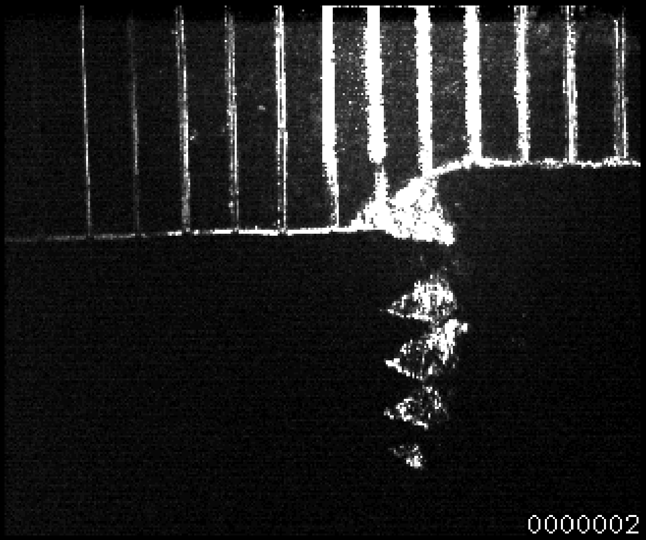
Inverse methods in metal plasticity
based on full-field deformation measurement

Professor Fabrice PIERRON

Faculty of Engineering and the Environment

Introduction

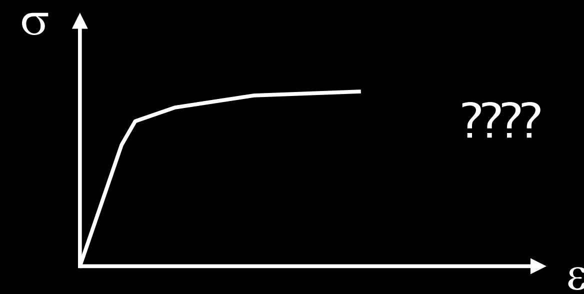
- Great progress in computational mechanics
  - Simulation of machining
    - Large strains elasto-plasticity
    - Large strain rates
    - Localization
    - Friction/thermal behaviour



- Problem
  - Many material parameters required
  - How to obtain them?

Introduction

- Standard tests: Tensile test on rectangular specimen
  - Uniform stress state
  - Uniaxial stress strain curve

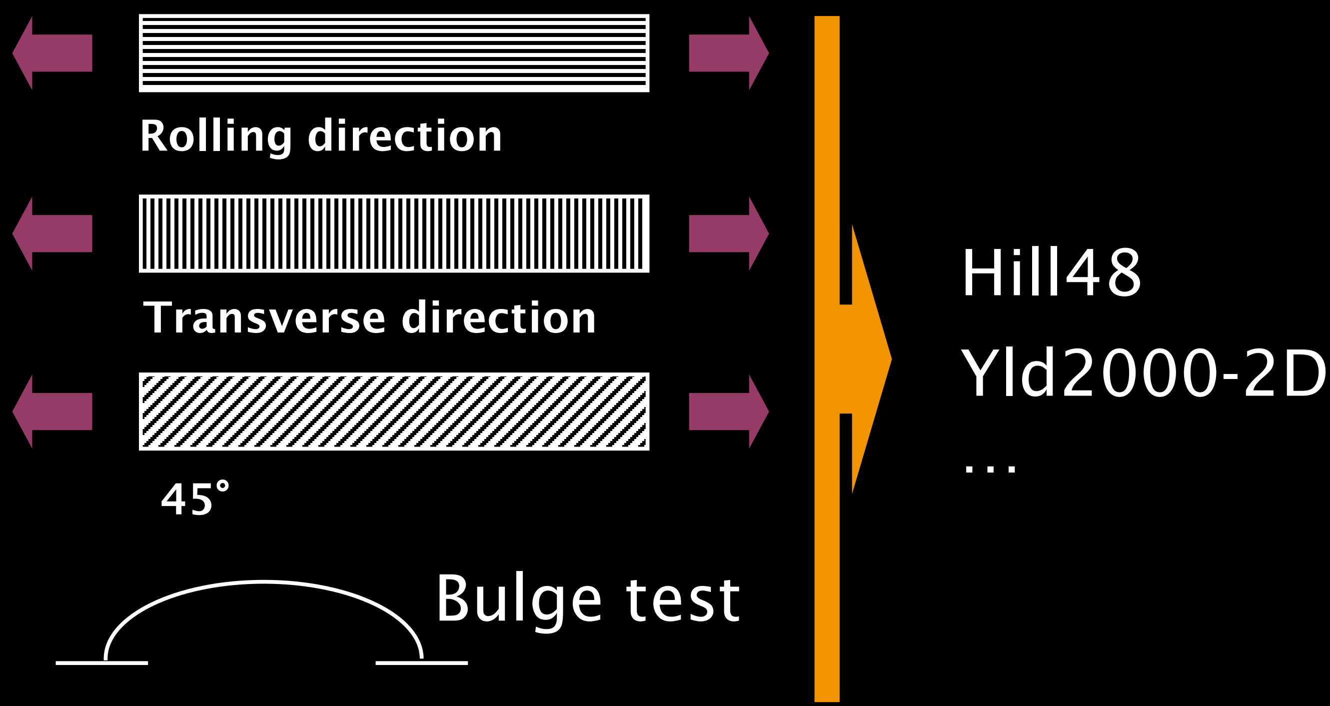


  - Very poor information (very boring!)
  - Very restrictive assumptions (constraints)

Develop the experimental identification procedures of the future !

