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Solid Mechanics and Materials Engineering

A novel modelling framework to predict the high rate response of soft materials: From polymers to particulate composites

British Society for Strain Measurement's 14th International Conference on Advances in Experimental Mechanics | Belfast, UK | 10 – 12 September 2019

Research conducted as part of a D.Phil. on the *High rate properties of particulate composites* at the University of Oxford.

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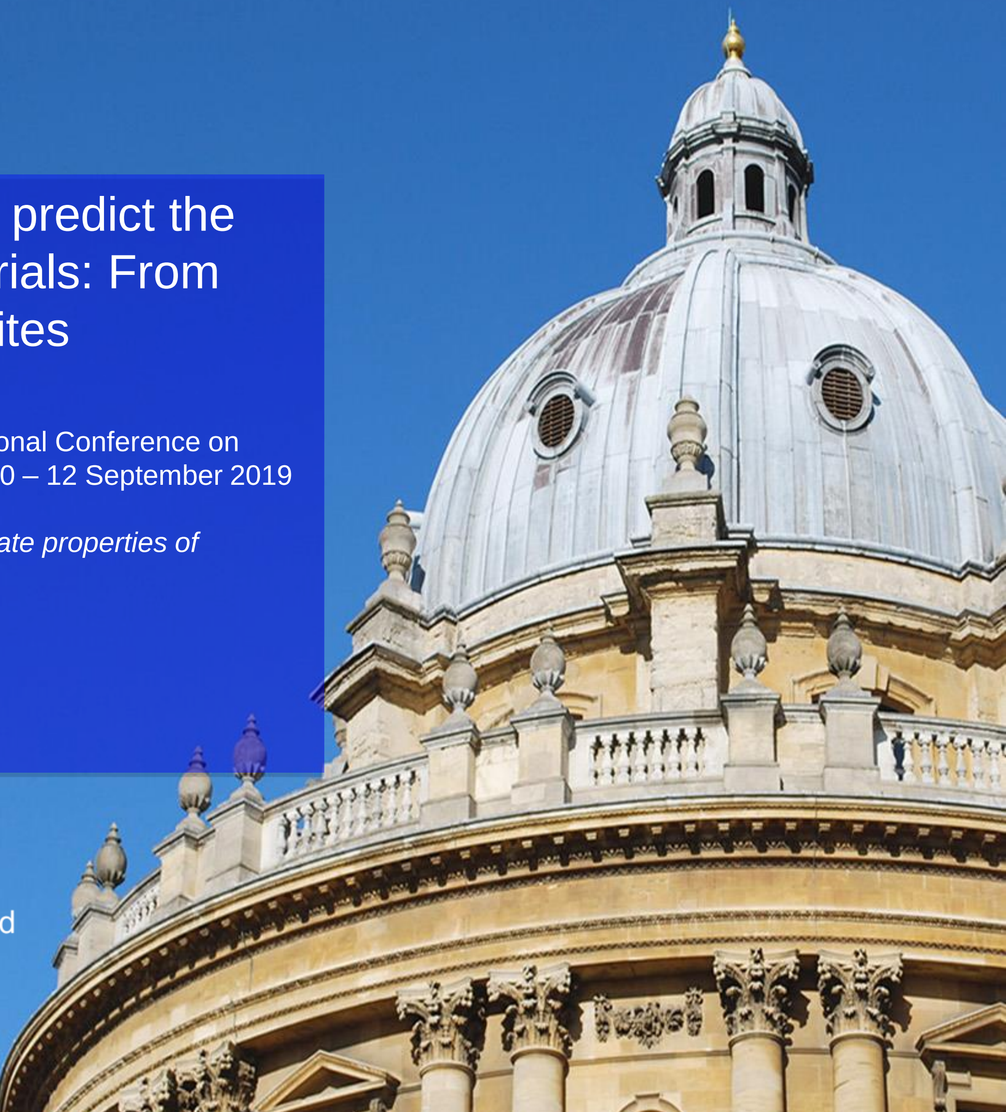
Take a **picture** for **contact details** and **more research**!



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<u>Aim:</u> To obtain the mechanical properties of soft polymers and their composites at high strain rates using simple, reliable, quasi-static experiments.

<u>Why?</u> Conventional techniques for high strain rate experimentation for soft materials do not give accurate measurements due to experimental artefacts.

<u>How?</u>

Experimental →  ← Constitutive Modelling

DMA: Dynamic Mechanical Analysis

TTS: time-temperature superposition

QS: Quasi-static

Get polymer → Use DMA and the TTS technique to obtain master curve/shift factors → Combine with QS experimental data to obtain relevant model parameters → Predict the material response at any suitable rate

Neoprene rubber test material to develop initial modelling framework [1,2]

↓

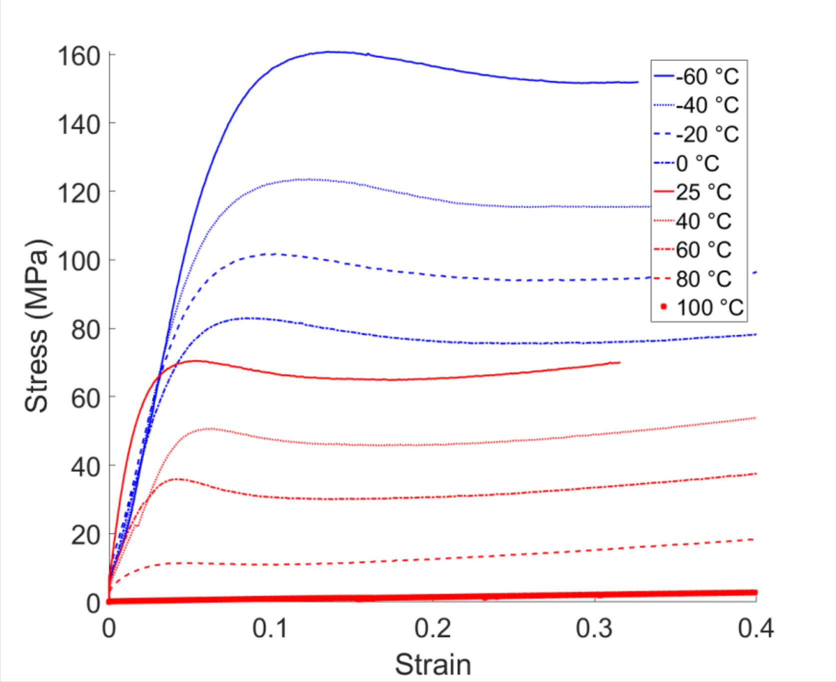
Plasticised PVC from a previous study [3] to refine the framework [4]

