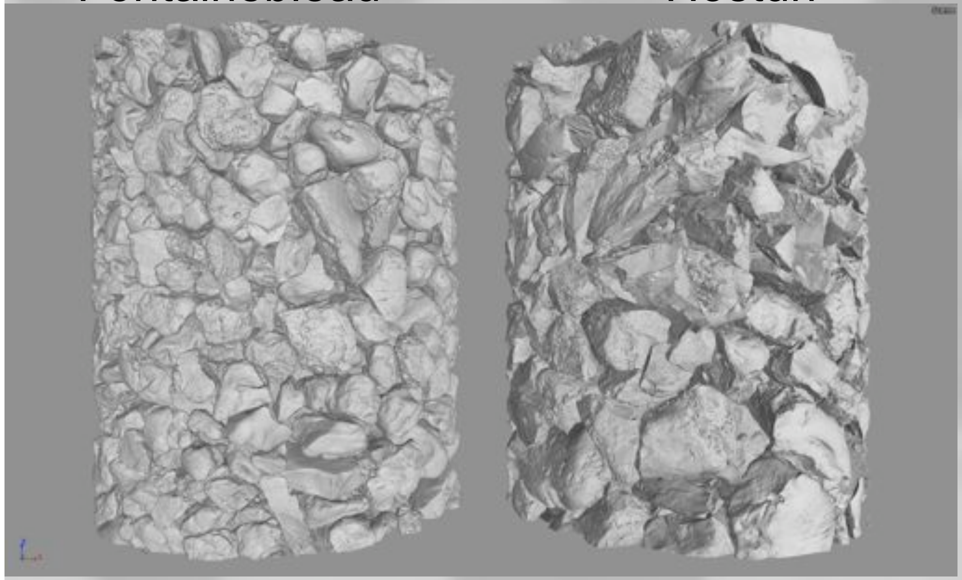
Example: hyromechanical couplings in granular materials

PhD J.F. Bruchon
(with M. Vandamme, J.M. Pereira
P. Delage)

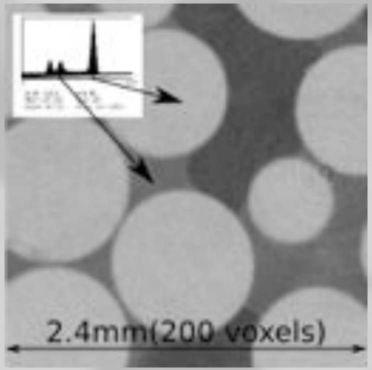
Sand

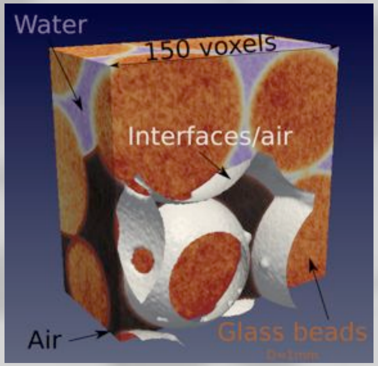
Fontainebleau

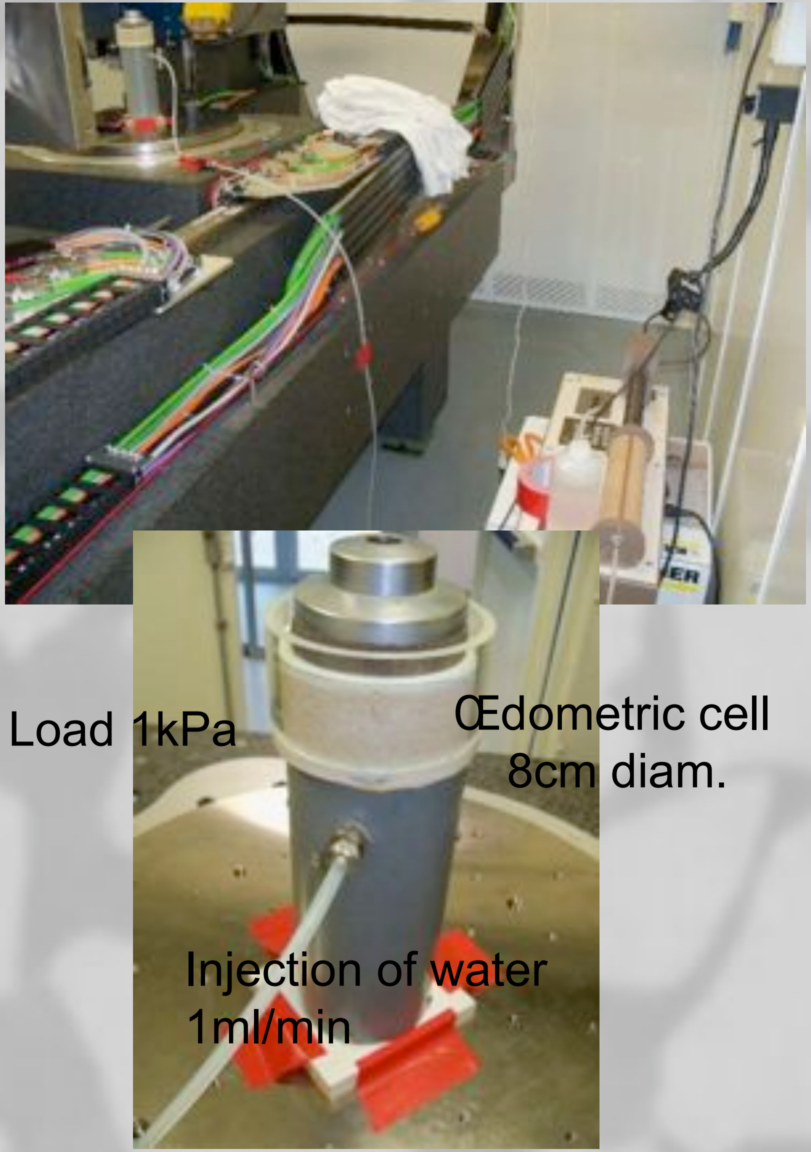
Hostun



Glass beads







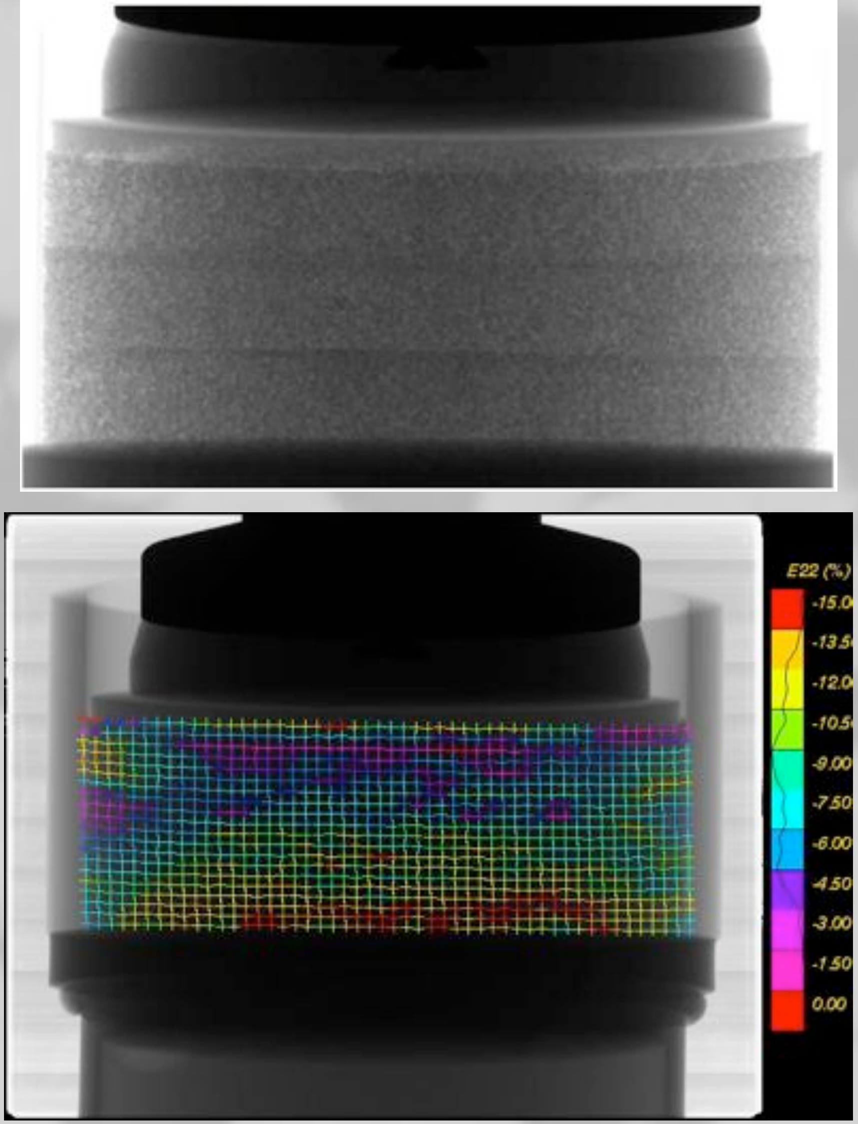
Load 1kPa

Œdometric cell
8cm diam.

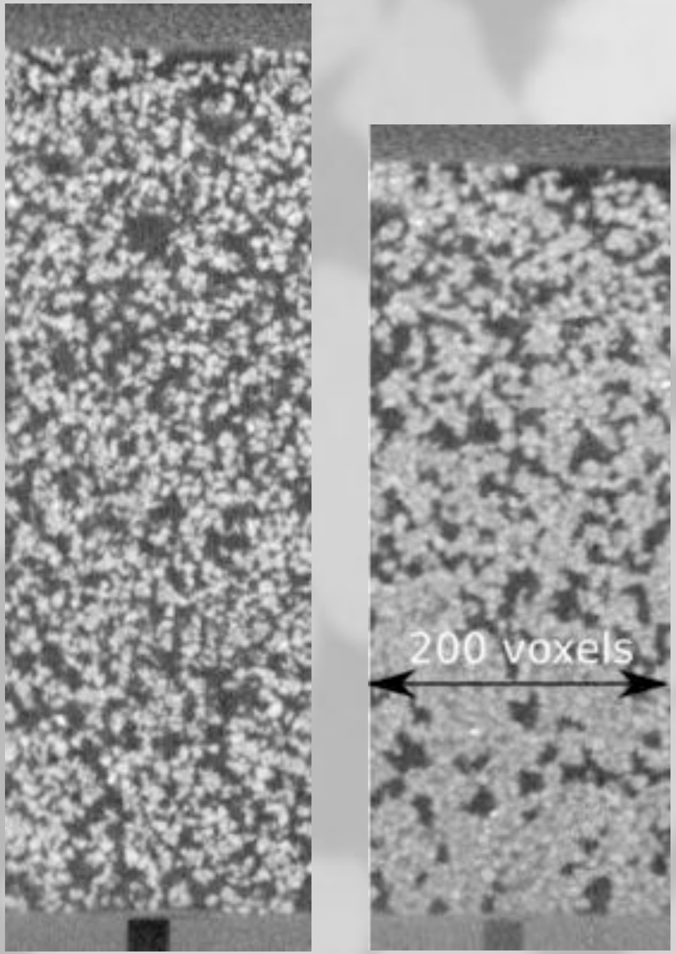
Injection of water
1ml/min

Preliminary test:

Radiographs movie + 2D-DIC

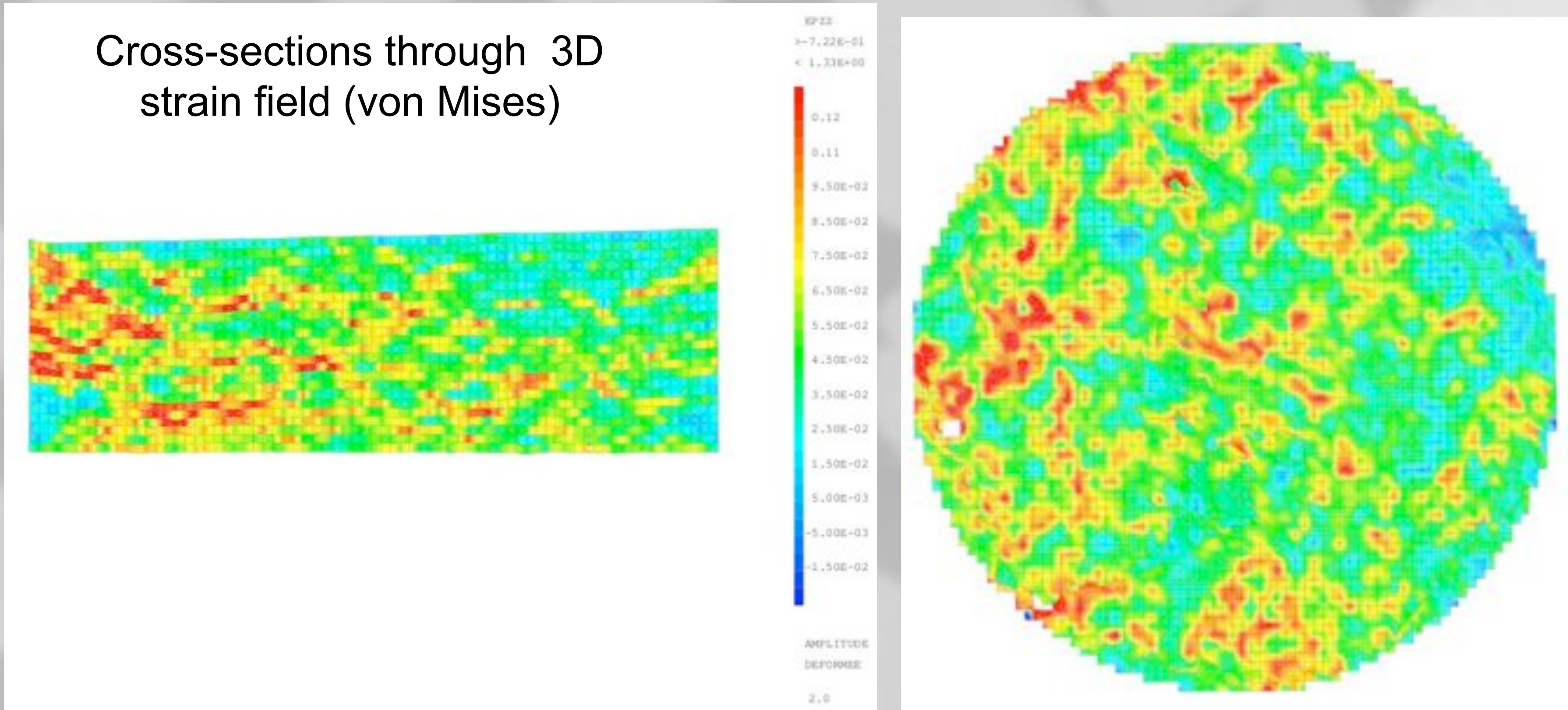


Cross-sections through 3D volumes before/after



Standard volumetric-DIC:

preliminary oedometric test on dry sand

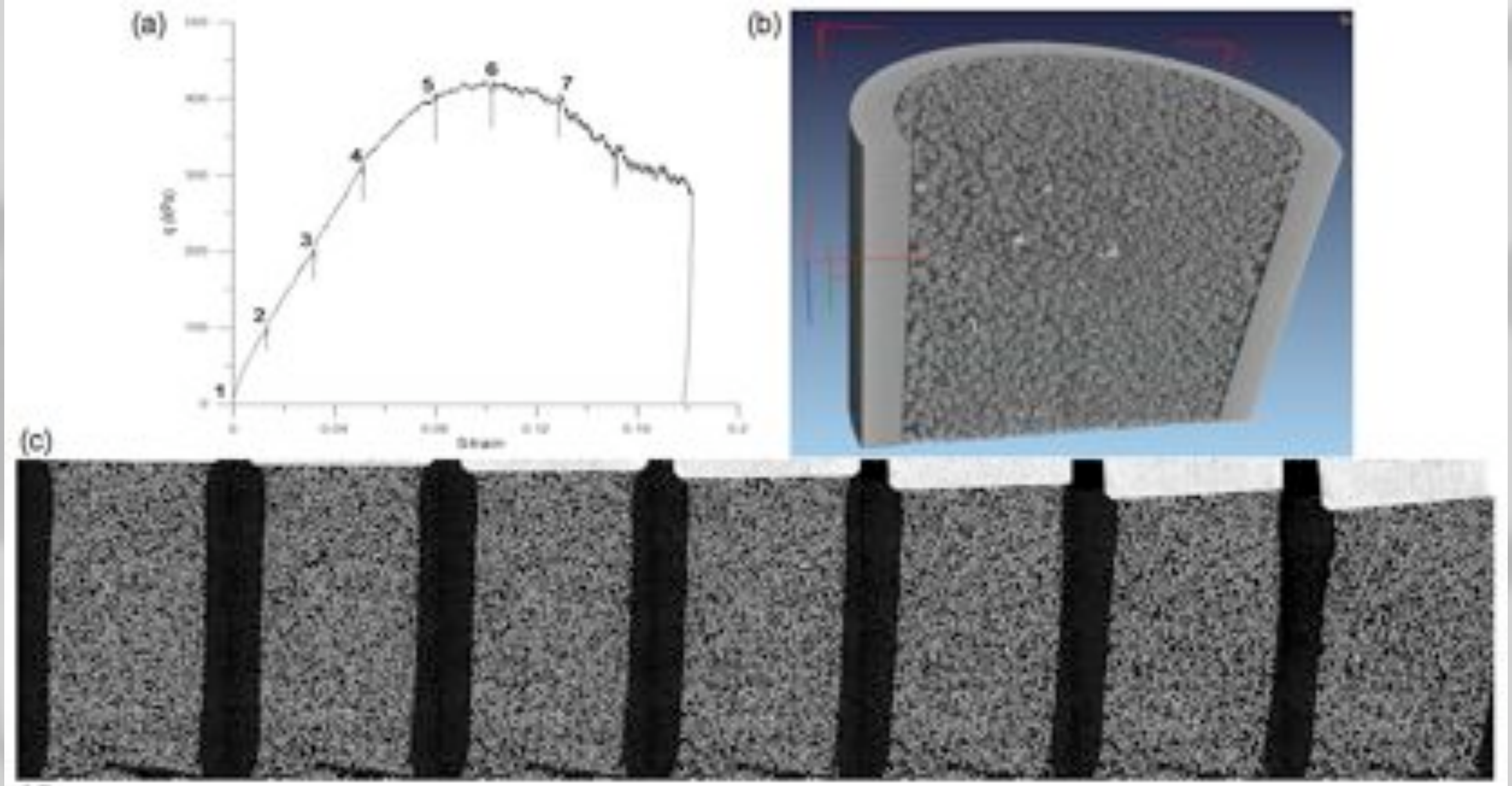


Cross-sections through 3D strain field (von Mises)