Motivation

- Classical identification for anisotropic plasticity

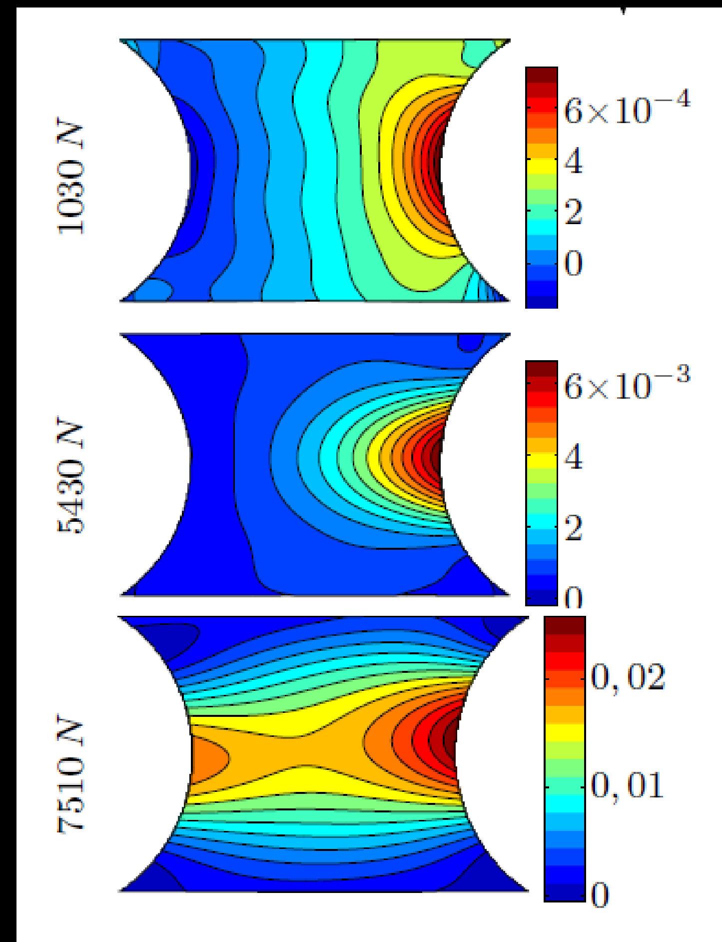


Local strain measurements

Closed-form solution (statically determinate)

Motivation

- Extract more information from 1 test



<u>Full-field measurements</u>

Inverse solution

Statement of the problem

Inverse resolution

- Basic equations

I Equilibrium equations (static)

$$\boldsymbol{\sigma_{ij,j} + f_i = 0}$$ + boundary conditions strong (local)

or

$$-\int_V \sigma_{ij}\varepsilon_{ij}^{*}dV + \int_{\partial V_f} T_i u_i^{*}dS + \int_V f_i u_i^{*}dV = 0$$ weak (global)

II Constitutive equations (elasticity) $\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$

III Kinematic equations (small strains/displacements)

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

Inverse resolution

	Known	Unknown
Direct problem	C_{ijkl} Geometry Boundary conditions	$\sigma_{ij}, \varepsilon_{ij}, u_i$

- Tools for solving this problem
  - Direct integration (closed-form solution)
  - Approximate solutions
  - Galerkin, Ritz
  - Finite elements, boundary elements…
  - etc…

Inverse resolution

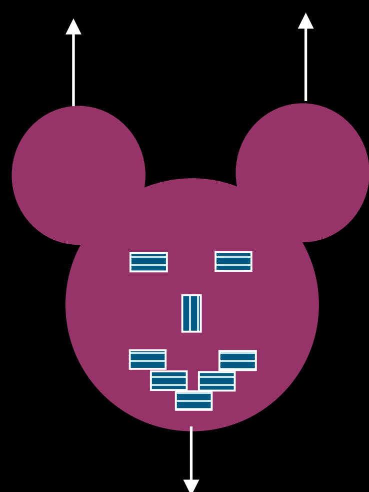
	Known	Unknown
Inverse problem	ε_{ij}, u_i (measured) Geometry Some information on the boundary conditions (load cell)	C_{ijkl} σ_{ij}