[1] Trivedi, A.R. & Siviour, C.R., 2018. *Framework for analyzing hyper-viscoelastic polymers.* In AIP Conference Proceedings
[2] Trivedi, A.R. & Siviour, C.R., 2017. *A framework for analysing hyper-viscoelastic polymers*, in Constitutive Models for Rubber X
[3] Kendall, M.J. & Siviour, C.R., 2014. *Rate dependence of poly(vinyl chloride), the effects of plasticizer and time-temperature superposition.* Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences.
[4] Trivedi, A.R. & Siviour, C.R., 2018. *Predicting the high strain rate response of plasticised poly(vinyl chloride) using a fractional derivative model.* EPJ Web of Conferences, 183, p.01013.

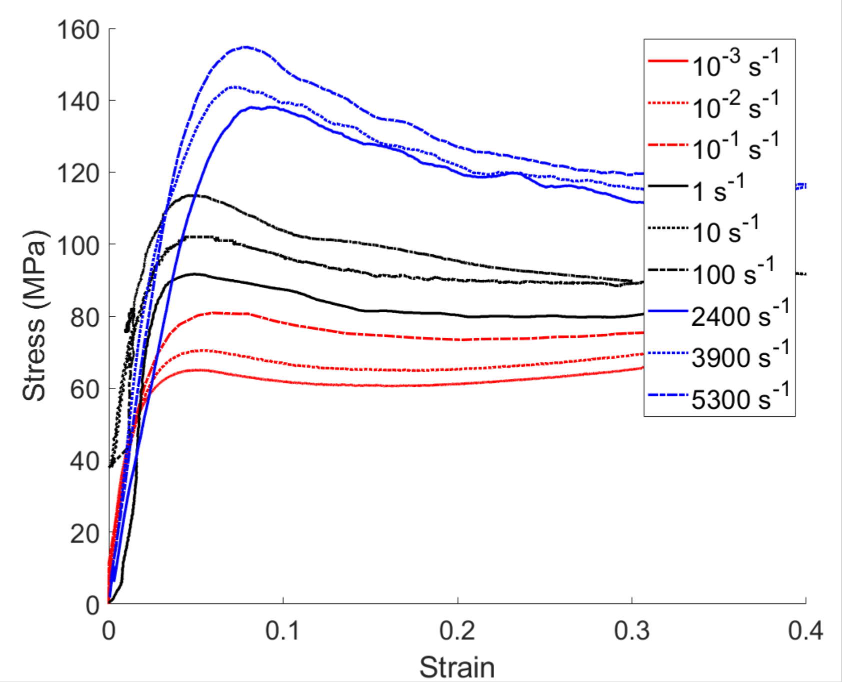
Plasticised and unplasticised PVC

Rate-temperature equivalence

Results of varying temperature tests



Results of varying rate tests



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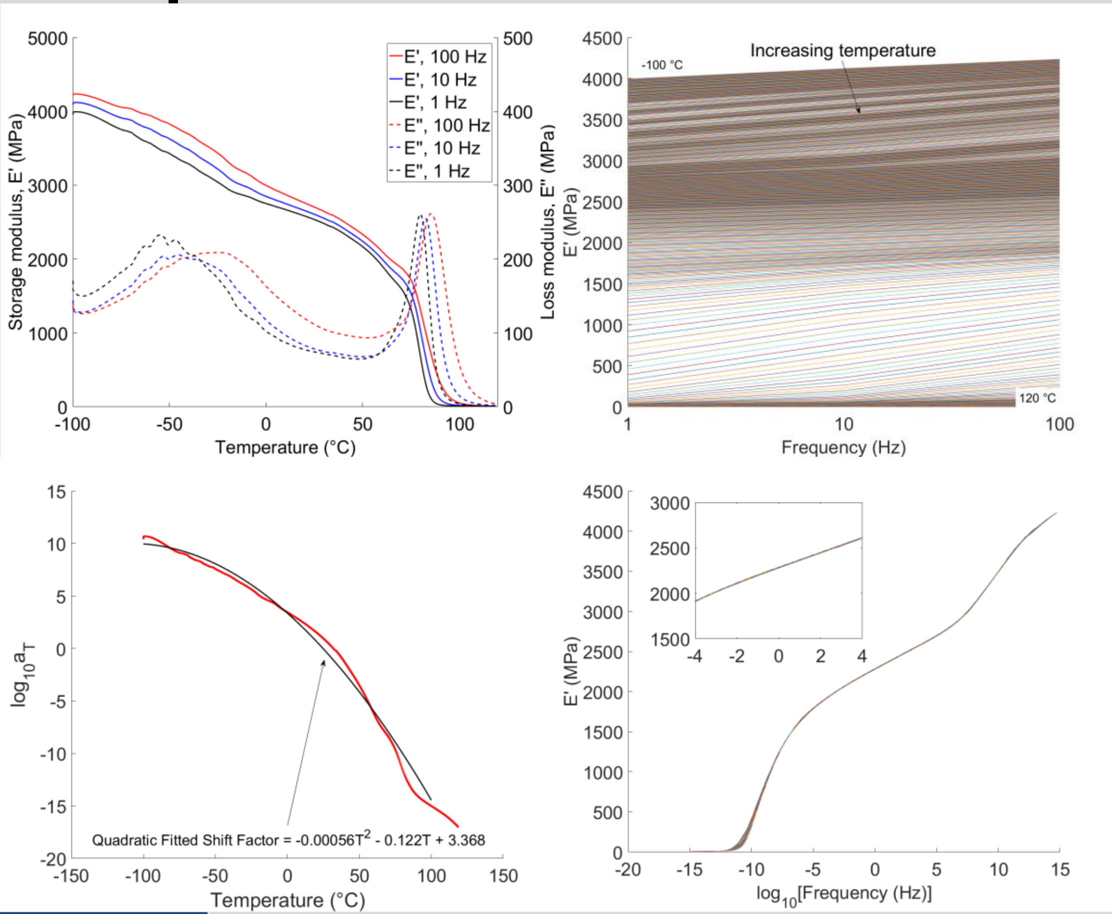
DMA experiments