Discrete volumetric-DIC: ongoing…

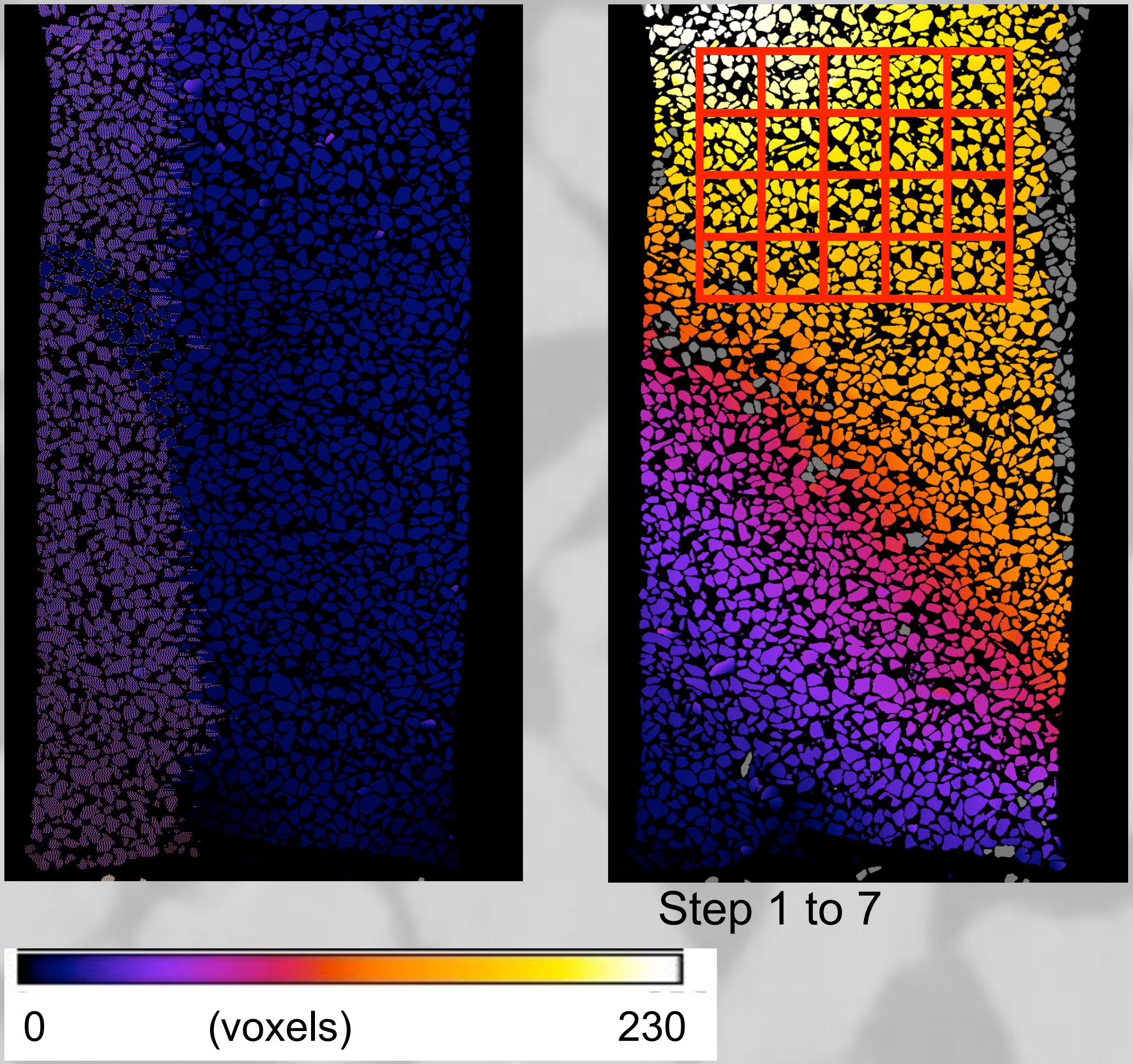
Older test

Hall et al, *Géotechnique*, 2010



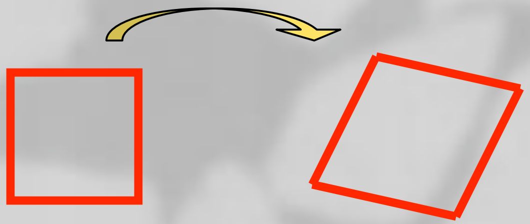
~65000 grains

Overall displacement field

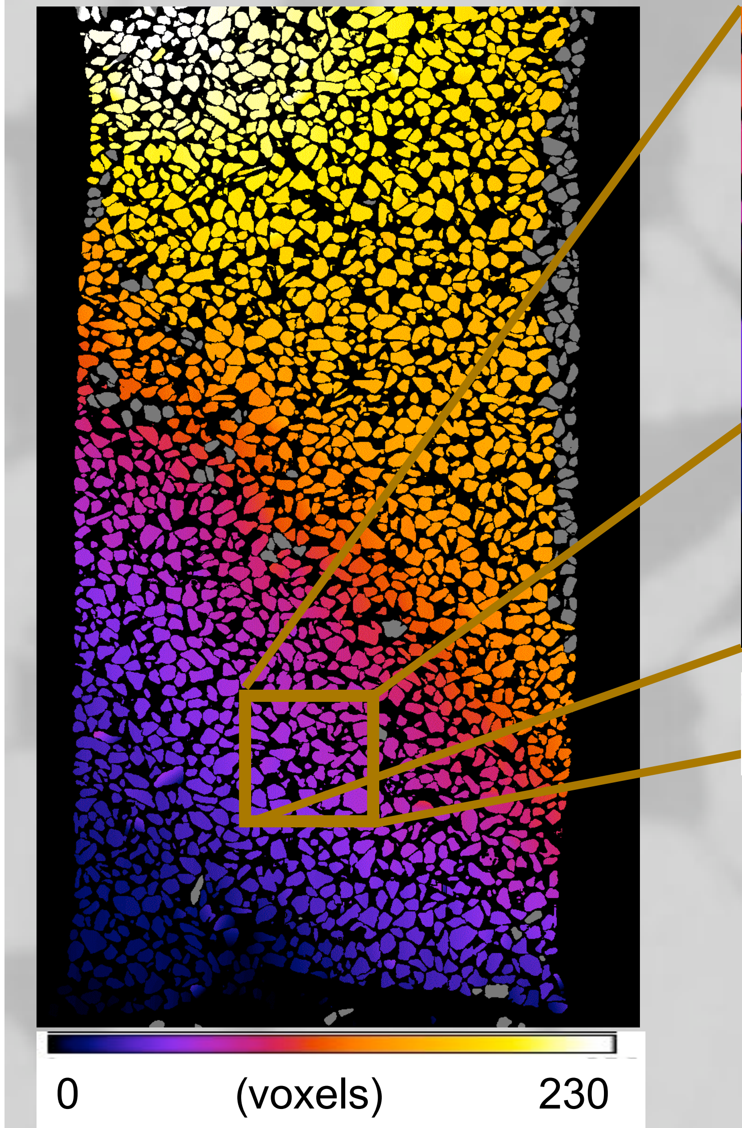


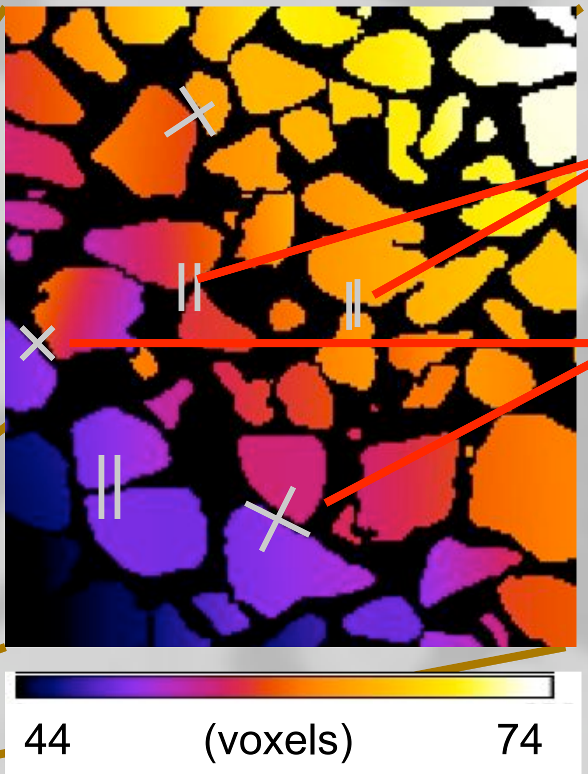
Step 1 to 7

Standard DIC OK,
with
regular subsets
and
(locally) continuous
shapes functions



Grain scale displacement field





Continuous

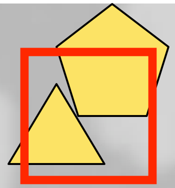
Discontinuous

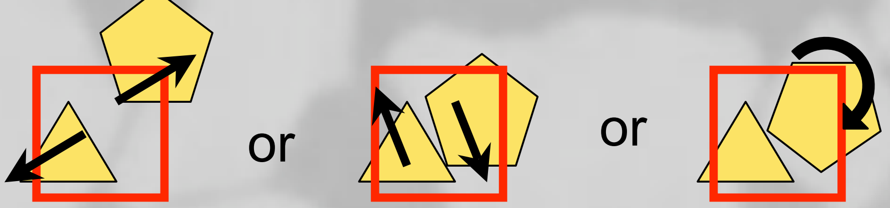
↓

Displacement fluctuations at grains scale can be strongly discontinuous

↓

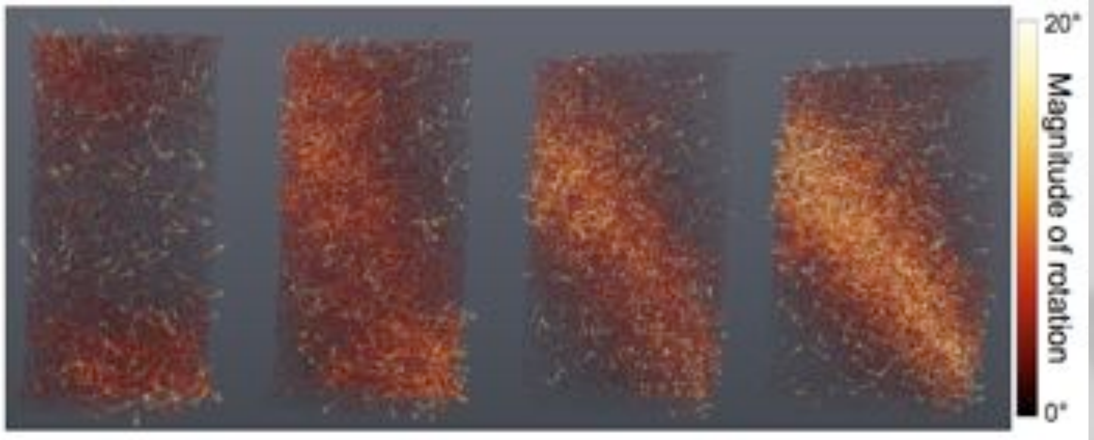
Standard DIC fails…

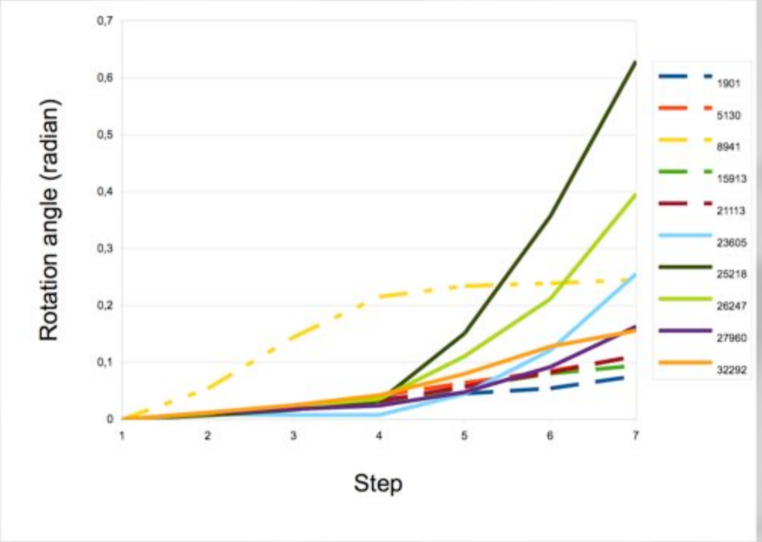


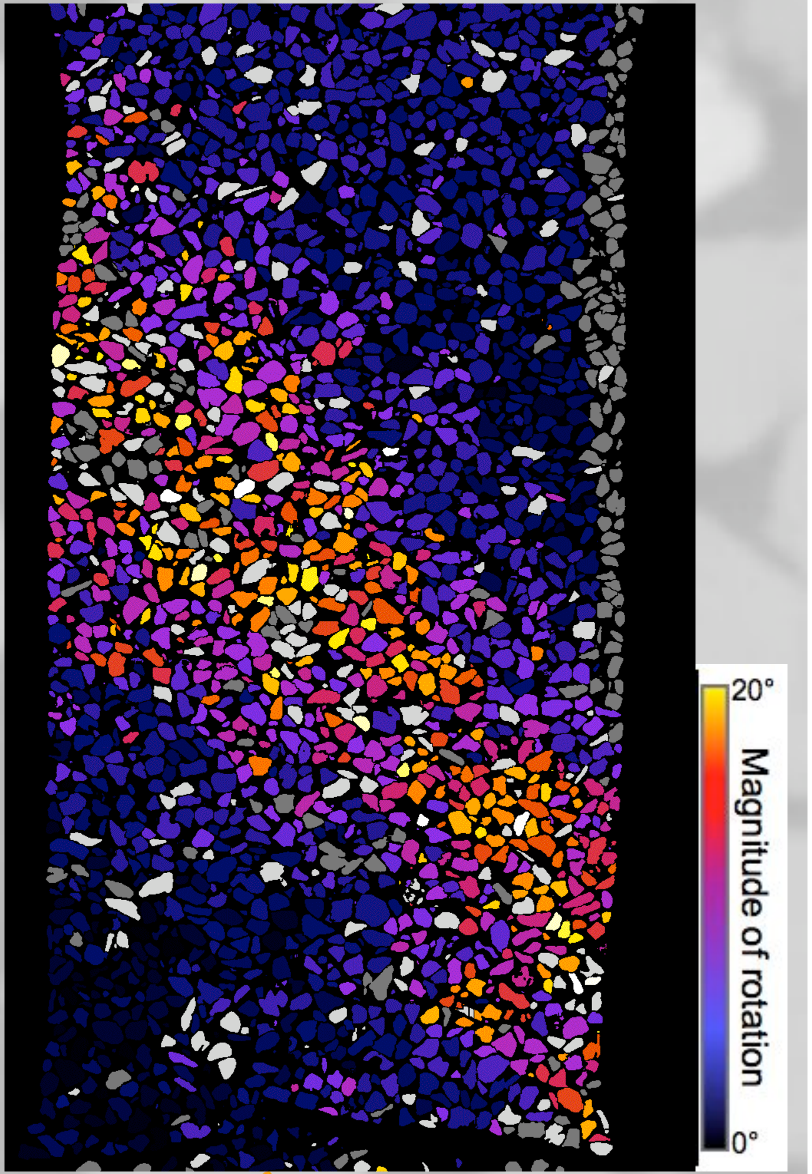


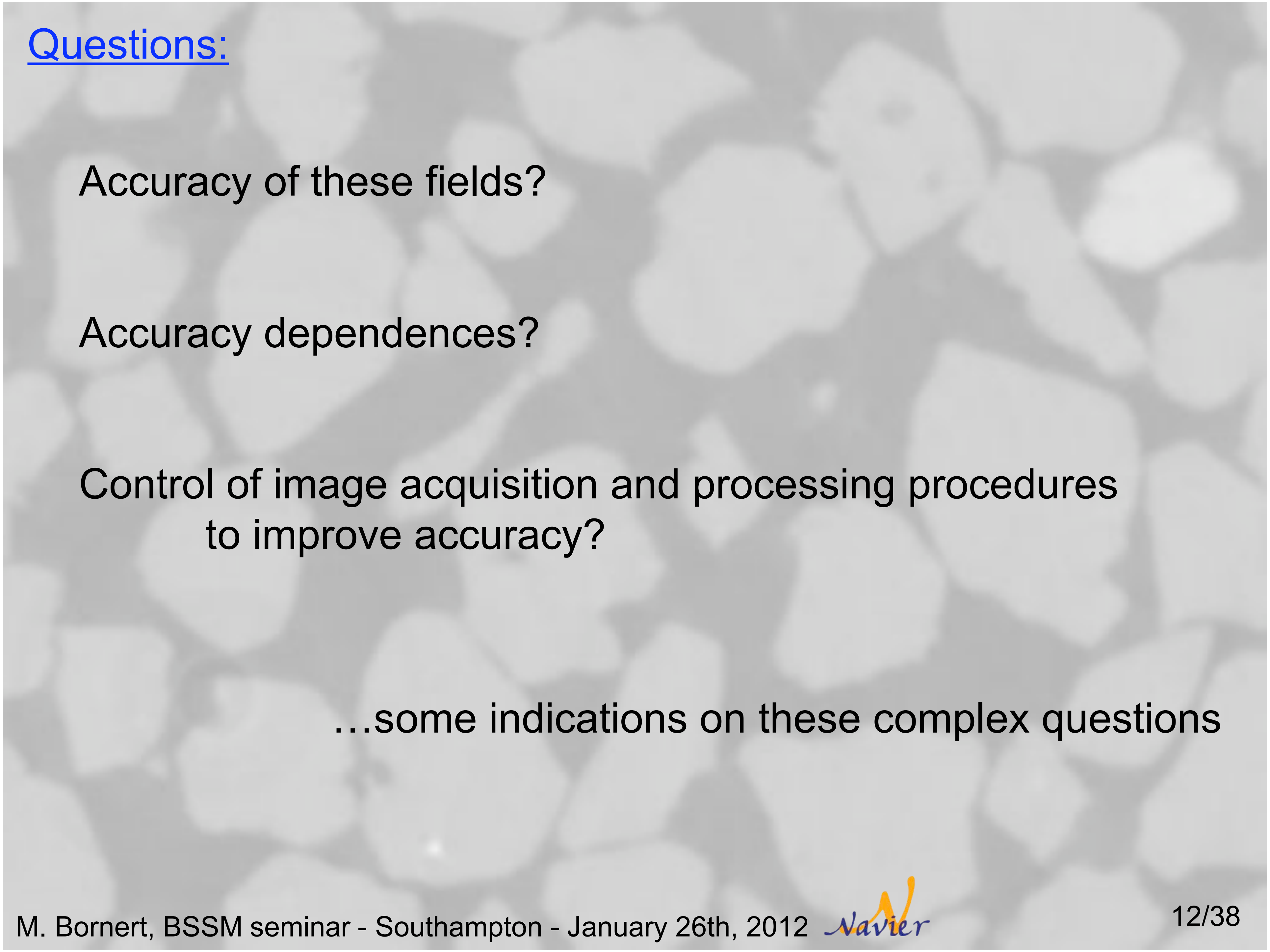
Discrete DIC, example of results: rotation angles

Incremental rotation angle









Questions:

Accuracy of these fields?

Accuracy dependences?

Control of image acquisition and processing procedures to improve accuracy?

…some indications on these complex questions

General DIC framework

(2D or 3D)

Estimated
local transformation
(output)

Correlation coefficient
(measure of similarity)

$$\boxed{\Phi_D \approx \underset{\Phi_0 \in V}{\text{Argmin}}\; C(D,\Phi_0,f,g)}$$

Set of possible local
transformation
(shape function)

Correlation window

Grey levels of
reference and current
images (input)

+ Repeat over all D's…

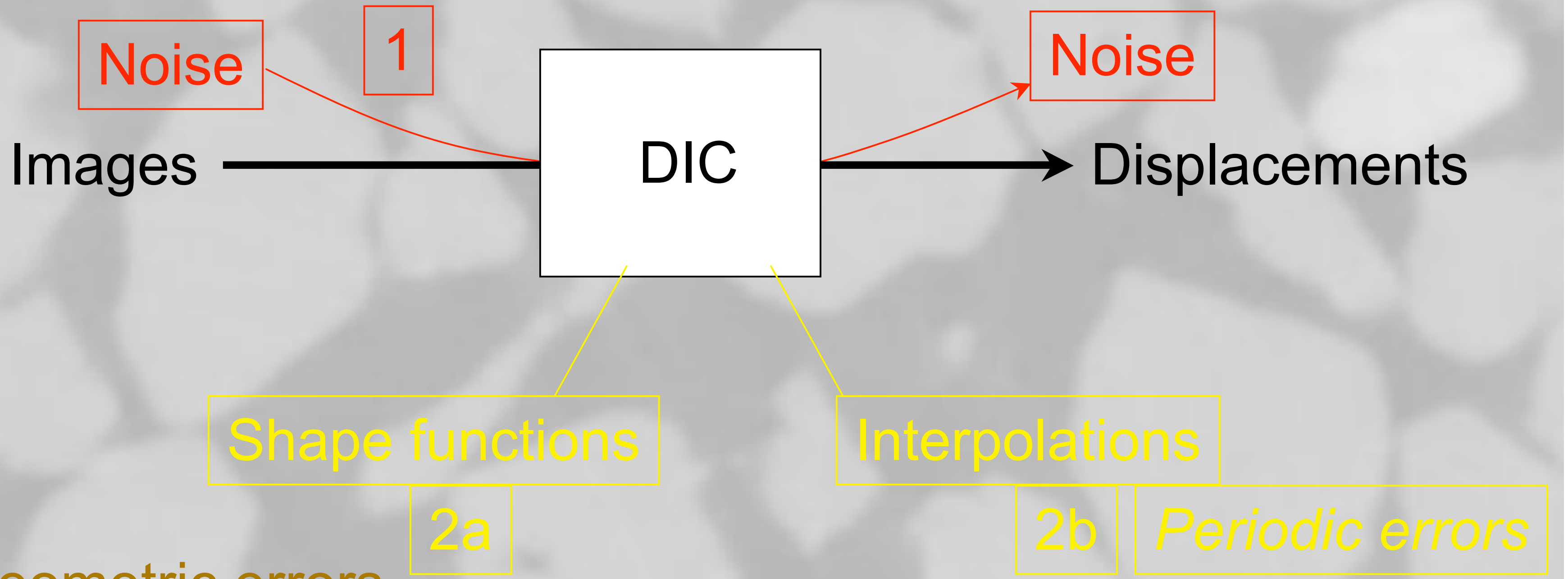
Standard DIC:
- D regularly shaped and spaced
- V = (0, 1, 2…)th order polynomial

Discrete DIC:
- D = grains
- V = rigid body motion

Fundamental assumption: convection of grey levels

Classification of DIC errors

(an attempt)



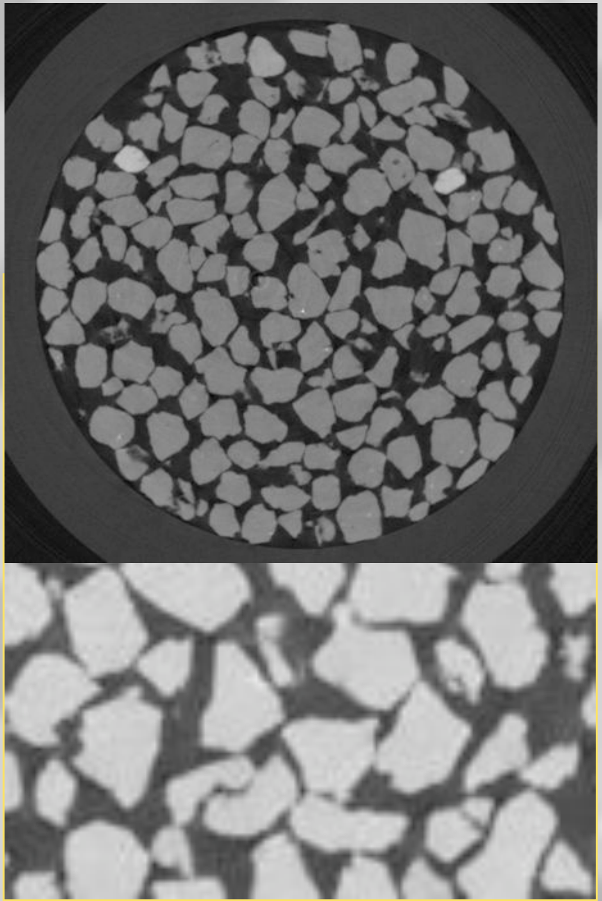
3 Geometric errors
(3D real space -> 2 or 3D image space)