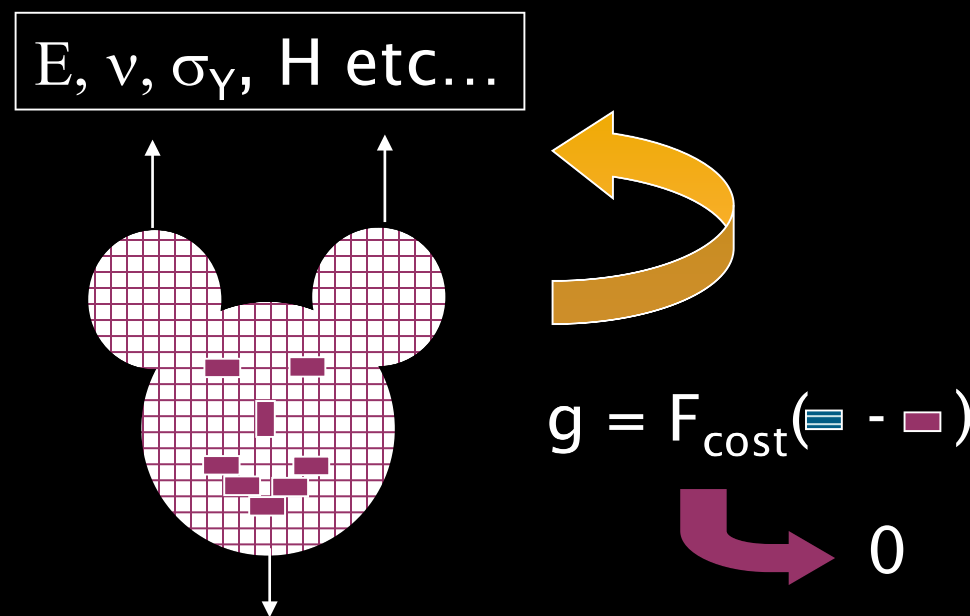
- Tools for solving this problem
  - Statically determined tests:
    Closed form solution of Eq. I (uncoupled system)
    Force BC, simple geometry
    Ex.: tensile test, bending tests (on rect. beams) etc…

Inverse resolution

- Tools for solving this problem
  - Finite element model updating (FEMU)
    Idea: iterative use of tool for direct problem (analytical or approximate)

Example





FEMU in plasticity

- Kajberg, J., & Lindkvist, G. (2004). Characterisation of materials subjected to large strains by inverse modelling based on in-plane displacement fields. *International Journal of Solids and Structures, 41(13), 3439-3459.*

- Bertin, M., Hild, F., Roux, S., Mathieu, F., Leclerc, H., & Aimedieu, P. (2016). Integrated digital image correlation applied to elastoplastic identification in a biaxial experiment. *The Journal of Strain Analysis for Engineering Design, 51(2), 118-131.*

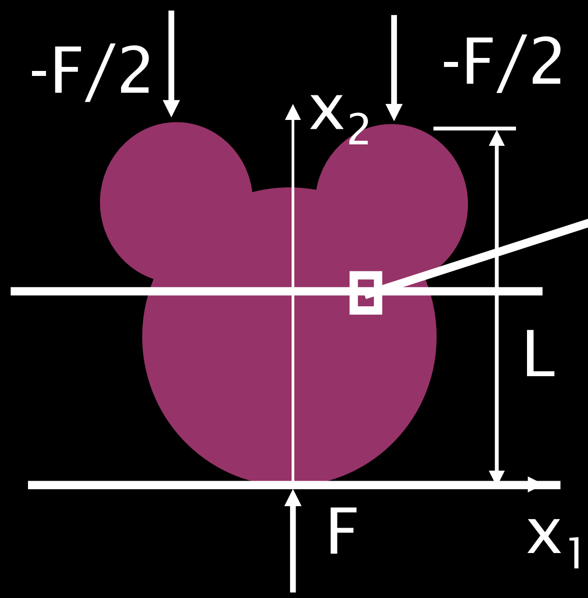
- FEMU: computationally intensive
  - Alternative: the Virtual Fields Method
  - Quoted 125 times faster than FEMU!!

Zhang, L., Thakku, S. G., Beotra, M. R., Baskaran, M., Aung, T., Goh, J. C. H., Girard, M. J. A. (2016). Verification of a virtual fields method to extract the mechanical properties of human optic nerve head tissues in vivo. *Biomechanics and Modeling in Mechanobiology, 1-17.*

The VFM in elasto-plasticity

The Virtual Fields Method

- Idea: use global equations (and not local)



$$\varepsilon_{22}(x_1^0, x_2^0) \quad \sigma_{22}(x_1^0, x_2^0)?$$

$$\int_S \sigma_{22} dx_1 dx_3 = -F$$

Integrate over x_2 $$\int_V \sigma_{22} dx_1 dx_2 dx_3 = -FL$$

The Virtual Fields Method

- Surface measurements only

Constant stress through the thickness

$$\int_V \sigma_{22} dx_1 dx_2 dx_3 = h \int_S \sigma_{22} dx_1 dx_2$$

Full-field measurement

$$\int_S \sigma_{22} dx_1 dx_2 \approx \sum_{i=1}^{n} \sigma_{22}^{i} s^{i}$$

$$\sum_{i=1}^{n} \sigma_{22}^{i} s^{i} + FL = 0$$

In plasticity

- First established in 2006
  - Grédiac, M., & Pierron, F. (2006). Applying the virtual fields method to the identification of elasto-plastic constitutive parameters. International Journal of Plasticity, 22(4), 602-627.