- Dual cantilever test from -100 °C to 120 °C
- Frequency sweep of 1, 10, 100 Hz
- Rectangular sample with dimensions 60 x 10 x 5 mm
- Master curve produced by shifting isotherms left or right in relation to the reference temperature of 25 °C
- Quadratic shift factor relationship observed



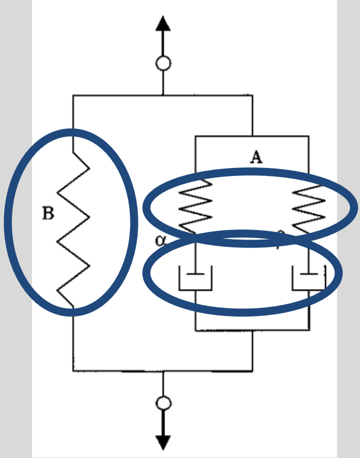
Modelling framework

Needed:

- Hyperelasticity for large strain behaviour
- Viscoplasticity for rate dependent plasticity
- Viscoelasticity for rate dependent elasticity
- Effects of adiabatic heating and subsequent temperature rise leading to thermal softening

Delivered by:

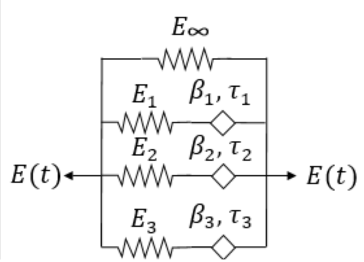
- Langevin chain statistics
- Mulliken-Boyce [5] model basis
- FD model fit to the DMA experiments
- Viscoelastic modulus changed based on shifts derived from temperature rise



[5] Mulliken, A.D. & Boyce, M.C., 2006. Mechanics of the rate-dependent elastic–plastic deformation of glassy polymers from low to high strain rates. *International Journal of Solids and Structures*

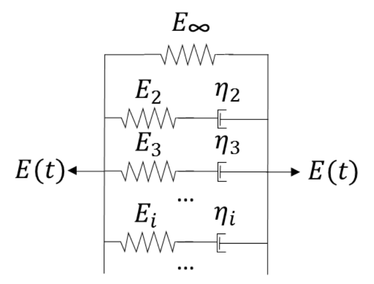
Fractional Derivative (FD) model

10-term fractional SLS model

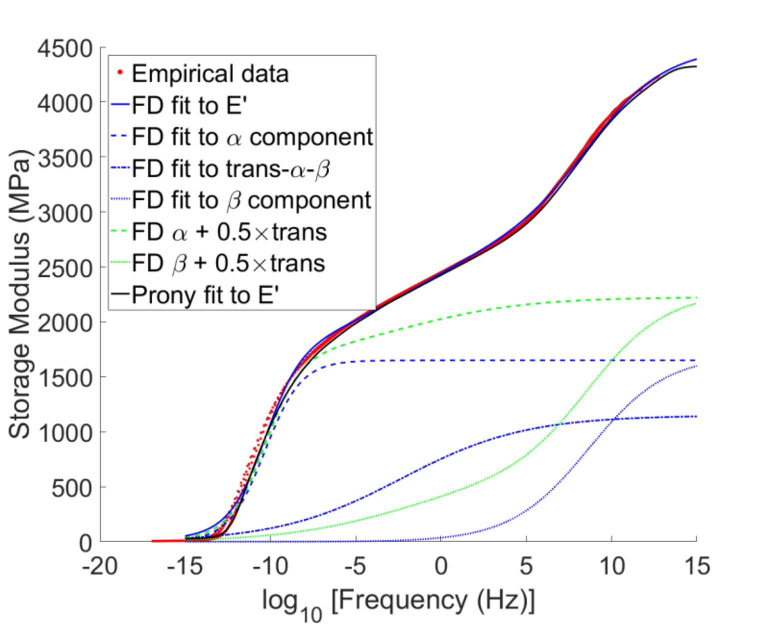


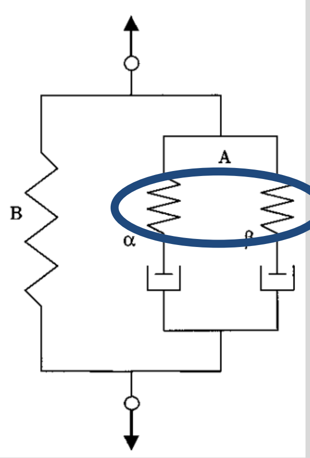
$$E^* = E' + iE'' = E_\infty + \sum_{i=1}^{M}\left[E_i \frac{(if)^{\beta_i}}{(if)^{\beta_i} + t_i^{-\beta_i}}\right]$$