4 Other errors: e.g. bad convection of grey levels…

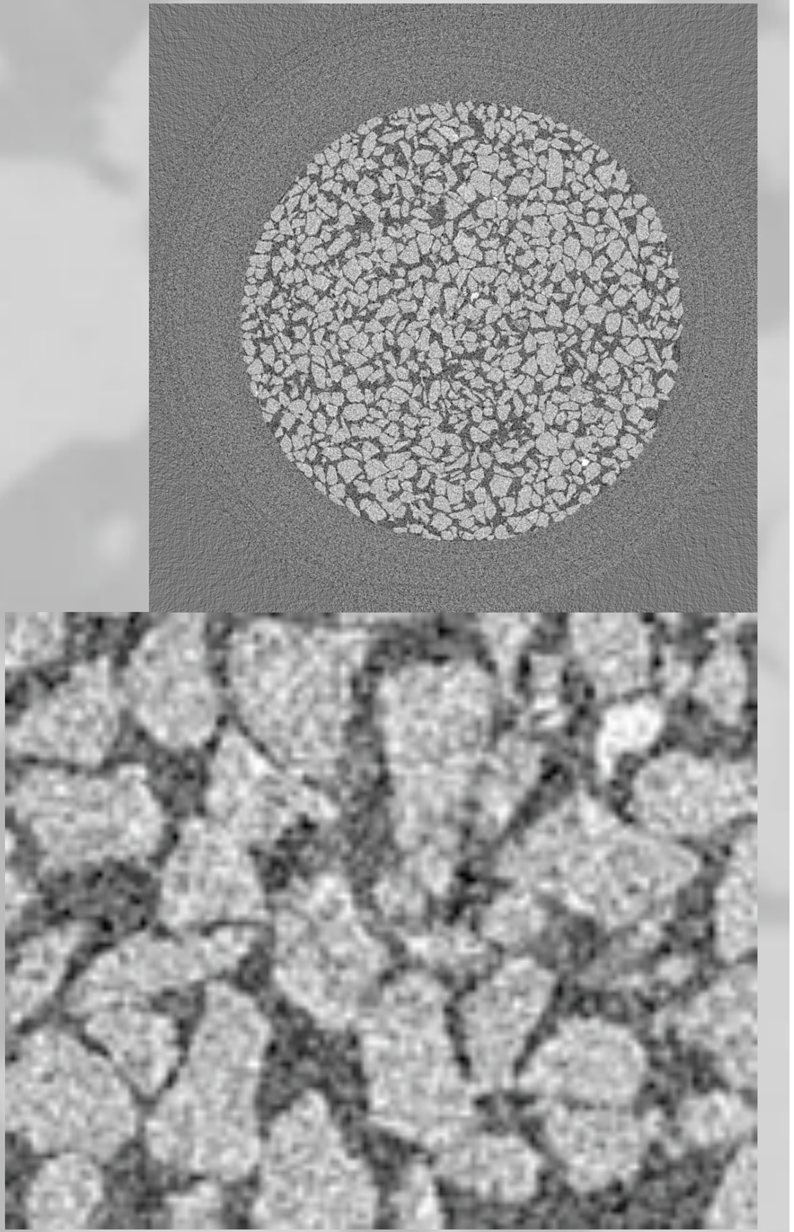
Here : focus on 1 and 2b

3

Theoretical modelling and experimental validation of angular error in discrete DIC



…related to image noise



(Bornert et al. ICEM14, Poitiers, 2010)

Theoretical analysis:
Perturbation of DIC minimum due to image noise ?

Correlation coefficient (SSD):

Rotation

(Derived from Hild & Roux 2006)

$$C(\underline{T},\underline{\underline{R}}) = \int_D \left[f(\underline{x}) - g(\underline{X}_i + \underline{T} + \underline{\underline{R}}.(\underline{x} - \underline{X}_i)) \right]^2 dx$$

Translation of center

Optimality condition:

$$dC = 0 = \int_D \left[f(\underline{x}) - g(\phi(\underline{x})) \right] \underline{\nabla g}(\phi(x)). \left[\underline{dT} + \underline{\underline{dR}}.(\underline{x} - \underline{X}_i) \right] dx \qquad \forall \underline{dT}, \forall \underline{\underline{dR}}$$

Perturbation of optimum due to noise:

(assuming $f(\underline{x}) \approx g(\phi(\underline{x}))$)
(such that $\underline{\nabla f}(\underline{x}) \approx \underline{\nabla g}(\phi(\underline{x})).\underline{\underline{R}}$)

noise

$$\int_D \left[\partial f(\underline{x}) - \partial g(\phi(\underline{x})) \right] \underline{\nabla g}(\phi(x)). \left[\underline{dT} + \underline{\underline{dR}}.(\underline{x} - \underline{X}_i) \right] dx \qquad \forall \underline{dT}, \forall \underline{\underline{dR}}$$

$$= \int_D \underline{\nabla g}(\phi(x)). \left[\underline{\partial T} + \underline{\underline{\partial R}}.(\underline{x} - \underline{X}_i) \right] \underline{\nabla g}(\phi(x)). \left[\underline{dT} + \underline{\underline{dR}}.(\underline{x} - \underline{X}_i) \right] dx$$

Induced perturbation

$$\begin{bmatrix} \underline{\underline{R}}^T.\underline{dT} \\ \underline{\underline{R}}^T.\underline{\underline{dR}} \end{bmatrix} . \begin{bmatrix} \int_D \left[\partial f(\underline{x}) - \partial g(\phi(\underline{x}))\right] \underline{\nabla f}(x) dx \\ \int_D \left[\partial f(\underline{x}) - \partial g(\phi(\underline{x}))\right] \underline{\nabla f}(x) \otimes (\underline{x} - \underline{X}_i) dx \end{bmatrix} = \qquad \forall\, \underline{dT}, \forall\, \underline{\underline{dR}}$$

$$\begin{bmatrix} \underline{\underline{R}}^T.\underline{dT} \\ \underline{\underline{R}}^T.\underline{\underline{dR}} \end{bmatrix} . \begin{bmatrix} \int_D \underline{\nabla f}(x) \otimes \underline{\nabla f}(x) dx & \int_D \underline{\nabla f}(x) \otimes \underline{\nabla f}(x) \otimes (\underline{x} - \underline{X}_i) dx \\ \int_D \underline{\nabla f}(x) \otimes (\underline{x} - \underline{X}_i) \otimes \underline{\nabla f}(x) dx & \int_D \underline{\nabla f}(x) \otimes (\underline{x} - \underline{X}_i) \otimes \underline{\nabla f}(x) \otimes (\underline{x} - \underline{X}_i) dx \end{bmatrix} . \begin{bmatrix} \underline{\underline{R}}^T.\underline{\partial T} \\ \underline{\underline{R}}^T.\underline{\underline{\partial R}} \end{bmatrix}$$

$\underline{\underline{R}}^T.\underline{\underline{dR}}$ is a skew-symmetric tensor such that $(\underline{\underline{R}}^T.\underline{\underline{dR}}).\underline{X} = \underline{dw} \wedge \underline{X}$

$\underline{dw}$ = infinitesimal rotation vector (in reference configuration)

$$\begin{bmatrix} \int_D \left[\partial f(\underline{x}) - \partial g(\phi(\underline{x}))\right] \underline{\nabla f}(x) dx \\ \int_D \left[\partial f(\underline{x}) - \partial g(\phi(\underline{x}))\right] \left[\underline{\nabla f}(x) \wedge (\underline{x} - \underline{X}_i)\right] dx \end{bmatrix} =$$

$$\underbrace{\begin{bmatrix} \int_D \underline{\nabla f}(x) \otimes \underline{\nabla f}(x) dx & \int_D \underline{\nabla f}(x) \otimes \left[\underline{\nabla f}(x) \wedge (\underline{x} - \underline{X}_i)\right] dx \\ \int_D \left[\underline{\nabla f}(x) \wedge (\underline{x} - \underline{X}_i)\right] \otimes \underline{\nabla f}(x) dx & \int_D \left[\underline{\nabla f}(x) \wedge (\underline{x} - \underline{X}_i)\right] \otimes \left[\underline{\nabla f}(x) \wedge (\underline{x} - \underline{X}_i)\right] dx \end{bmatrix}}_{\text{6x6 matrix } \underline{\underline{M}}} . \begin{bmatrix} \underline{\partial t} \\ \underline{\partial w} \end{bmatrix}$$

$\underline{\partial t} = \underline{\underline{R}}^T.\underline{\partial T}$

Computation of covariance tensor of errors

(assuming white noise on pixels)

$$\left\langle \left[\frac{\partial t}{\partial \underline{w}}\right] \otimes \left[\frac{\partial t}{\partial \underline{w}}\right] \right\rangle :_{2}^{3} \left[\underline{\underline{M}} \otimes \underline{\underline{M}}\right] = 2p^{3}\sigma_{f}^{2}\,\underline{\underline{M}}$$

Correlation length of noise (~voxel size)

Standard deviation of image noise

General procedure: diagonalize $\underline{\underline{M}}$ …

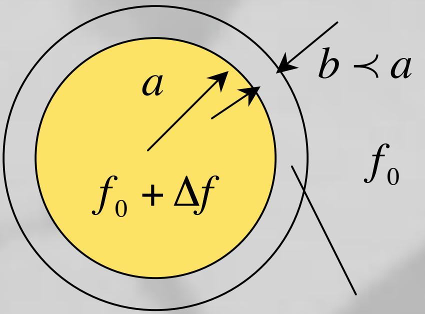
(See Bornert et al. ICEM14, Poitiers, 2010)

If $\underline{\underline{M}}$ diagonal:

$$\left\langle \left[\frac{\partial t}{\partial \underline{w}}\right] \otimes \left[\frac{\partial t}{\partial \underline{w}}\right] \right\rangle = 2p^{3}\sigma_{f}^{2}\,\underline{\underline{\mathrm{Diag}}}\left(\frac{1}{\mu_1},\cdots,\frac{1}{\mu_6}\right)$$

eigenvalues

Example 1: spherical grain

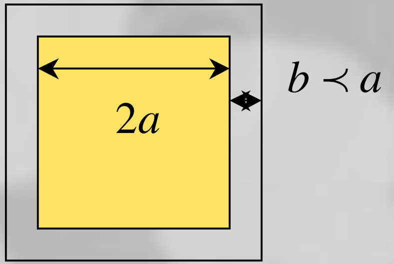


Linear variation of grey levels

$$\underline{\underline{M}} = \mu^t \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mu^t = \frac{4\pi a^2 (\Delta f)^2}{3b}$$

$$\sigma_t = p.\frac{\sqrt{6pb}}{2\sqrt{\pi a}}.\frac{\sigma_f}{\Delta f} \qquad \sigma_w = +\infty$$

Example 2: cubic grain



$$\underline{\underline{M}} = \begin{bmatrix} \mu^t & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu^t & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu^t & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu^w & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu^w & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu^w \end{bmatrix}$$

$$\mu^t = \frac{8a^2 (\Delta f)^2}{b}$$

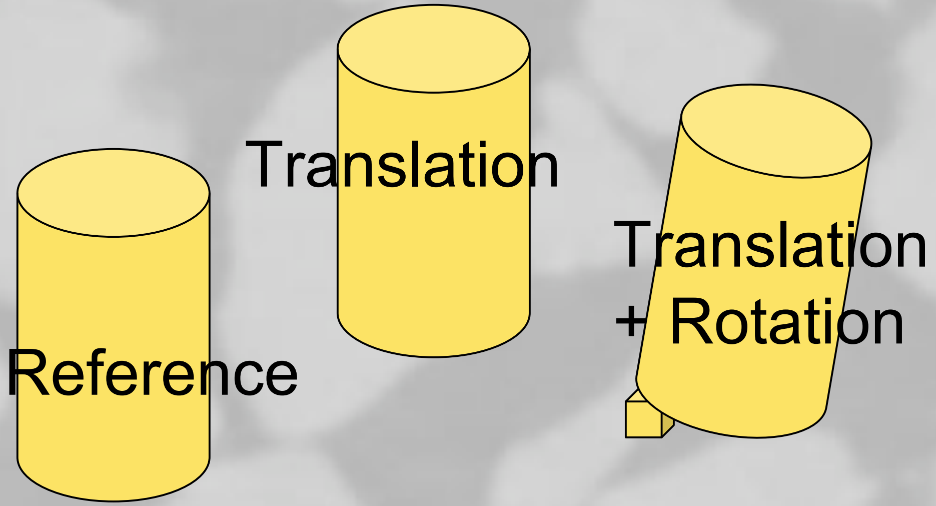
$$\mu^t = \frac{(2a)^4 (\Delta f)^2}{3b}$$

$$\sigma_t = p.\frac{\sqrt{pb}}{2a}.\frac{\sigma_f}{\Delta f} \qquad \boxed{\sigma_w = \frac{p}{a}\frac{\sqrt{6pb}}{a}.\frac{\sigma_f}{\Delta f}}$$

(NB: *isotropic* sensitivity to rotation)

Experimental validation:

“Zero deformation” experiment

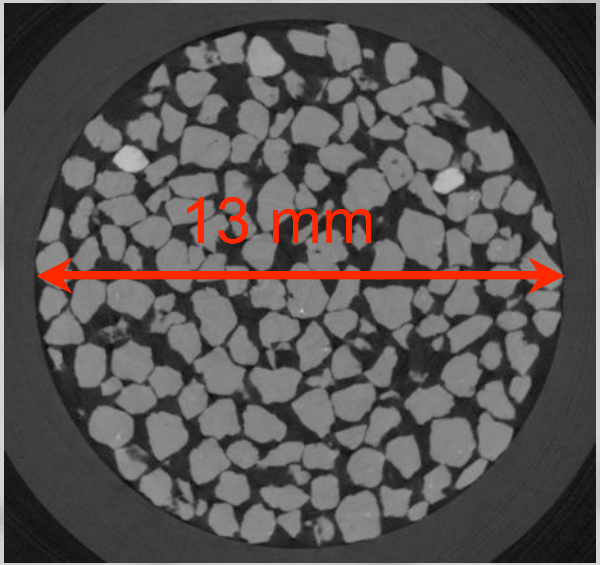


Sample of
Hostun sand
with large grains

(D50 = 900µm)

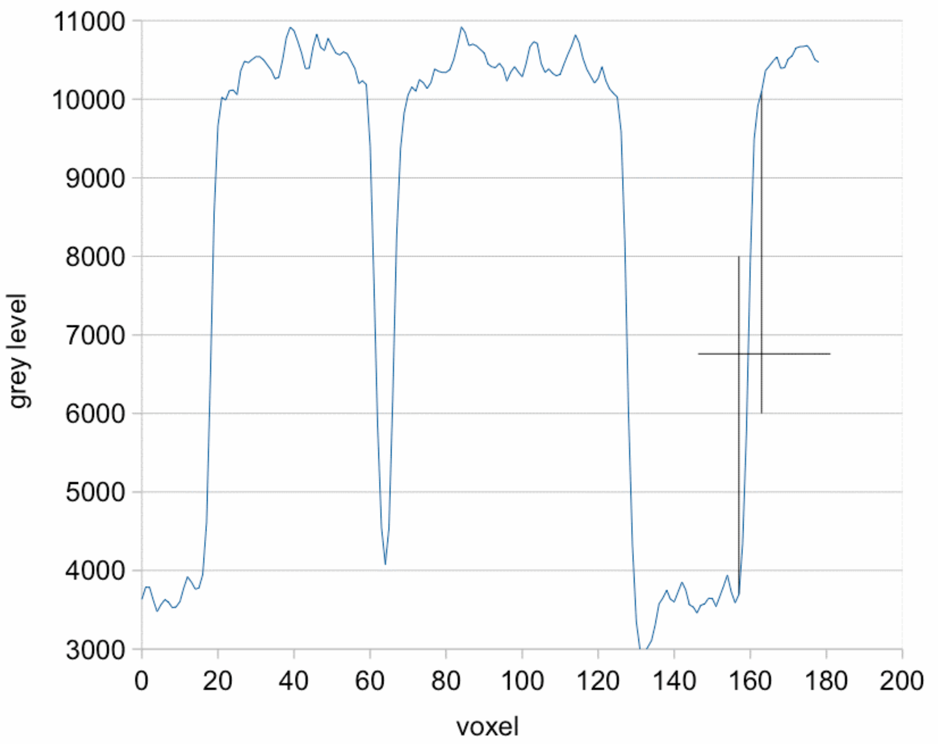
1150x1150
x351 voxels

1 voxel = 15µm



Laboratory
CT scanner
at L3SR

RX-Solutions



Profile in 16bis CT section

Theoretical error:

$$\sigma_f \approx 300$$

$$\Delta f \approx 7000$$

$$\frac{\sigma_f}{\Delta f} \approx \frac{1}{23}$$

$$b \approx 6$$

$$a \approx 30$$

Spheres : $\sigma_t \approx 0.003 \text{ vox}$

Cubes : $\sigma_t \approx 0.002 \text{ vox}$

$$\sigma_w \approx +\infty$$

$$\boxed{\begin{aligned}\sigma_w &\approx 1/3000 \text{ rad}\\ &\approx 0.02 \text{ degrees}\end{aligned}}$$