$$\sigma_{22}(t) = f(\varepsilon, Y_1^0, \cdots, Y_m^0) \qquad F(t)$$

Measured:

$\varepsilon(t), F(t)$

$$\sum_{i=1}^{n} \sigma_{22}^{i}(\varepsilon, Y_1 \ldots Y_n) s^i + FL = 0$$

$$\min\left(\sum_{i=1}^{n} \sigma_{22}^{i}(\varepsilon, Y_1 \ldots Y_n) s^i + FL\right)^2$$

$$Y_1, \cdots, Y_m$$

General framework

- Principle of virtual work

$$-\int_V \sigma_{ij}\varepsilon^*_{ij}dV + \int_{\partial V} T_i u^*_i dS = 0$$ No volume forces, static

Valid for any continuous and differentiable virtual field : infinity of equations

1st virtual field: virtual compression field

$$u^*_1 = 0 \; ; \; u^*_2 = -x_2$$
$$\varepsilon^*_{11} = 0 \; ; \; \varepsilon^*_{22} = -1 \; ; \; \varepsilon^*_{12} = 0$$

$$\int_V \sigma_2 dV = -FL$$

$$-\int_V \sigma_{ij}\varepsilon^*_{ij}dV = \int_V \sigma_{22}dV$$

$$\int_{\partial V_f} T_i u^*_i dS = FL$$

VFM in large deformation

- Most common route

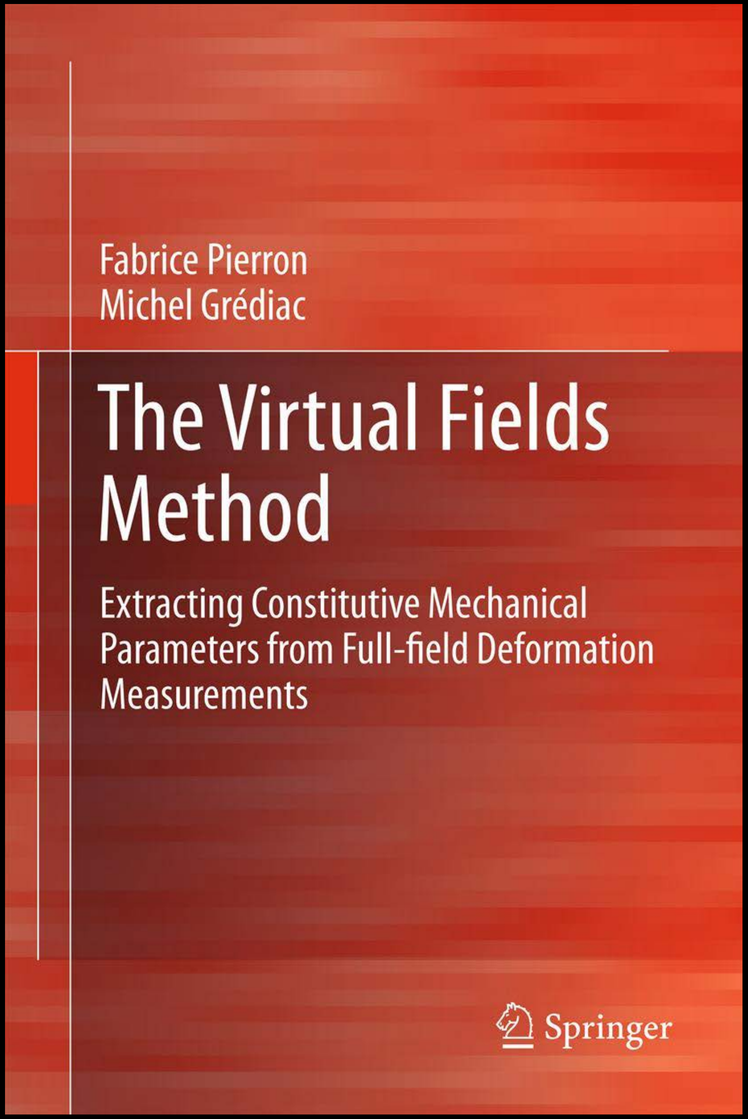
$$-\int_V \Pi_{ij} \frac{\partial U_i^*}{\partial X_j} dV + \int_{\partial V} \Pi_{ij} N_j U_i^* dS = 0$$

- Strains expressed in current configuration
- Cauchy stress obtained from strains
- Π calculated from σ

$$\Pi = J \sigma F^{-t}$$

The Virtual Fields Method

- Theory
- Applications
- Training



Stress calculation

- At each step

$$\varepsilon^t = \boxed{\varepsilon^e + \varepsilon^p} \longrightarrow$$ Needed to calculate the stress