26-term Prony SLS model



$$E^* = E' + iE'' = E_\infty + \sum_{i=1}^{N}\left[E_i \frac{if}{if + t_i^{-1}}\right]$$



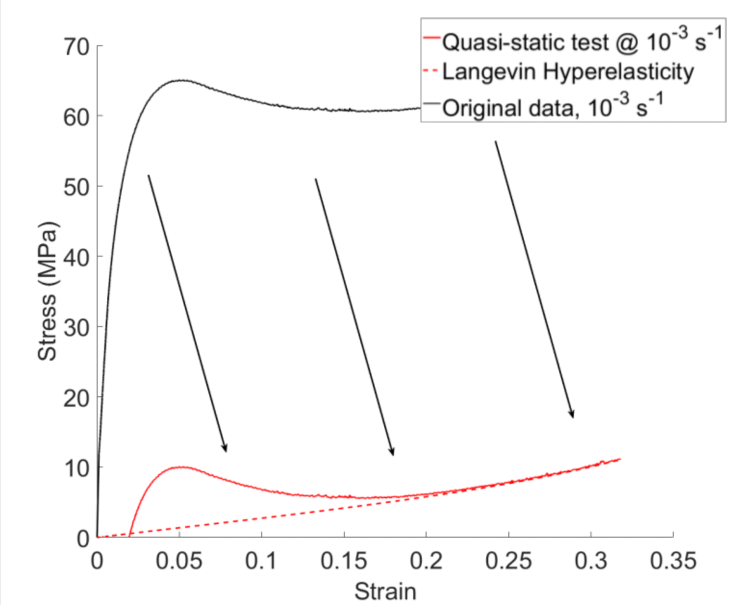


Prony series (MPa)	E_∞	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
	30.39	113.9	114.9	127.7	160.7	207.3	210.8	209.1	191.1	138.2
	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}	E_{16}	E_{17}	E_{18}	E_{19}
	107.9	90.19	82.97	81.04	81.8	80.81	86.94	72.88	110.2	82.12
	E_{20}	E_{21}	E_{22}	E_{23}	E_{24}	E_{25}				
	127.0	178.0	219.3	347.9	362.5	598.6				

Fractional model	E_∞ (MPa)	E_1 (MPa)	β_1	t_1 (s)	E_2 (MPa)	β_2	t_2 (s)	E_3 (MPa)	β_3	t_3 (s)
	3	1650	0.41	4×10^{7}	1150	0.12	4×10^{-1}	1700	0.19	4×10^{-12}

Modelling results: Langevin

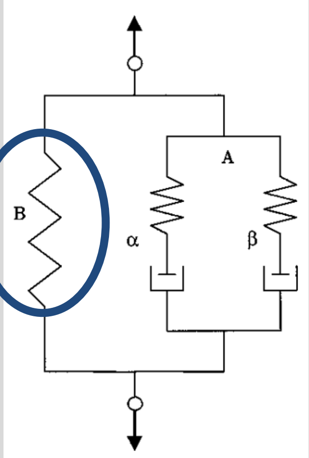
- Two parameter Langevin hyperelasticity
- Fit to quasi-static compression test



$$\mathcal{L}(\beta) \equiv \coth(\beta) - \frac{1}{\beta}$$

$$\lambda_{chain}^{p} = \sqrt{\frac{1}{3}\left(\varepsilon_n^2 + \frac{2}{\varepsilon_n}\right)}$$

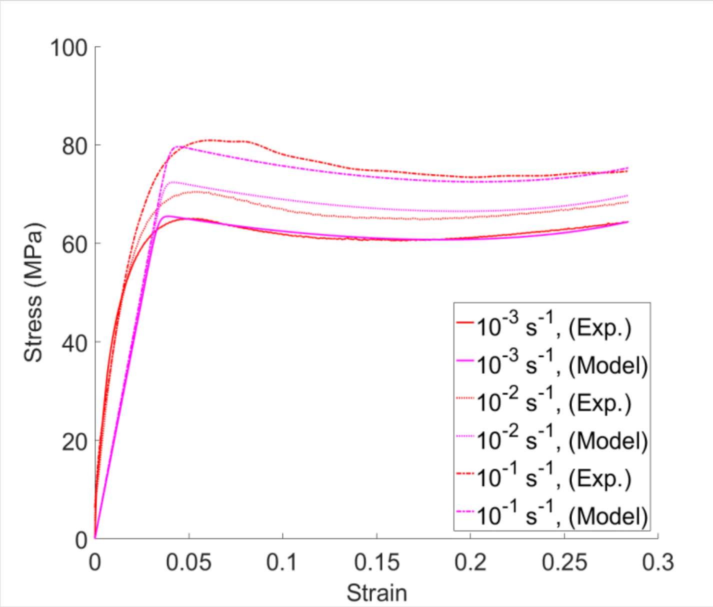
$$\sigma_L = \frac{C_R}{3}\frac{\sqrt{N}}{\lambda_{chain}^{p}}\mathcal{L}^{-1}\left(\frac{\lambda_{chain}^{p}}{\sqrt{N}}\right)(\varepsilon_n^2 - \varepsilon_n^{-1})$$

