# Acoustic response of thermally stressed plates using a temperature dependent finite element material model

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**Abstract.** Development of a finite element model with temperature dependent material characteristics, capable of predicting the nonlinear behaviour of thermally stressed ideal plates up to buckling point, is presented.

## Introduction

The highly complex and transient environment at hypersonic speeds in aircraft produces combined high temperatures and acoustic loads that reduce the fatigue life of components and influence overall aircraft design and analysis. Thin-gauge structures are especially susceptible to friction-induced high temperatures that can promote buckling. It is difficult to make measurements under these conditions so that computational models of material and structural responses become important.

The majority of prior analytical and computational research used temperature independent material characteristics to model the buckling behaviour of thin plates. Early work found a clear relationship between resonant frequencies and buckling loads [1,2]. It has been shown that the first mode shape of ideal plates at a reference temperature ($\Delta T_{ref} = 0$) becomes the buckled shape at a critical buckling temperature ($\Delta T_{ref} = \Delta T_{cr}$) [2-5]. Moreover, frequencies of certain modes move across the frequency spectrum during the thermal loading, in a phenomenon called vibrational mode shifting [2,5]. Chen and Virgin [3], developed a finite element (FE) model that showed this behaviour occurred after the first buckling event but not after a second buckling event.

Jeyaraj et al.'s [4] FE model focused on the vibration and acoustic response of an ideal isotropic plate at several spatially uniform temperatures. For all boundary conditions studied, a decrease in frequency of each resonant mode was demonstrated to occur with an increasing uniform temperature. However, in many real aircraft components, non-uniform temperature distributions will impart both compressive and tensile stresses on components, leading to nonlinear conditions. By analysing non-uniform temperature distributions, Mead [5] verified that buckling of a specific region of the plate is associated with compressive stresses in the same region.

Contrary to other work, Ko's [6] FE model included temperature dependent material characteristics to study the critical buckling temperature and shape of rectangular plates subjected to several types of parabolic temperature distributions. Results were compared to analytic predictions and a good agreement was found. In the present work, the development of an FE model using temperature dependent material characteristics is presented and an ideal rectangular plate of aspect ratio 1.5 analysed.

## Methodology

The present model simulated experimental conditions described by Berke et al. [7]. The subject of study is an ideal 130x80x1.016mm Hastelloy X plate, discretized into 9600 quadrilateral shell elements and fully constrained at the single central node. Despite assuming small strain elastic material behaviour, large deflections associated with the buckling of plates required the use of a nonlinear solver for the stiffness formulations. In order to better deal with the snap-through associated with the buckling event, displacements were not defined as a function of time (implicit analyses). Hyperworks was used for pre-/post-processing and LS-Dyna was the chosen solver.

**Temperature dependent material model**: the model included temperature dependent values for the Young's modulus, thermal conductivity, mean coefficient of thermal expansion and specific heat capacity of the material. All material properties were provided by the manufacturer [8] and interpolated using a second degree polynomial in order to extract the material property values over the relevant temperature range.

**Buckling and modal analyses:** a custom-written Matlab script was used to replicate the temperature distribution used by Berke et al. and assign the appropriate discrete temperature values to the model nodes shown in Fig. 1. An eigenvalue buckling analysis was then set-up to predict the critical buckling temperature ($T_{cr}$). An increasing thermal load was applied to the plate, starting at a uniform 22°C (nominally room temperature) up to the temperature distribution shown in Fig. 1. After determining $T_{cr}$, a modal analysis was performed in which the plate's temperature was increased in eight equal increments from a uniform room temperature to 80% of the overall temperature difference between room temperature and the critical temperature ($T_{cr}$ – 22°C) and eigenvalues extracted at each increment. So as to better understand the resonant frequency changes near the buckling point, two modelling decisions were made: a) four smaller, equal increments were specified between 80% of ($T_{cr}$ – 22°C) and $T_{cr}$; b) the analysis was extended into the early post-buckling regime, up to 110% of ($T_{cr}$ – 22°C). Even smaller increments were considered but would not have allowed for simple differentiation of modes between consecutive extraction points for eigenvalues.

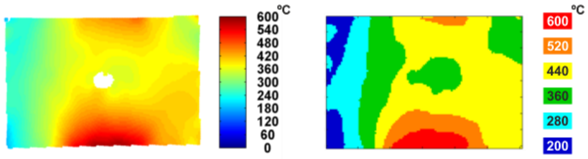


**Figure 1. Berke et al. temperature distribution (left) and discretised map (right)**

The buckling and modal models were first validated against the literature [4]. The percentage mean relative differences were below 1% for the buckling model and below 3% for the modal analyses, except for the first mode at 99% of $T_{cr}$ due to the nonlinearities of the buckling phenomenon.

## Results and Discussion

The temperatures at which buckling occurs and the corresponding buckled shape (z displacements) of the plate when exposed to the temperature distribution applied by Berke et al. are shown in Fig. 2.