↑ measured

- How to do this in practice?
  - Return mapping algorithm
    - Sutton, M. A., Deng, X., Liu, J., & Yang, L. (1996). Determination of elastic-plastic stresses and strains from measured surface strain data. Experimental Mechanics, 36(2), 99-112.
    - Grédiac, M., & Pierron, F. (2006). Applying the virtual fields method to the identification of elasto-plastic constitutive parameters. International Journal of Plasticity, 22(4), 602-627.
    - Avril, S., Pierron, F., Pannier, Y., & Rotinat, R. (2008). Stress reconstruction and constitutive parameter identification in plane-stress elasto-plastic problems using surface measurements of deformation fields. Experimental Mechanics, 48(4), 403-419.

Stress calculation

- How to do this in practice?
  - Tangent stiffness matrix
    - Pierron, F., Avril, S., & Tran, V. T. (2010). Extension of the virtual fields method to elasto-plastic material identification with cyclic loads and kinematic hardening. International Journal of Solids and Structures, 47(22-23), 2993-3010.
    - Kim, J. H., Barlat, F., Pierron, F., & Lee, M. G. (2014). Determination of Anisotropic Plastic Constitutive Parameters Using the Virtual Fields Method. Experimental Mechanics, 54(7), 1189-1204.
  - Direct approach
    - Rossi, M., & Pierron, F. (2012). Identification of plastic constitutive parameters at large deformations from three dimensional displacement fields. Computational Mechanics, 49(1), 53-71.
    - Rossi, M., Pierron, F., & Štamborská, M. (2016) Application of the virtual fields method to large strain anisotropic plasticity. International Journal of Solids and Structures. In press.

First experimental application of the VFM to elasto-plasticity

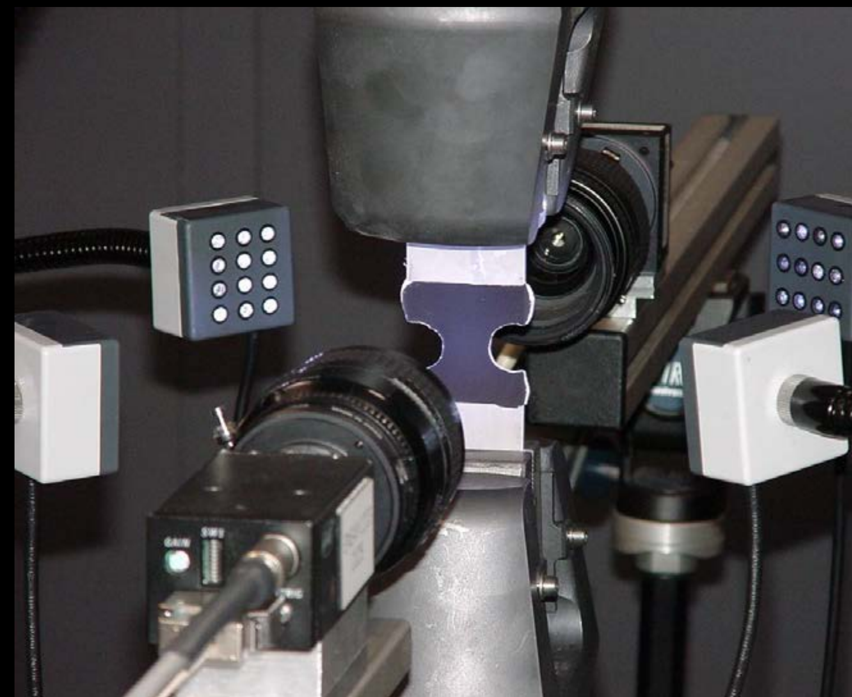
Pannier, Y., Avril, S., Rotinat, R., & Pierron, F. (2006). Identification of elasto-plastic constitutive parameters from statically undetermined tests using the virtual fields method. Experimental Mechanics, 46(6), 735-755.

Avril, S., Pierron, F., Pannier, Y., & Rotinat, R. (2008). Stress reconstruction and constitutive parameter identification in plane-stress elasto-plastic problems using surface measurements of deformation fields. Experimental Mechanics, 48(4), 403-419.