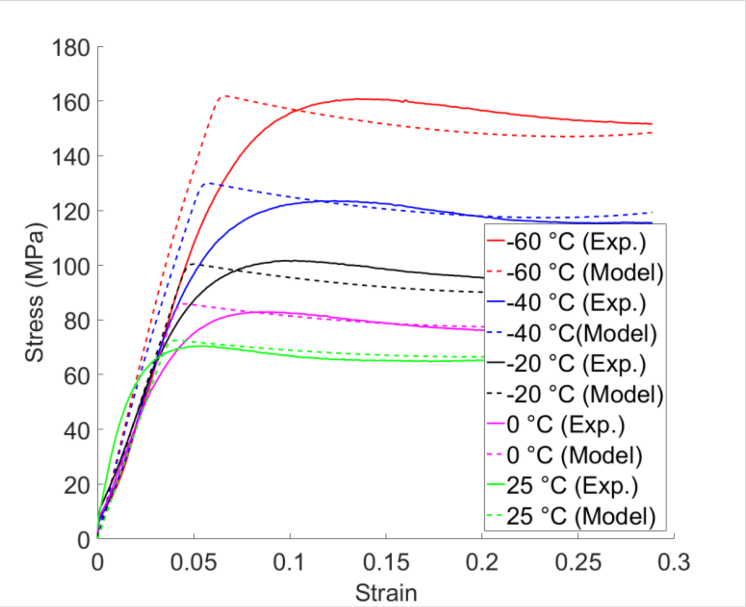


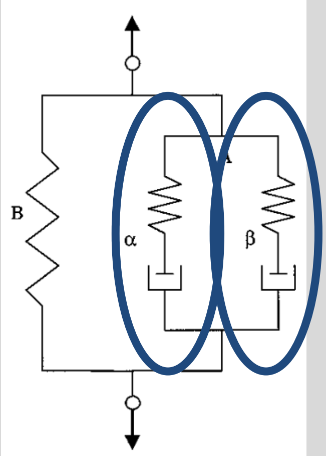
C_R, rubbery modulus
$\sqrt{N}$, limiting chain extensibility
ε_n, nominal strain

Modelling results: Alpha + Beta

- Alpha parameters are fit only to low rate data
- Beta parameters are fit to low temperature data