Macroscopic DIC:

$$F - i = \begin{bmatrix} -9.31.10^{-4} & -1.41784.10^{-3} & 3.0182.10^{-2} \\ 6.04.10^{-4} & -7.50814.10^{-4} & 3.1939.10^{-2} \\ -3.0518.10^{-2} & -3.2247.10^{-2} & -1.447.10^{-3} \end{bmatrix}$$

(error ~0.0005)

$$\alpha \approx 2.53(\pm 0.01)\,\mathrm{deg}$$

$$\underline{n} \approx 0.726\underline{e}_x + 0.687\underline{e}_y - 0.02288\underline{e}_z$$

$$\longrightarrow \alpha_x = 1.840, \alpha_y = 1.740, \alpha_z = -0.058\ (\mathrm{deg})$$

Individual discrete-DIC grain analysis

(on ~700 grains, >95% success)

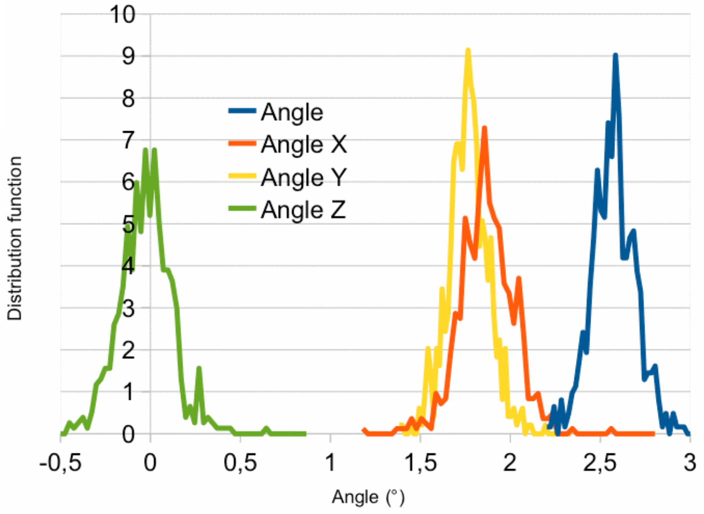
		t_x	t_y	t_z	α_x	α_y	α_z	α
		(voxels)			(degrees)			
Translation	Av.	-	-	-	0.023	-0.023	-0.104	0.109
	σ	0.148	0.177	0.111	0.133	0.118	0.138	0.126
Rotation	Av.	-	-	-	1.855	1.759	-0.044	2.565
	σ	0.094	0.129	0.0651	0.148	0.111	0.151	0.124

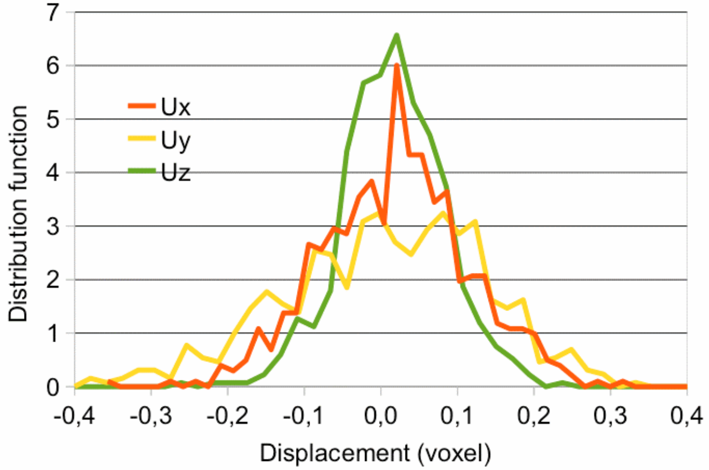
Statistical distribution functions

Errors on rotations

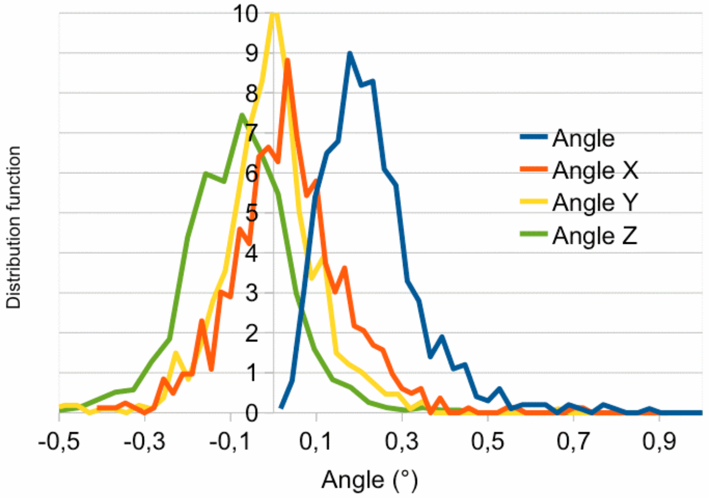
Errors on translation

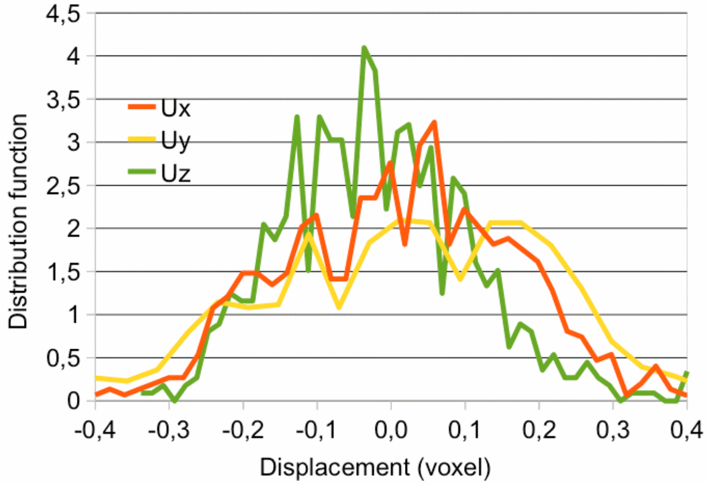
Motion =
Rotation
+ translation





Motion =
Translation





Comments

Consistency exp/theory on σ_w

$$\sigma_w^{th} \in [0.02\,;+\infty] \quad \text{(deg)}$$

$$\sigma_w^{\exp} \approx 0.1$$

While apparently $\sigma_t^{\exp} >> \sigma_t^{th}$

But : X_i is not the exact center of grains

$$\sigma_t(X_i) \approx \sqrt{\left[\sigma_t(X_{center})\right]^2 + \left[\sigma_w.\left\|X_i - X_{center}\right\|\right]^2}$$

Application to other images

(D50 = 280µm)

$$\sigma_f \approx 22 \qquad \frac{\sigma_f}{\Delta f} \approx \frac{1}{4}\,!$$

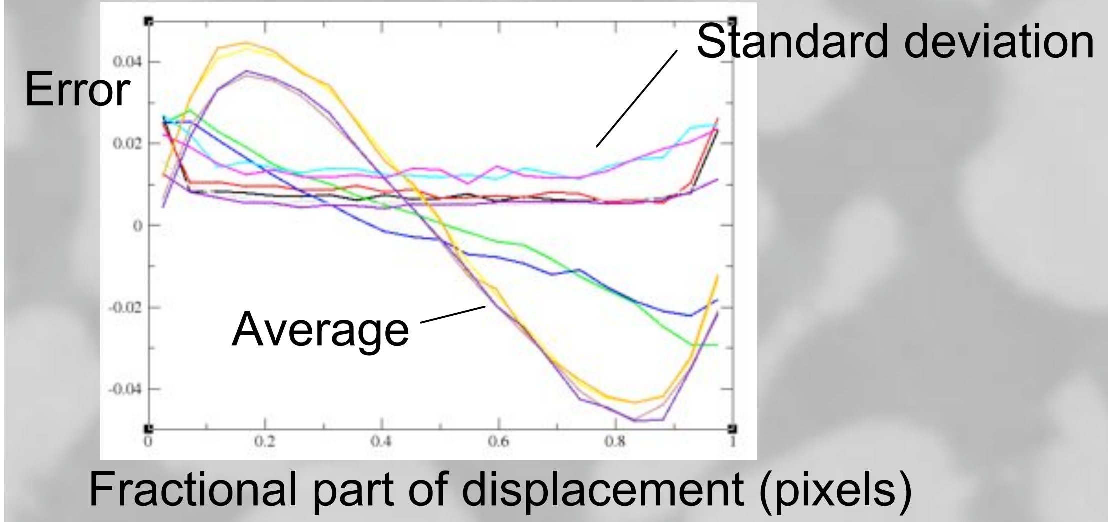
$$\Delta f \approx 90$$

$$a \approx 10 \qquad b \approx 1$$

$$\boxed{\sigma_w^{th} \in [0.35\,;+\infty]} \ \text{(deg)}$$

$$\longrightarrow \boxed{\sigma_w^{\exp} \approx 1\,\deg}$$

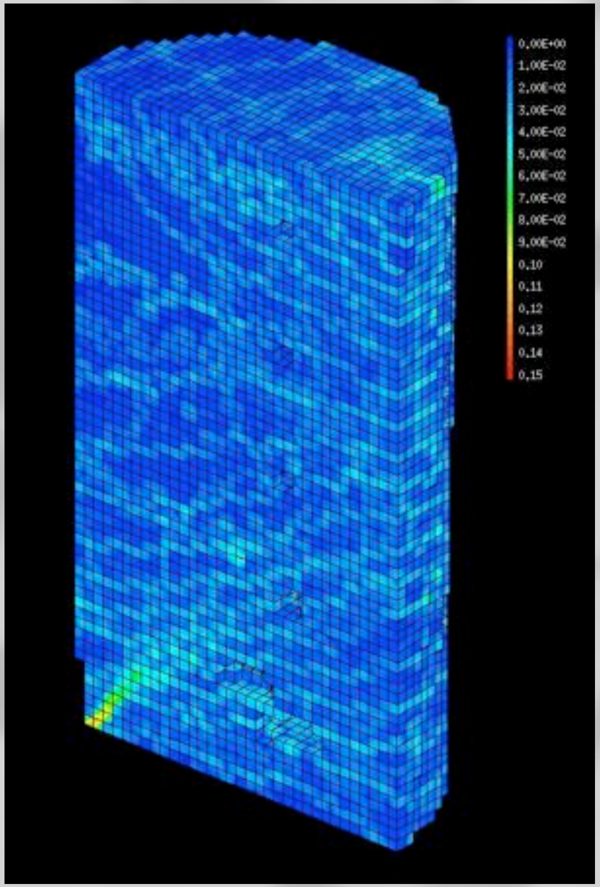
4) Quantification of full systematic error curve with just two images



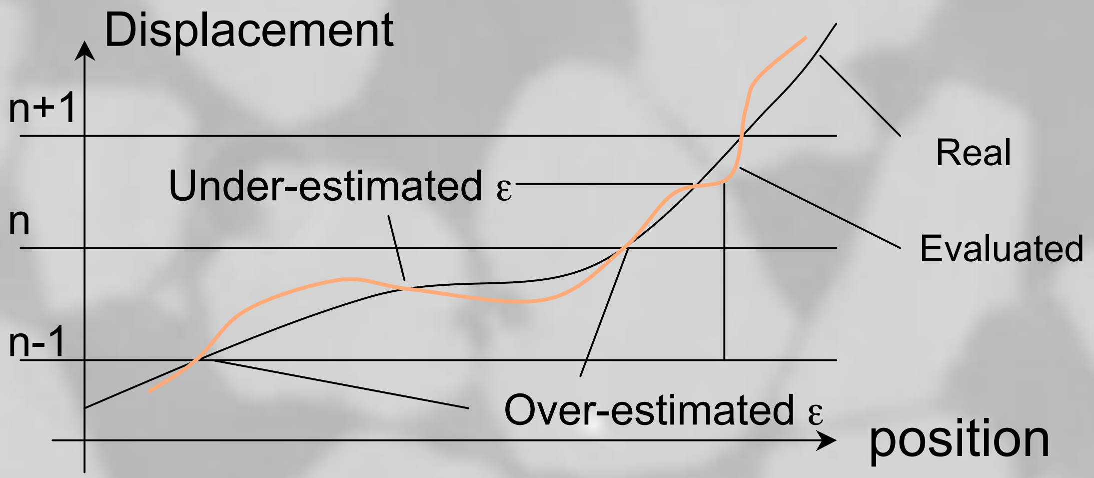
Images are discrete data

$$\int_{u\in D} \alpha(u)\,du \Rightarrow \sum_{(ij)\in D} \alpha_{ij}$$

→ Interpolations



Lenoir et al, Strain 2007



Experimental evaluation of S-shaped systematic error curve

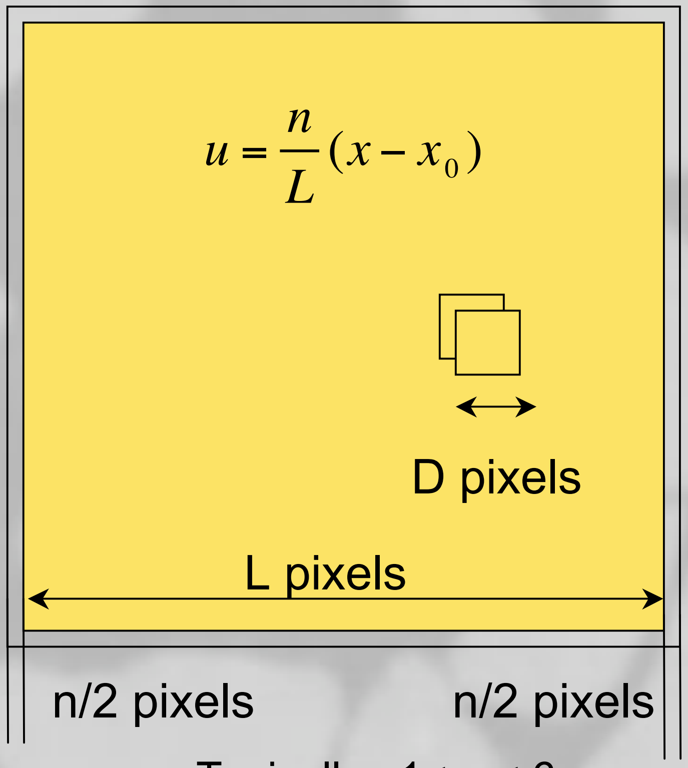
- Standard procedure:

Prescribe several real subpixel translations of sample and compare with DIC measurements

Long and difficult to perform in practice

- More efficient procedure:

Prescribe motions to sample or imaging system that generate locally in image an apparent translation with known characteristics



$$u = \frac{n}{L}(x - x_0)$$

Rigid rotation or

<u>magnification variation</u>

Simple and fast

If $\frac{n}{L} <\sim \frac{0,2}{D}$ displacement is sufficienlty uniform in correlation window

$\frac{n}{L}$ and x_0 evaluated (accurately) from overall (apparent) strain

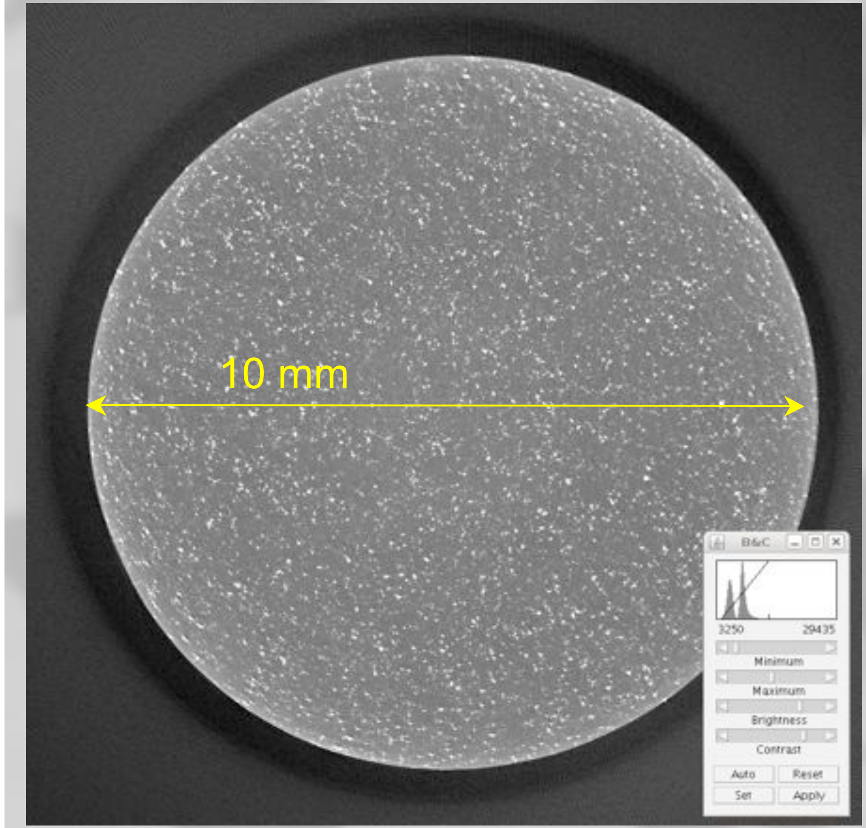
Typically : 1< n < 6

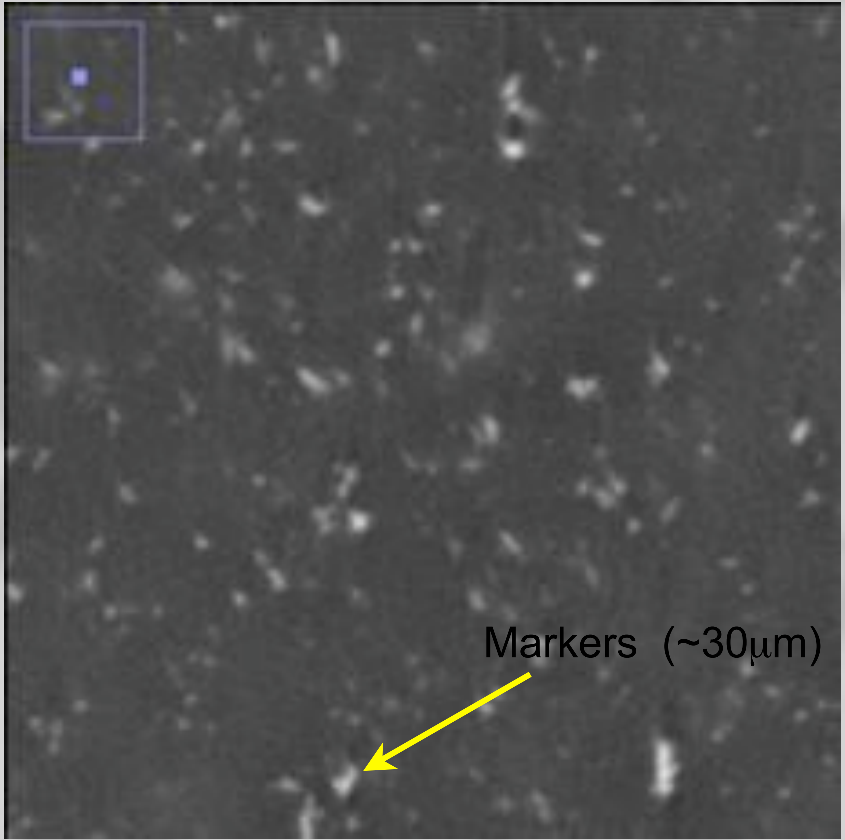
Yang et al, 2010, ICEM14

3D application: Virtual homogeneous isotropic straining of cylindrical halite sample with Cu markers

(with M. Bourcier, A. Dimanov, LMS
ANR Project « MicroNaSel »)

GE X-ray 160kV nanofocus tube
@ 67kV / 100 µA / 6,5 W (mode 1)
Flat Panel Varian 2520,
@1920x1536, 1s/image, average 30
1440 projections (13h scan)
Images 1840x1840x992 voxels





Sample: 10mm Diameter x 20mm Height
(imaged zone 6,5mm in height)

Virtual straining:

Virtually deformed image:
Same sample, same conditions
with imager shifted by 0.9mm
+ sample shift ΔY = 100µm

Initial voxel size = 6.5 µm

New voxel size = 6.48µm

MORE PRECISELY (according to geometry of system) :
apparent dilatation = 1.0031962 = ratio of voxelsizes = 6.50022 / 6.47951

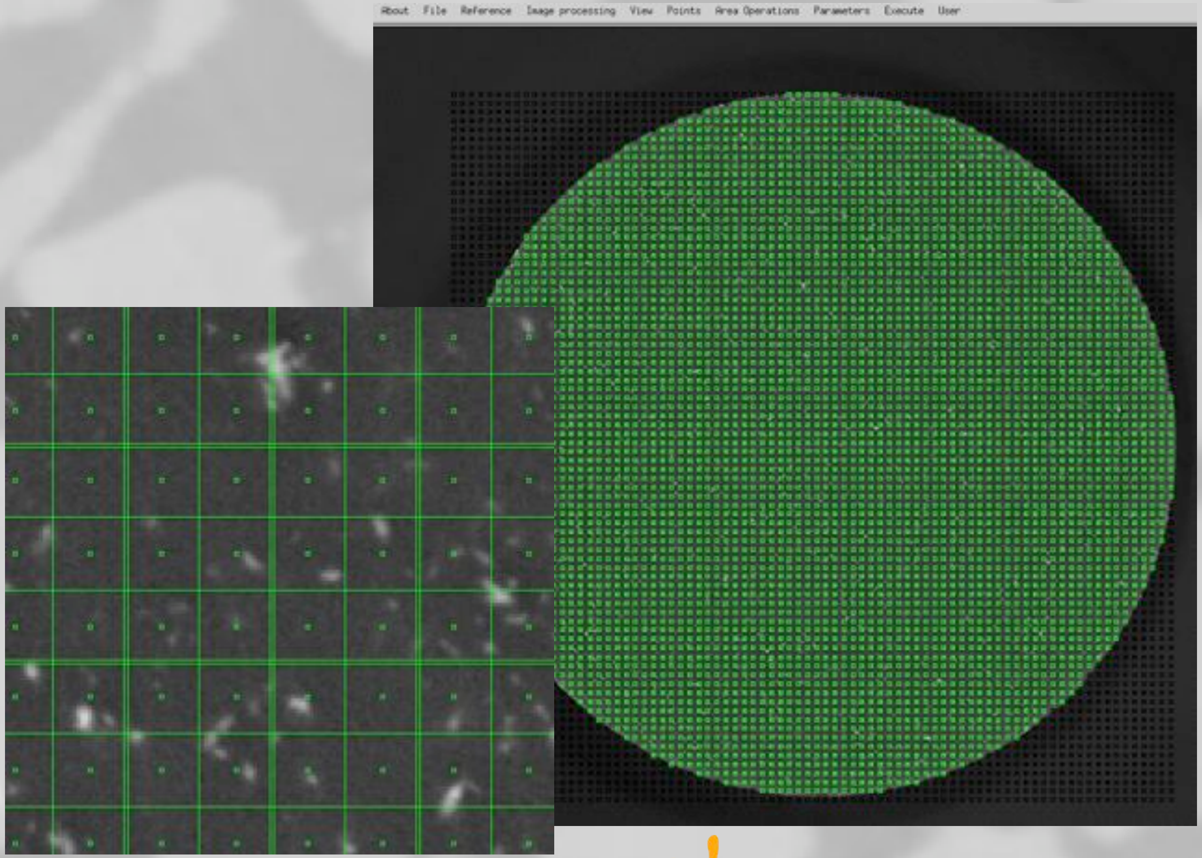
This corresponds to ~5 voxels increase in sample diameter

Vol-DIC analysis:

Grid:
20 voxels steps,
80x80x47 points
= 300800 points,
232683 in sample

Trilinear g.l. interpolation
Rigid transformation
In-house code (CMV3D)

Various window sizes
From20^3 to 50^3
Fixed or adjustable



A) Direct processing of original images

Average deformation gradient:
(example of result)

(very close to prescribed
magnification variation)

0.003196

0.003198	0.000441	-0.000067
-0.000087	0.003191	-0.000009
-0.000118	0.000076	0.003268

(Accuracy better than 0.0001)

Statistical analysis of local evaluations of displacement

Compare DIC measurements with theoretical displacement

1) Global analysis

Standard deviation on 3
displacement components

2) Local analysis
as a function of fractional part
of theoretical displacement

Standard deviation + bias
on 3 displacement components

1) Global analysis

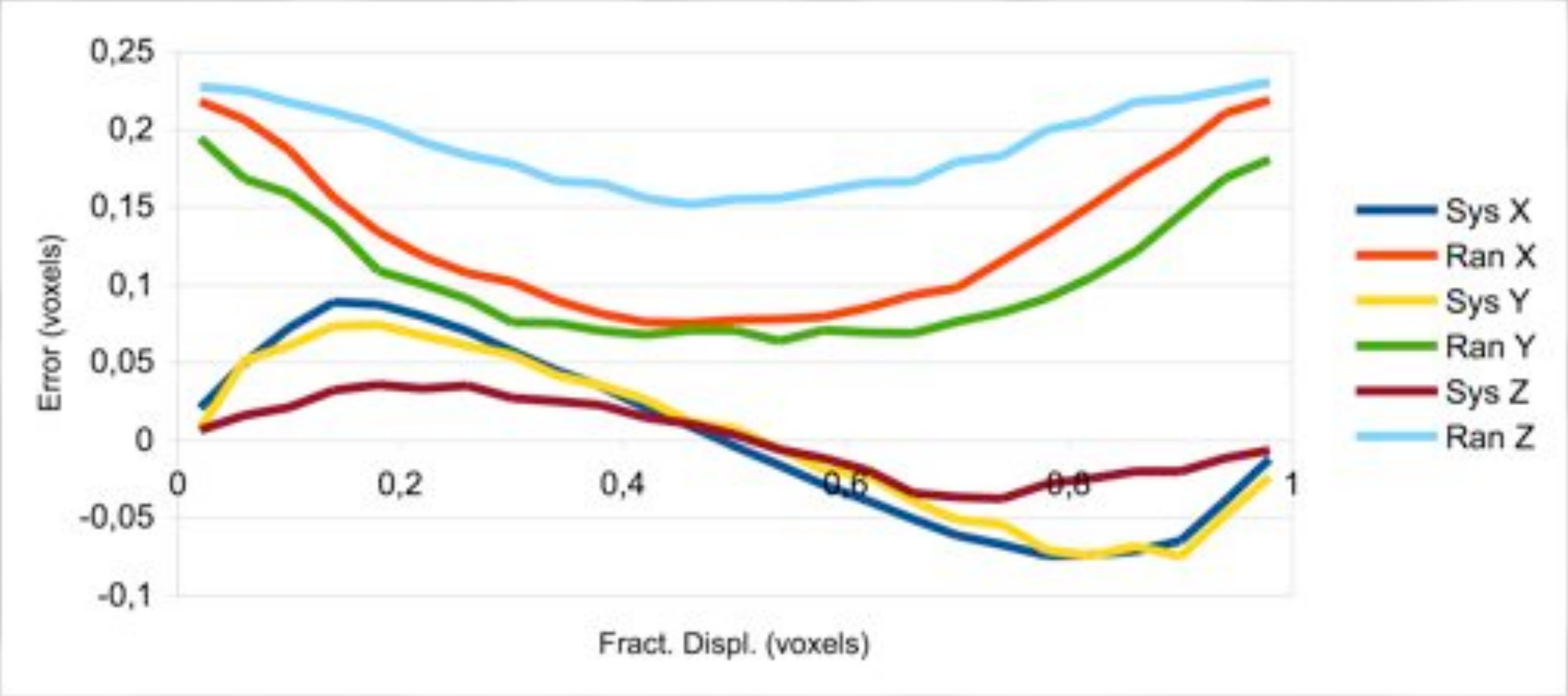
Window Size	Std. Dev. X	Std. Dev. Y	Std. Dev. Z
20^3 constant	0.158734	0.128389	0.254016
20^3 variable	0.146954	0.121243	0.195838
30^3 variable	0.133726	0.106016	0.185546
40^3 constant	0.128136	0.100171	0.182437
50^3 constant	0.126381	0.098852	0.181389