Pannier Y., PhD thesis, ENSAM, France, 2006

First attempt

Quasi-uniaxial



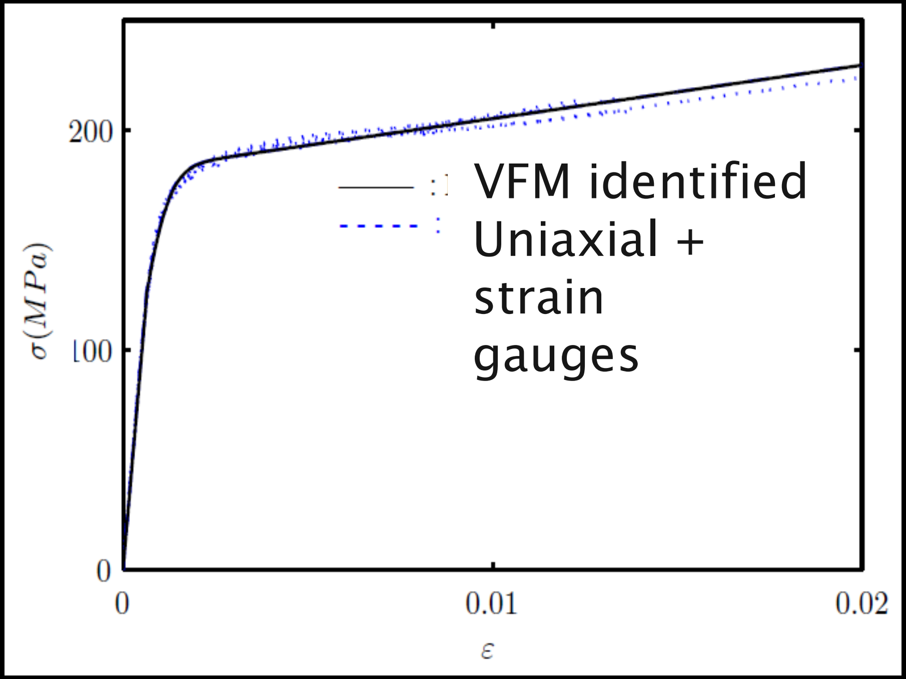
Fully multiaxial

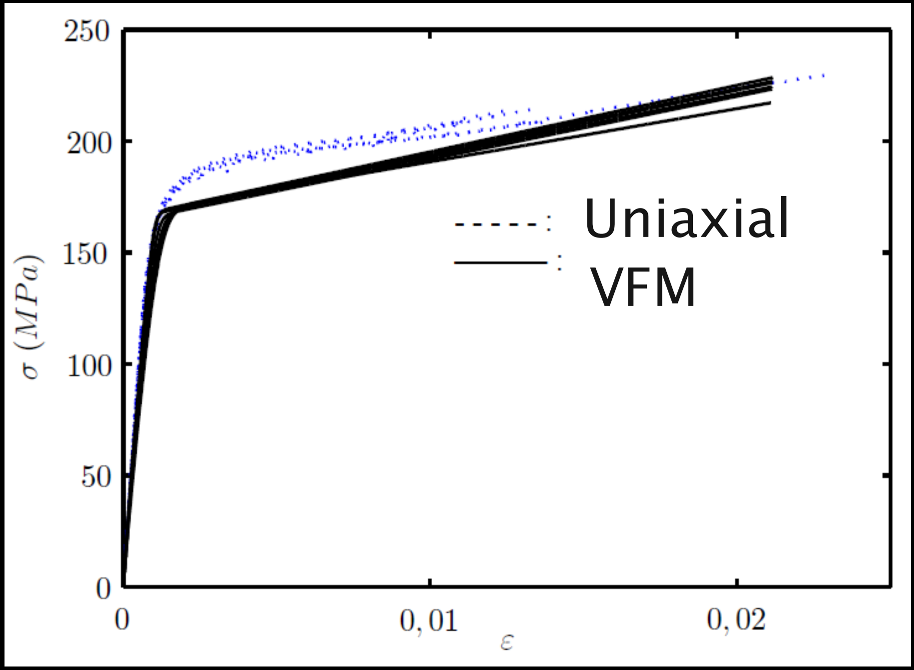
Isotropic plasticity, simple hardening

$$\sigma = \sigma_0 + R_0 \varepsilon^p + R_{\text{inf}} \left[1 - \exp(-b\varepsilon^p) \right]$$

First attempt

- With n=100 slices (1 per row of data)



