Measurement of the whole strain tensor by a 6-axis embedded sensor

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Abstract. We propose a concept of a 6-axis sensor allowing the measurement of the complete deformation tensor (6 components). This sensor is weakly intrusive and only minimally disturbs the strain field around it. It is based on six regularly spaced rings, which can be resistive, optical or even acoustic. The proposed study is for the moment theoretical, but gives the mathematical relations necessary for the measurement and details the expected performance of the sensor in terms of accuracy.

Introduction

Strain measurements are usually made on the surface, with the exception of X-ray tomography (3D-DIC) measurements. However, they require an intense energy source and an internal microstructure which is not always available. As a result, there is a need for physical deformation sensors embedded in the material.

Existing embedded transducers are mostly 1D and generally have a halter-shaped test body that strongly disturbs the strain (or stress) field around them. We have previously proposed and experimentally tested [1, 2] a 6-axis, spherical-shaped sensor that allows the measurement of the complete strain tensor. While Eshelby's theory allowed us to analytically trace the strain tensor "at infinity", *i.e.* in the absence of the sensor, this spherical inclusion nevertheless implied a high concentration of stress and a risk of disbonding at the interface.

Recently, we have shown the feasibility of deformation measurement using fibre optic rings [3]. This new arrangement can also be applied to other sensor technologies (resistive wires, acoustic measurement...). In this communication we show the operating principle, the equations allowing to pass from the transducers information to the deformation tensor of the matrix, we prove the isotropy of the measurement as well as an evaluation of the perturbation generated by the sensor.

Principle

The sensor consists of six rings arranged in the symmetry of an icosahedron:

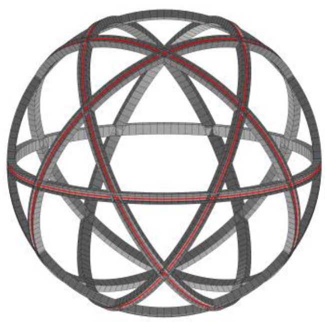

Fig. 1 Sketch of the 6-rings strain sensor

The elongation ΔL_i of each ring i of length L is measured by a transducer whose technology is not specified. The ring may be the transducer itself (*e.g.* with resistive wire technology) or it may be a test body supporting the transducer (*e.g.* a strain gauge or a fibre bonded to an elastic ring).

Assuming that the matrix undergoes a homogeneous deformation at the scale of the sensor and that the rings do not disturb this deformation of the matrix, the link between the components of the deformation tensor and the relative elongations of the rings is:

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \varepsilon_{23}\\ \varepsilon_{31}\\ \varepsilon_{12}\end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 2 & \varphi-1 & \varphi-1 & -\varphi & -\varphi\\ -\varphi & -\varphi & 2 & 2 & \varphi-1 & \varphi-1\\ \varphi-1 & \varphi-1 & -\varphi & -\varphi & 2 & 2\\ 1-2\varphi & 2\varphi-1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1-2\varphi & 2\varphi-1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1-2\varphi & 2\varphi-1 \end{bmatrix} \bullet \begin{bmatrix}\Delta L_1/L\\ \Delta L_2/L\\ \Delta L_3/L\\ \Delta L_4/L\\ \Delta L_5/L\\ \Delta L_6/L\end{bmatrix} \quad (1)$$

Where $\varphi=(1+R^{1/2})/2$ is the golden ratio.

Isotropy of the sensitivity

A strain sensor must be equally sensitive regardless of the direction of the strain. By direction, we mean the direction of the principal strains. This requires that the magnitude of the response of the transducers depends only upon the magnitude, *i.e.* the Euclidean norm of the measured strain. As a consequence, we define the internal sensitivity of the sensor as:

$$s(\boldsymbol{\varepsilon}) = \frac{\left|\frac{\Delta L_I}{L}\right|}{\|\boldsymbol{\varepsilon}\|} \tag{2}$$

From (1) it follows that:

$$\|\boldsymbol{\varepsilon}\|^2 s(\boldsymbol{\varepsilon})^2 = \varepsilon_J C_{JK} \varepsilon_K \tag{3}$$

where ε_K denotes the Bechterew (or Kelvin) representation of the component of the strain tensor: $\varepsilon_{K}= \varepsilon_{ii}$ and $\varepsilon_{K}= 2^{1/2}\varepsilon_{ij}$ if i≠j. Furthermore, the expression of C is:

$$\boldsymbol{C} = 2\,\boldsymbol{P}^H + 0.2\,\boldsymbol{P}^D \tag{4}$$

Where $\boldsymbol{P}^D$ and $\boldsymbol{P}^H$ are respectively the deviatoric and hydrostatic fourth-rank projectors. Being both isotropic, this proves the isotropy of the sensor as defined above. However, this also implies that the sensor is more sensitive, thus - in a user point of view - more precise, to a hydrostatic strain (*s=1.*414) that to a deviatoric one (*s=0,447*).

Matrix strain perturbation

The minimalist shape of the sensor already suggests a weak perturbance of the strain field in the matrix. It is shown, with a semi-analytical method based on the Kelvin point-force solution, that the relative perturbation is, at a distance *2R* from the center of the sensor:

$$p \leqslant 0.2121\,\frac{S}{R^2}\,\frac{|E_r - E|}{E} \tag{5}$$

Where E_r is the Young modulus of a ring, *E* the one of the matrix, *S* the cross section of a ring, and *R* its radius. With a thin wire technology, *p* can be easily close to 10^{-3}. A similar study also shows that the singularity generated by the presence of the sensor is limited in the close neighborhood of the rings. As a consequence, the is no need for this sensor to know the elasticity of the matrix as it was the case with the previous Eshelby-type spherical sensor.

Conclusion

The proposed 6-rings sensor architecture allows the measurement of the whole strain tensor within a bulk body with a minimal perturbation and an isotropic sensitivity. Various type of transducers can be used however it is better to minimize the ring section and prefer transducers which measure the mean ring deformation (*i.e.* its elongation) than ones that measure a local deformation. However, many technological aspects remain to be solved, such as the ring adherence, the prevention of any micro-buckling which may occur in compressive states, although this is helped by the low volume fraction if the rings are thin [3].

References

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