↓ Improvement

↓ No significant change

~ <

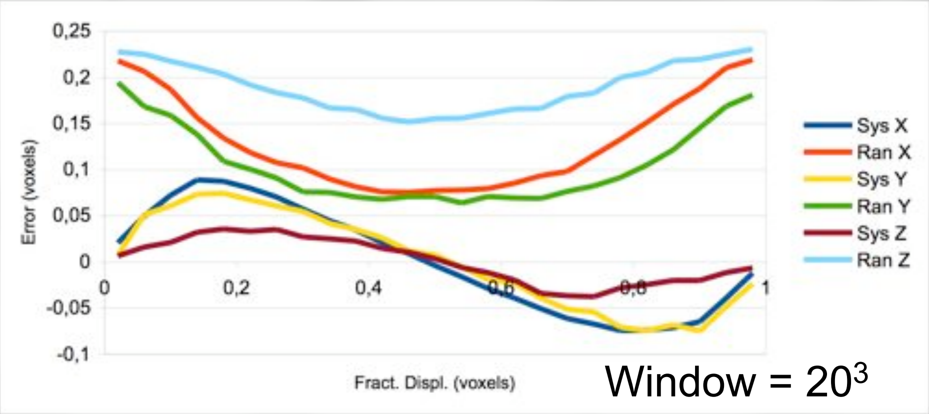
2) Local analysis



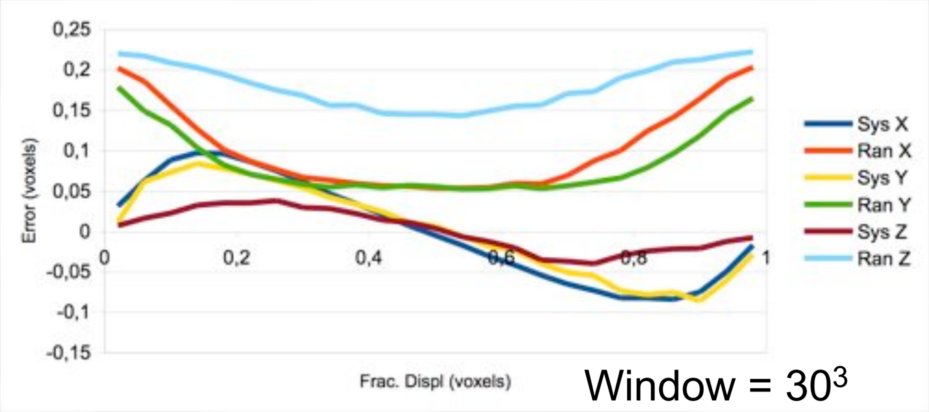
Full S-shaped curve obtained with two images

Similar X and Y behaviour (as expected), consistent with 2D observations

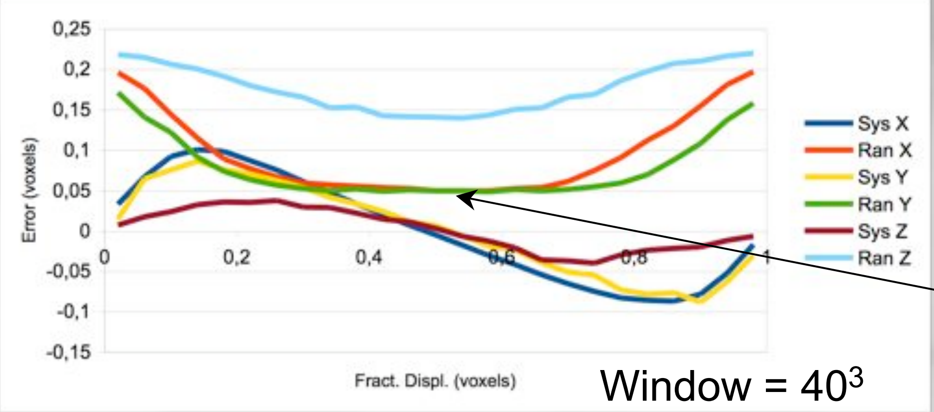
Behaviour along Z is again quantitatively different



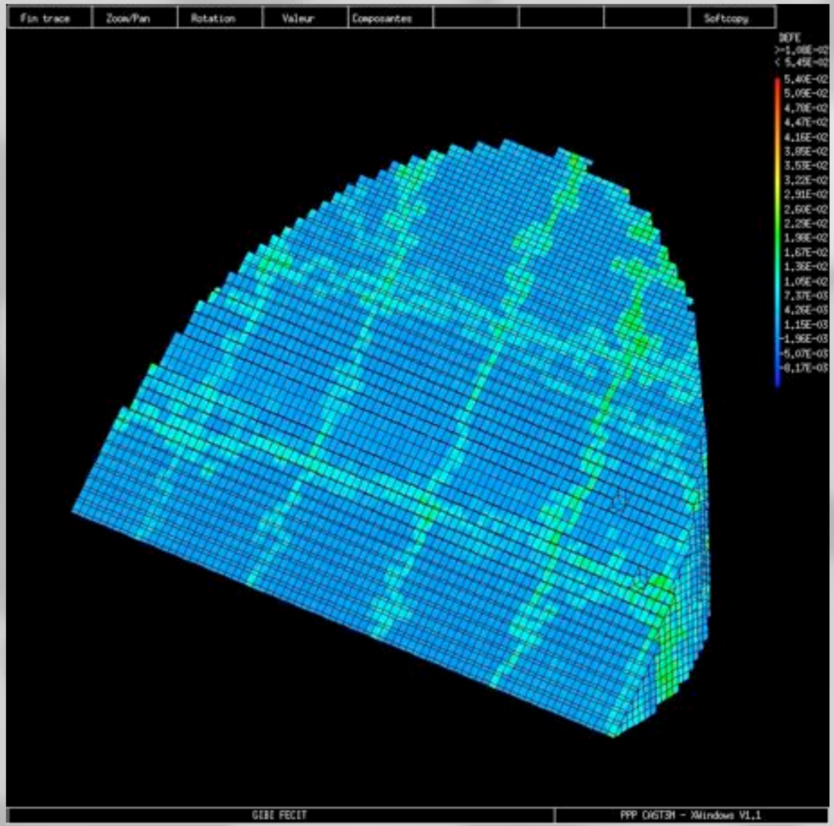
Window = 20^3



Window = 30^3



Window = 40^3



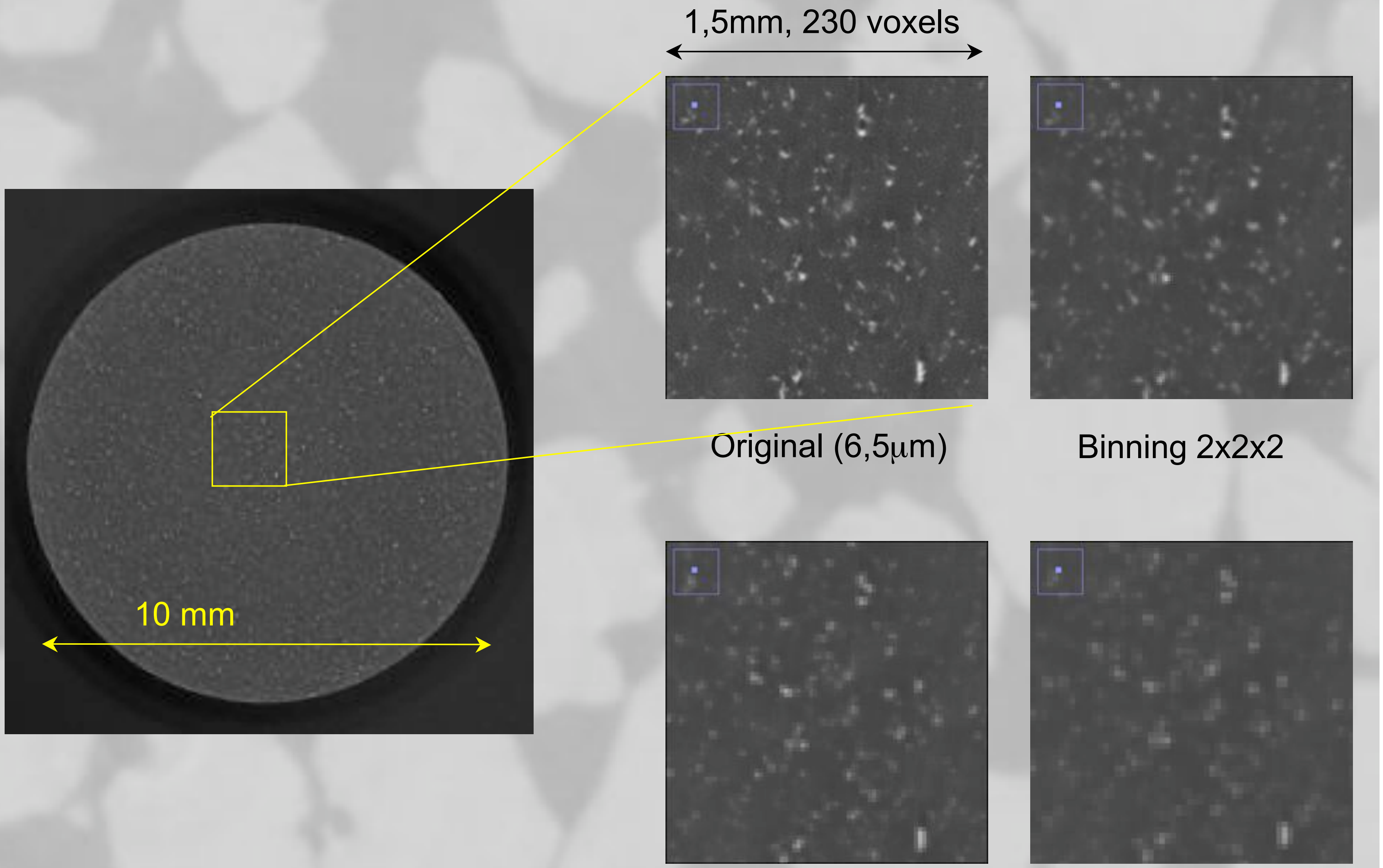
2,6%

0

Von Mises Strain
Window = 40^3
(FE derivatives)

Residual shape function mismatch error
(can be shown to be ~0.05 voxels)

A) Processing of binned images



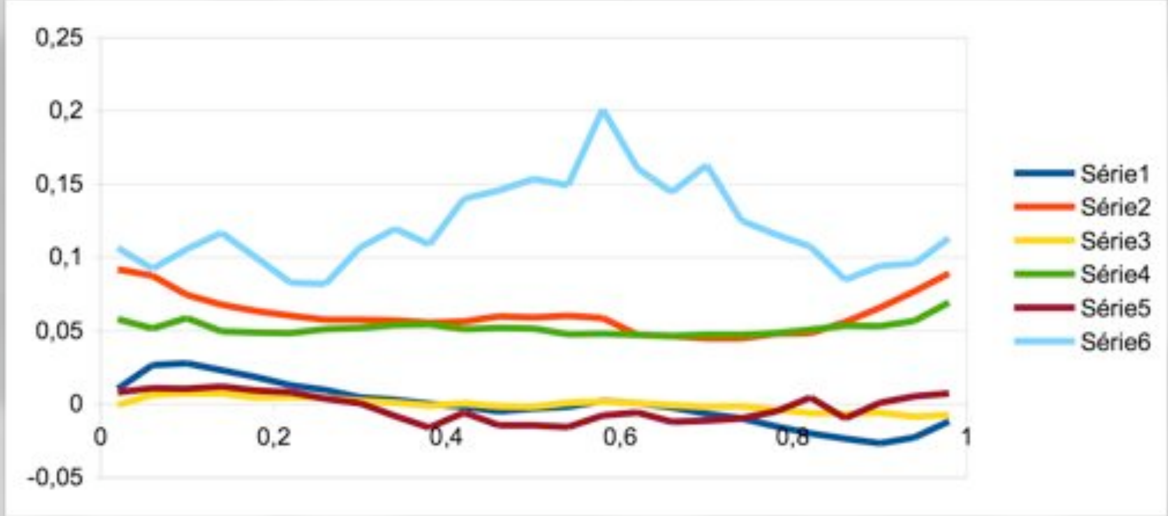
Original (6,5µm)

Binning 2x2x2

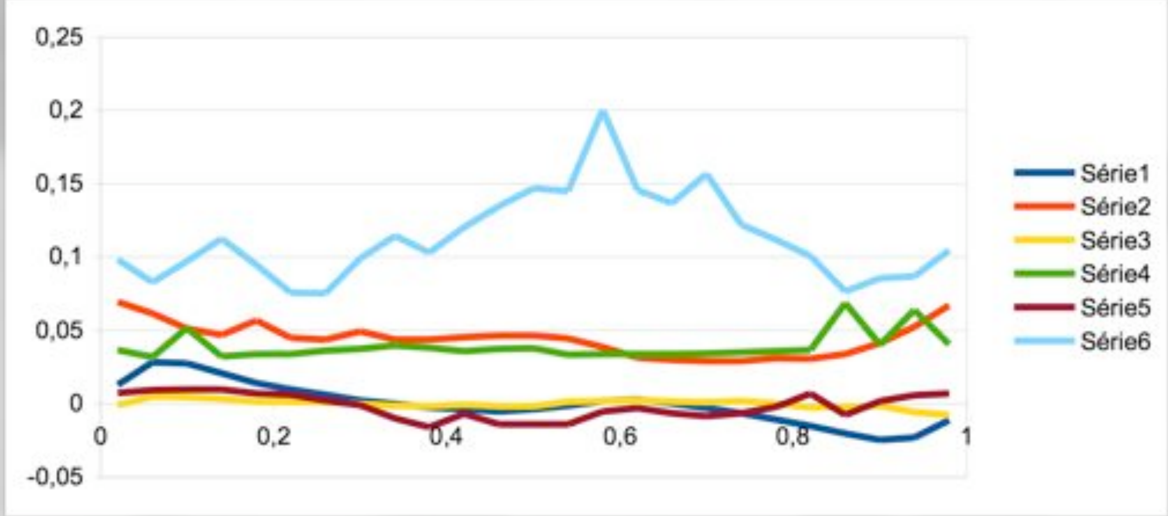
Binning 3x3x3

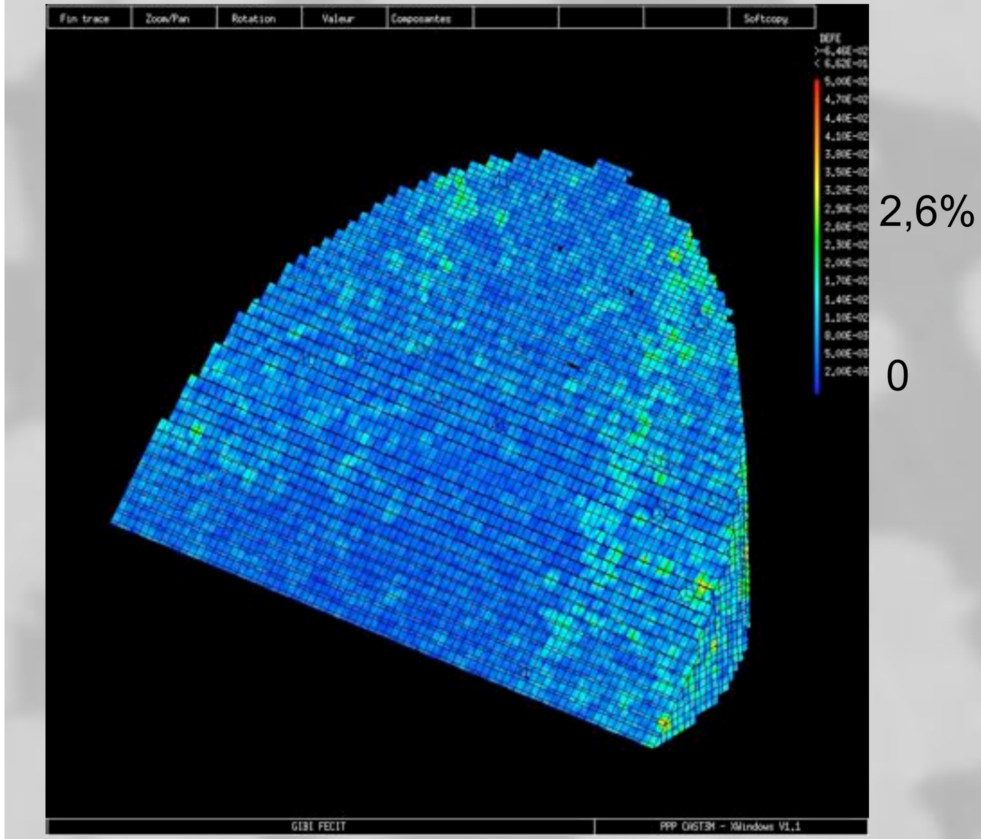
Binning 4x4x4
(voxel 26µm)