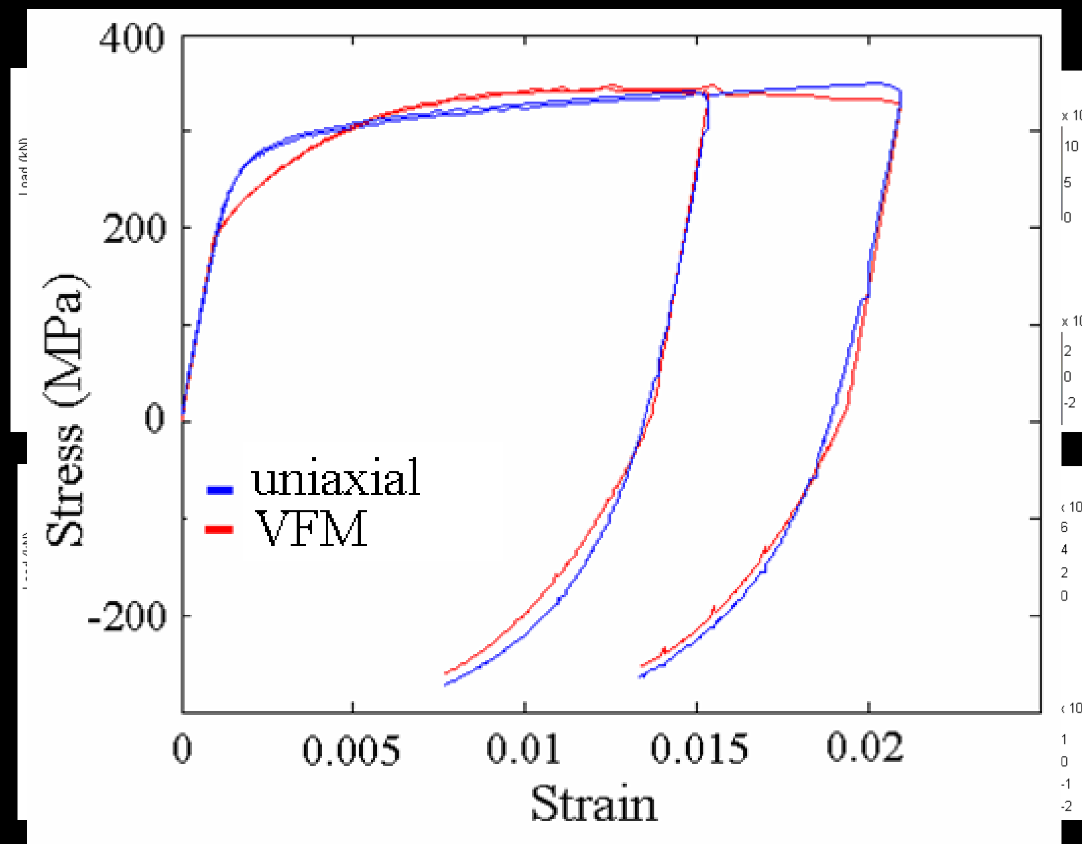


$$\sigma = \sigma_0 + R_0 \varepsilon^p + R_{\inf} \left[1 - \exp(-b\varepsilon^p) \right]$$

Uniqueness issue: needs better virtual fields (see later)

Kinematic hardening

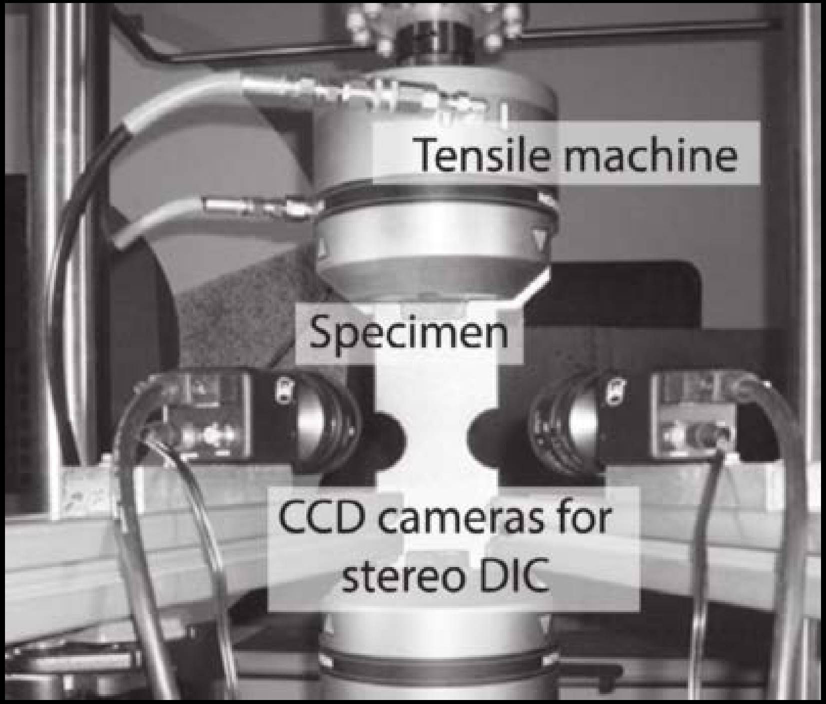
- Homogeneous elasto-plasticity
  - Non-linear kinematic hardening, loading-unloading, Virtual fields selection