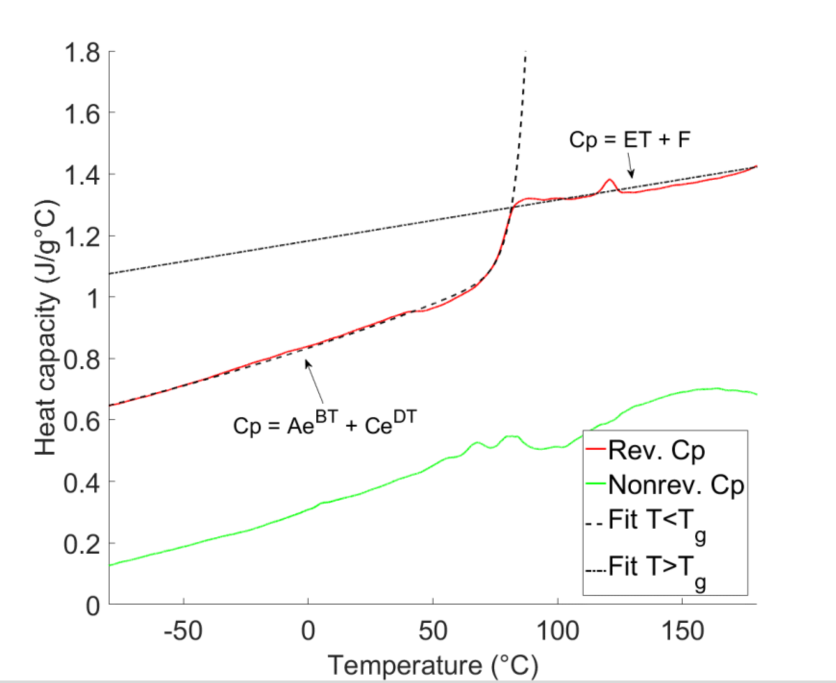




Time-Temperature Superposition principle is key to this approach

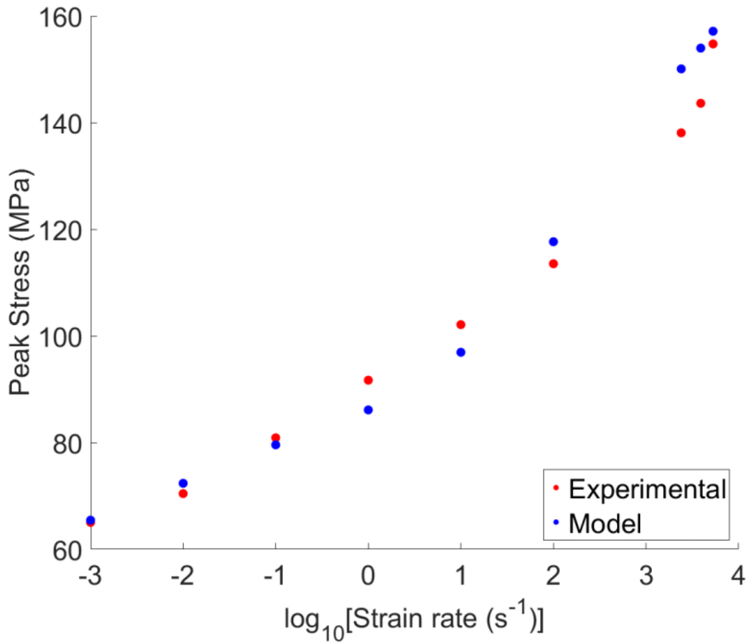
Adiabatic effects

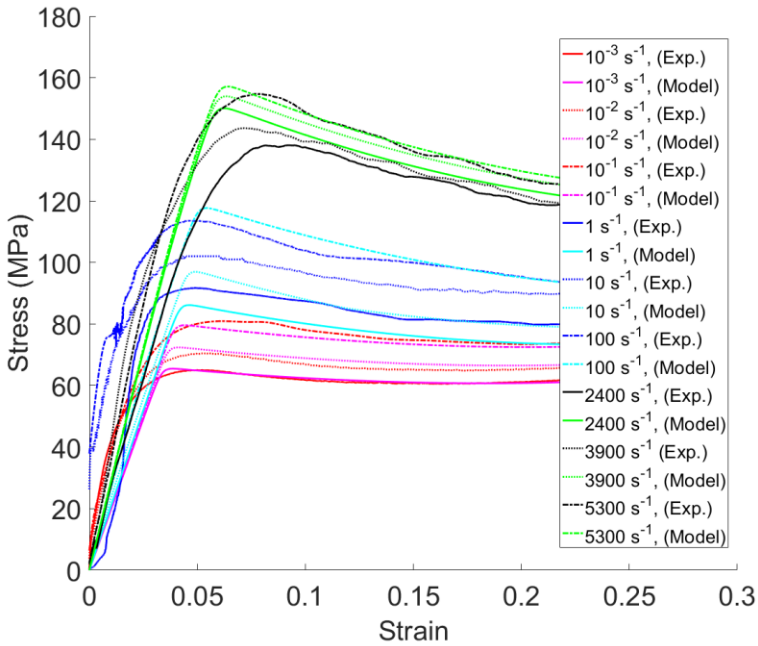
- At higher rates, compression transitions from isothermal to adiabatic
- Two fits either side of the Tg on the DSC results were used to approximate the heat capacity of the PVC
- All mechanical work assumed to be converted to heat; temperature rise calculated assuming adiabatic process
- The temperature rise leads to thermal softening of the modulus as shown



DSC: Differential Scanning Calorimetry

High rate prediction and validation





Virgin natural rubber

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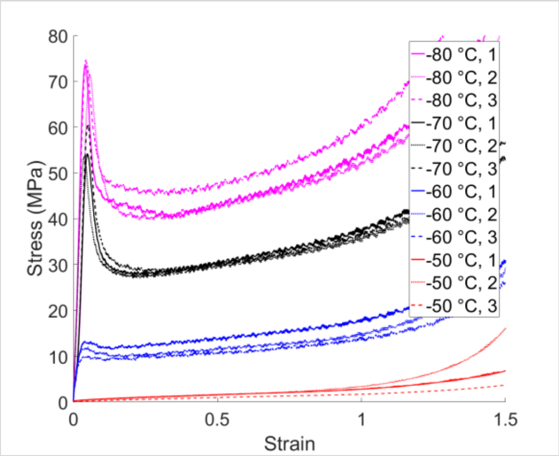
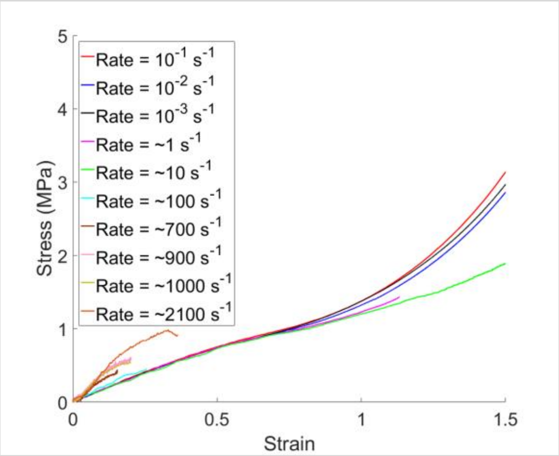
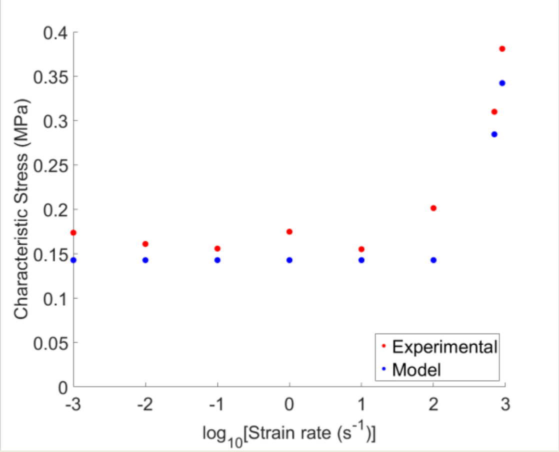
Results of varying temperature tests



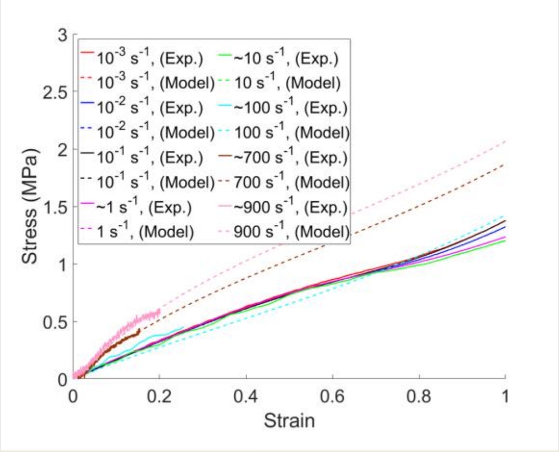
Results of varying rate tests



Characteristic stress (at $\varepsilon = 0.1$)