Bining 2x2x2
Window = 10^3 (=20^3)



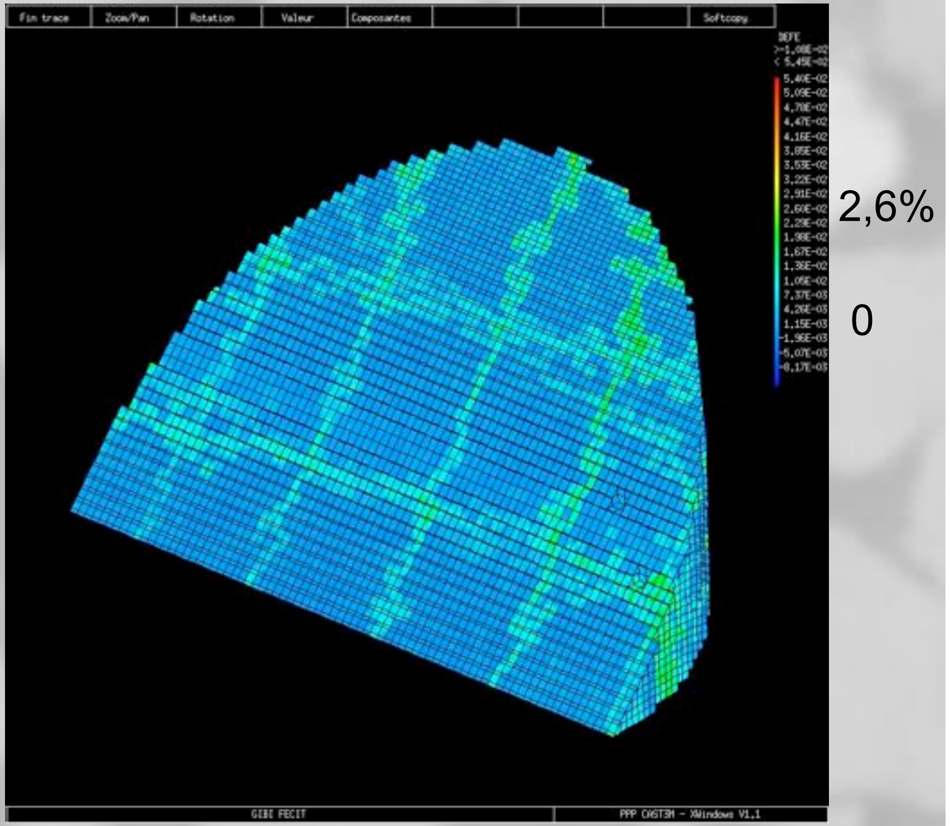
Bining 2x2x2
Window = 15^3 (=30^3)





Von Mises Strain
Bining 2x2x2
Window = 10^3 (=20^3)

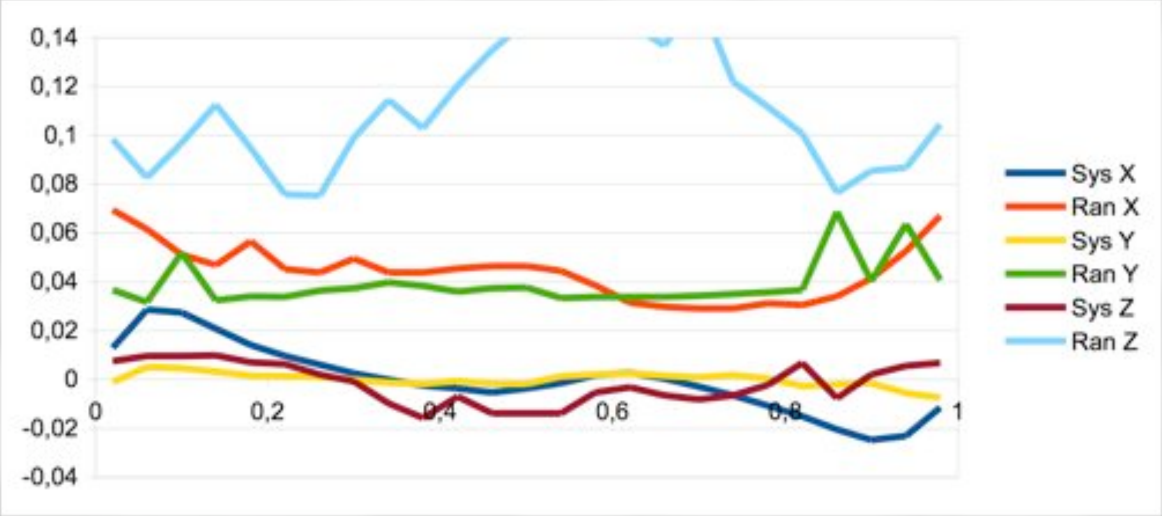
(Computation time = minutes)
(Acquisition time divided by 2)



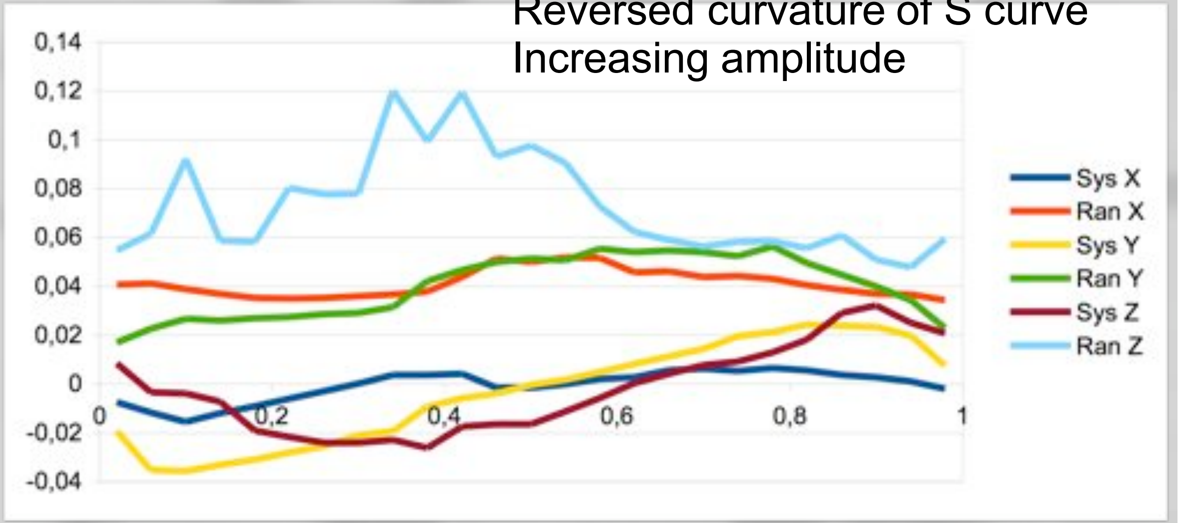
Von Mises Strain
Window = 40^3

(Computation time = hours)

Bining 2x2x2
Window = 15^3 (=30^3)



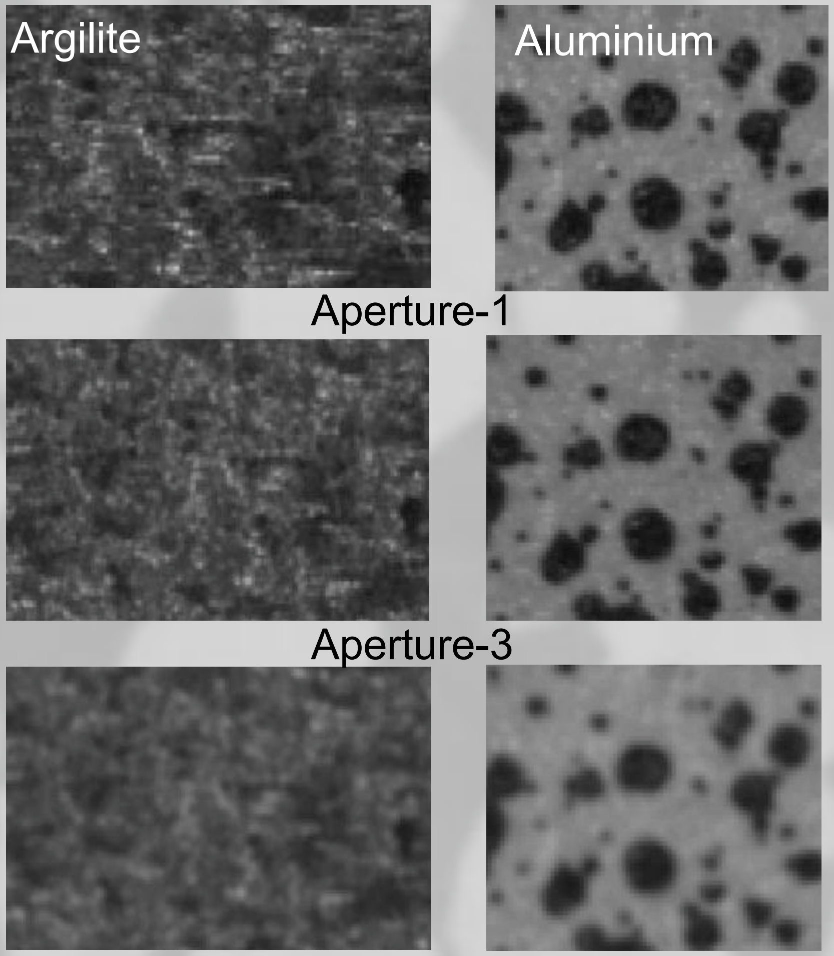
Bining 3x3x3
Window = 10^3 (=30^3)
Reversed curvature of S curve
Increasing amplitude



OPTIMAL COMBINATION OF
(SAMPLE + IMAGING + DIC)
TO REDUCE SYSTEMATIC
ERRORS

Simular results in 2D-DIC (with contrast controlled by lens aperture)

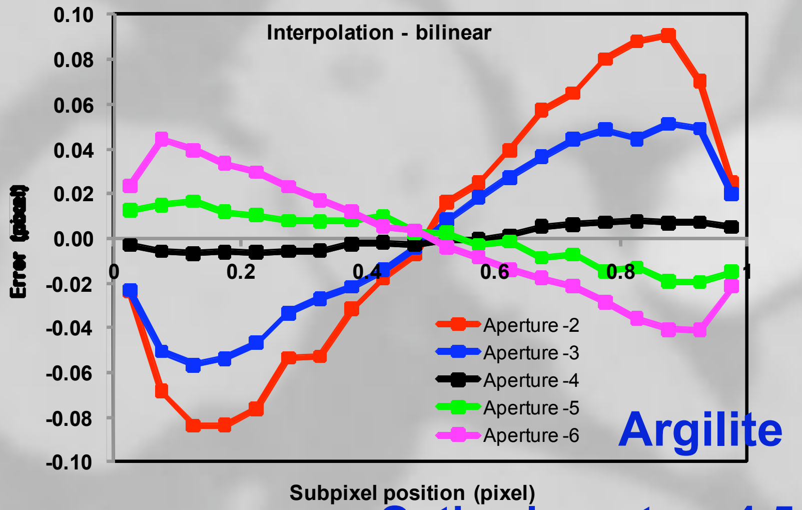
Lens F4,5/90mm, G=1, 1pixel = 7,4µm



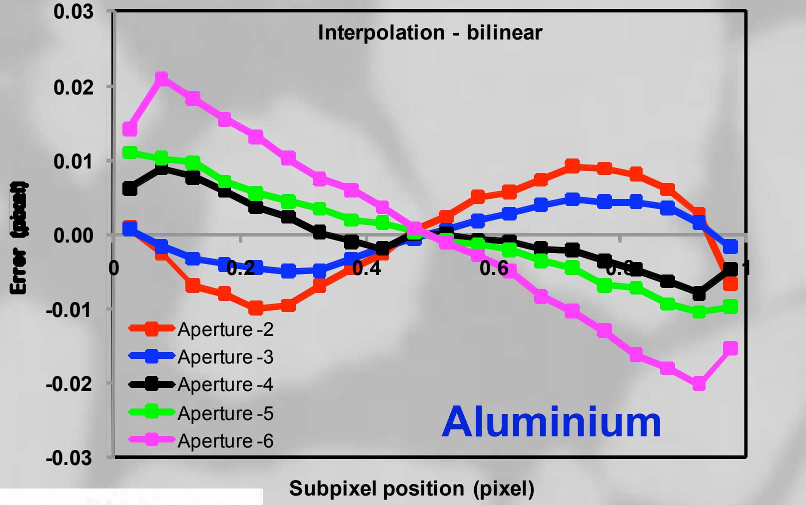
Aperture-1

Aperture-3

Aperture-5

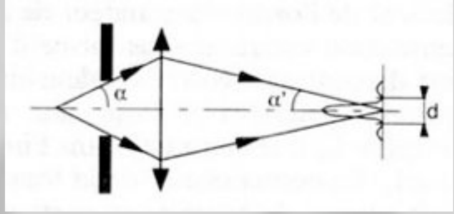


Optimal aperture 4-5



Optimal aperture 3-4

(Yang et al, ICEM14, Poitiers, 2010)



$$\text{Airy} \approx 0,6\frac{\lambda}{\alpha}$$

Concluding remarks

MicroCT in situ test combined with (Discrete-)DIC
provide highly valuable insights
for the micromechanics of (geo)materials

Several DIC error sources

We need to understand them, to model them
and to quantify them *for real experimental conditions*

Some simple and accurate procedures are proposed

But still a lot to do….