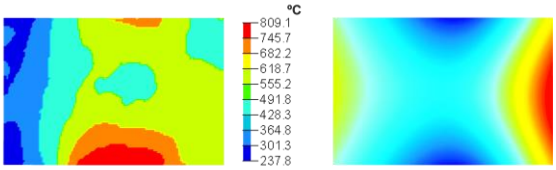

**Figure 2. Buckling temperatures (left) and shape (right)**

Fig. 3 shows the change in the resonant frequencies predicted to occur as the temperature field changes from a uniform room temperature to the temperature distribution at buckling shown in Fig. 2. The resonant frequency of the first non-rigid body mode (Mode 3) reaches zero at between 95 and 100% of $T_{cr}$, which agrees with Bailey's [2] studies. Modes 1 and 2 represent rigid-body motions and the change in material stiffness with temperature does not yield a significant frequency change.

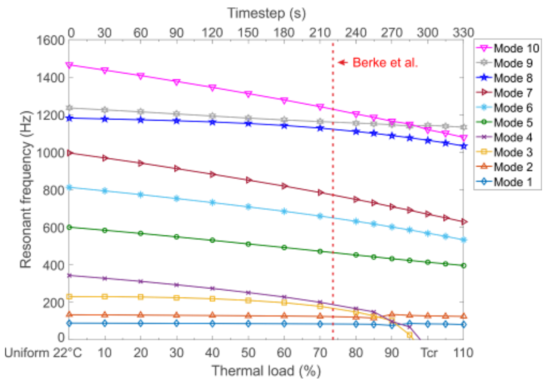

**Figure 3. Resonant frequency change with $T_{cr}$**

Mode shifting is present between Modes 9 and 10 in Fig. 3. This happens shortly after the resonant frequency of Mode 3 reaches zero, which marks the moment of buckling and confirm Chen and Virgin's findings.

In the pre-buckling regime, 73.6% of ($T_{cr}$ – 22°C) corresponds to the temperature distribution calculated by Berke et al. (Fig. 1) and a direct comparison to their results is possible. This has proven difficult as, in their tests, Berke et al. have evaluated their temperature distribution at the start of their experiment and assumed it was constant throughout their test, which might have not been the case.

The mean relative error in the predictions of Berke et al. was 19.32%, compared to the 16.57% for the present model. While this is an improvement, the analysed ideal plate did not account for initial imperfections (i.e. material or geometric) that can affect experimental results. An FE model with temperature independent material characteristics and an initial geometric imperfection of the plate developed by Murphy et al. [9] yielded results which did not show the disappearance of resonant frequencies. Imperfections were proven to increase the structural stiffness of the post-buckling regime.

## Conclusions

The comparison of FE resonant frequency results from Berke et al. and the present temperature dependent material model shows that the latter yields a lower mean relative error against experimental data. The current model also successfully depicts phenomena such as vibrational mode disappearance and mode shifting in the analysis of thermally stressed plates up to buckling temperature.


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## References


[1] H. Lurie, "Lateral vibrations as related to structural stability," *J. Appl. Mech.*, vol. 19, pp. 195–204, 1952.

[2] C. D. Bailey, "Vibration and Local Instability of Thermally Stressed Plates," *Comput. Methods Appl. Mech. Eng.*, no. 25, pp. 263–278, 1981.

[3] H. Chen and L. N. Virgin, "Dynamic analysis of modal shifting and mode jumping in thermally buckled plates," *J. Sound Vib.*, vol. 278, no. 1–2, pp. 233–256, 2004.

[4] P. Jeyaraj, C. Padmanabhan, and N. Ganesan, "Vibration and Acoustic Response of an Isotropic Plate in a Thermal Environment," *J. Vib. Acoust.*, vol. 130, no. 5, p. 51005, 2008.

[5] D. J. Mead, "Vibration and buckling of flat free-free plates under non-uniform in-plane thermal stresses," *J. Sound Vib.*, vol. 260, pp. 141–165, 2003.

[6] W. L. Ko, "Thermal buckling analysis of rectangular panels subjected to humped temperature profile heating," *NASA/TP*, vol. 2120241, 2004.

[7] R. B. Berke, C. M. Sebastian, R. Chona, E. A. Patterson, and J. Lambros, "High Temperature Vibratory Response of Hastelloy-X: Stereo-DIC Measurements and Image Decomposition Analysis," *Exp. Mech.*, vol. 56, no. 2, pp. 231–243, 2015.

[8] Haynes International, "HASTELLOY® X ALLOY," 1997. [Online]. Available: http://www.haynes.ch/doc/HASTELLOY_X.pdf. [Accessed: 25-Jan-2017].

[9] K. Murphy, L. N. Virgin, and S. A. Rizzi, "Characterizing the Dynamic Response of a Thermally Loaded, Acoustically Excited Plate," *J. Sound Vib.*, vol. 196, no. 5, pp. 635–658, 1996.