Stress-strain curves at varying rates



Experimental | Constitutive Modelling

Glass filled natural rubber

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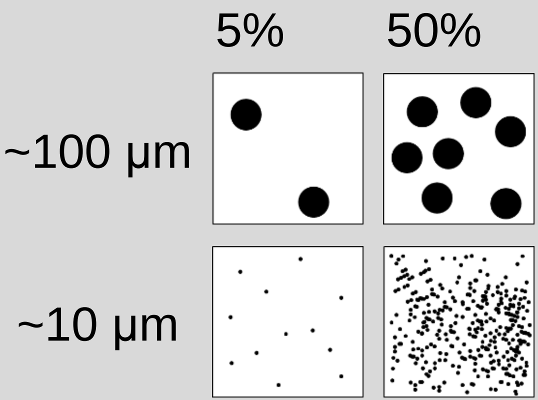
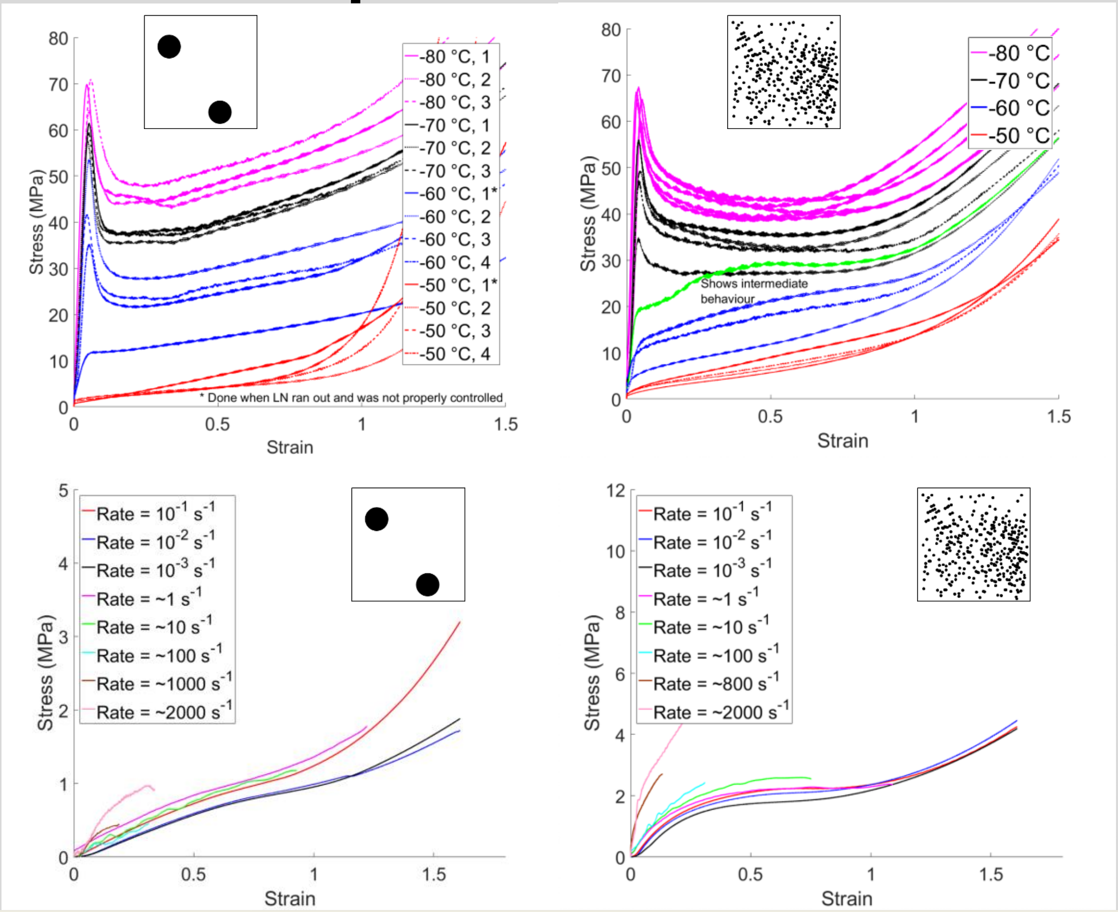
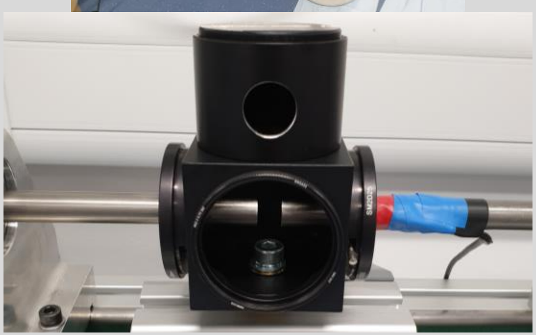
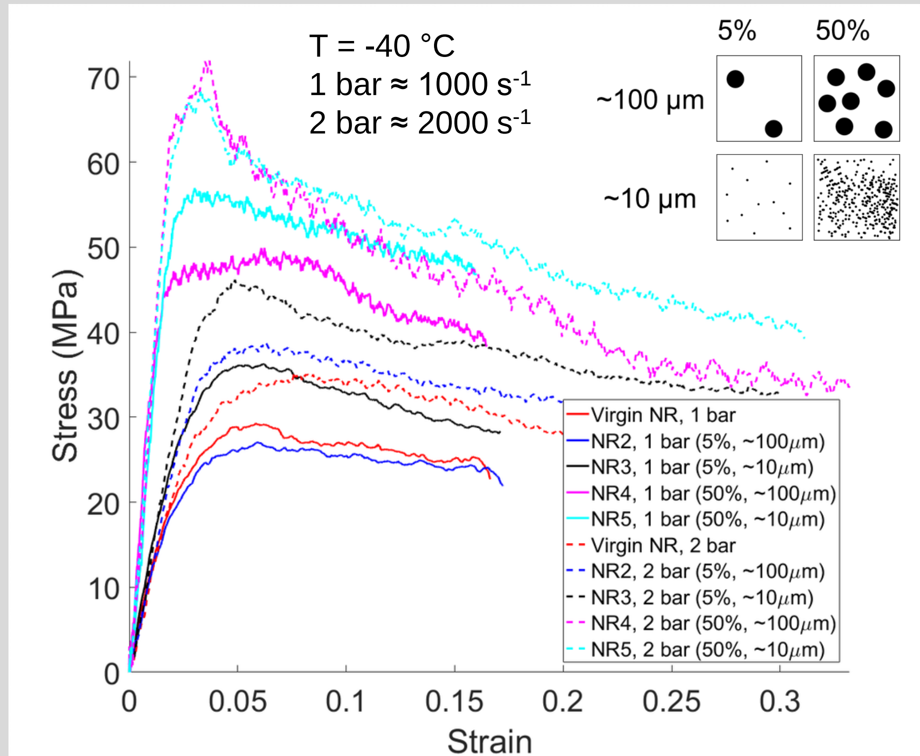
Composite experiments





LN Immersion Chiller for SHPB





LN: Liquid nitrogen

SHPB: Split Hopkinson pressure bar

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Conclusions

- Modelling framework has been effective at *predicting* not only the yield stresses the full stress-strain response of high rate compression experiments

- A new modelling framework has been presented that minimises parameterisation to purely simple, quasi-static reliable experiments.

- A Fractional Derivative model has been used to enable a reduction from 25 parameters of the conventional Prony series to only 10 parameters

- The Mulliken Boyce model has been used as a foundation and modified with the addition of:
    - Dynamically adjusted heat capacity
    - Temperature dependent modulus reflected in modelling process with use of DMA data

Challenges (i.e. Future research opportunities):
- Constant activation energy in model for α and β components despite spectrum of relaxations
- Very sensitive to C_p and Taylor-Quinney coefficient values
- Adapt model to (un)filled natural rubber incorporating a damage effect

Acknowledgements

- Alan Muhr and colleagues at TARRC for processing the natural rubber samples
- Richard Duffin and the workshop staff for helping to prepare samples for experimentation
- Nick Hawkins and Marzena Tkaczyk for assistance with the DMA and DSC experiments
- Igor Dyson for helping with the Instron experiments

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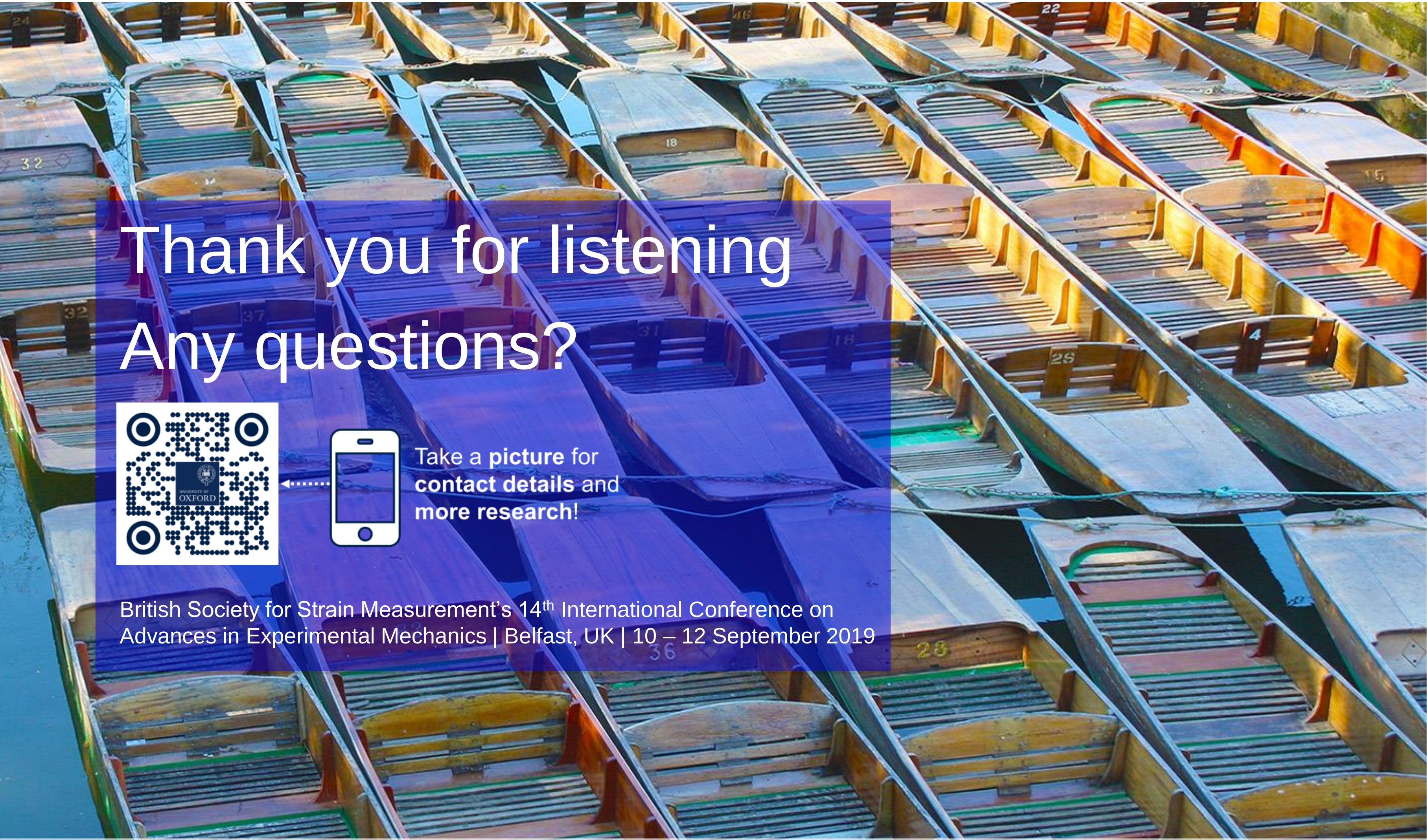

This material is based upon work supported by the Air Force Office of Scientific Research, Air Force Materiel Command, USAF under Award No. FA9550-15-1-0448. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the Air Force Office of Scientific Research, Air Force Materiel Command, USAF.