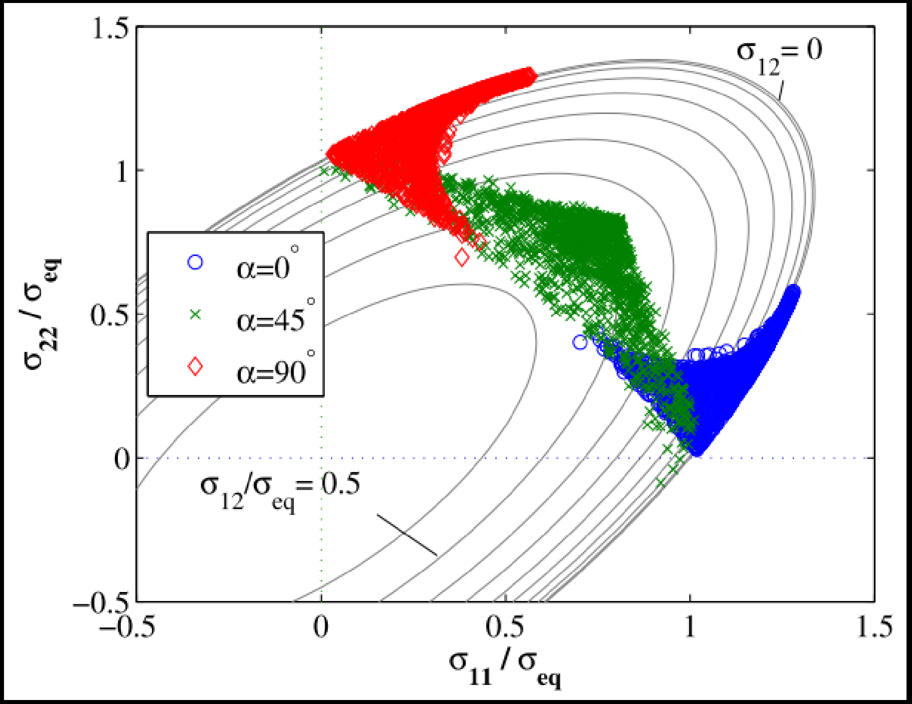


Pierron F., Avril S., Tran T.V., International Journal of Solids and Structures, 2010.

Anisotropic plasticity

- Stainless steel





Rossi, M., Pierron, F., & Štamborská, M. (2016). Application of the virtual fields method to large strain anisotropic plasticity. Int. J. Sol. and Struct., 97-98, 322-335.

Anisotropic plasticity

- Three manually-defined virtual fields
  - Hardening: Swift Law
- Combining tests at 0, 45 and 90°

	R_0	R_{45}	R_{90}	Cost function
Hill48	1.21	1.32	1.12	576
Yld2000-2D	2.00	1.63	2.07	153
Reference (uniaxial)	*1.88*	*1.54*	*2.18*	

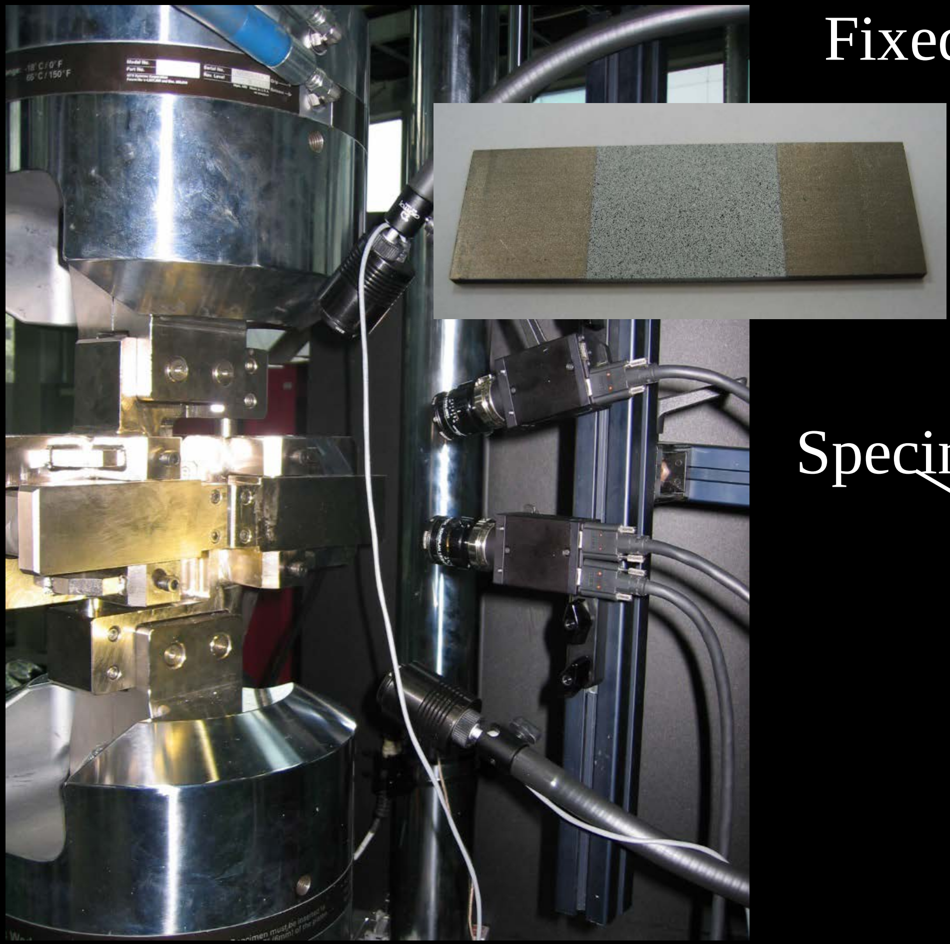
More complex hardening

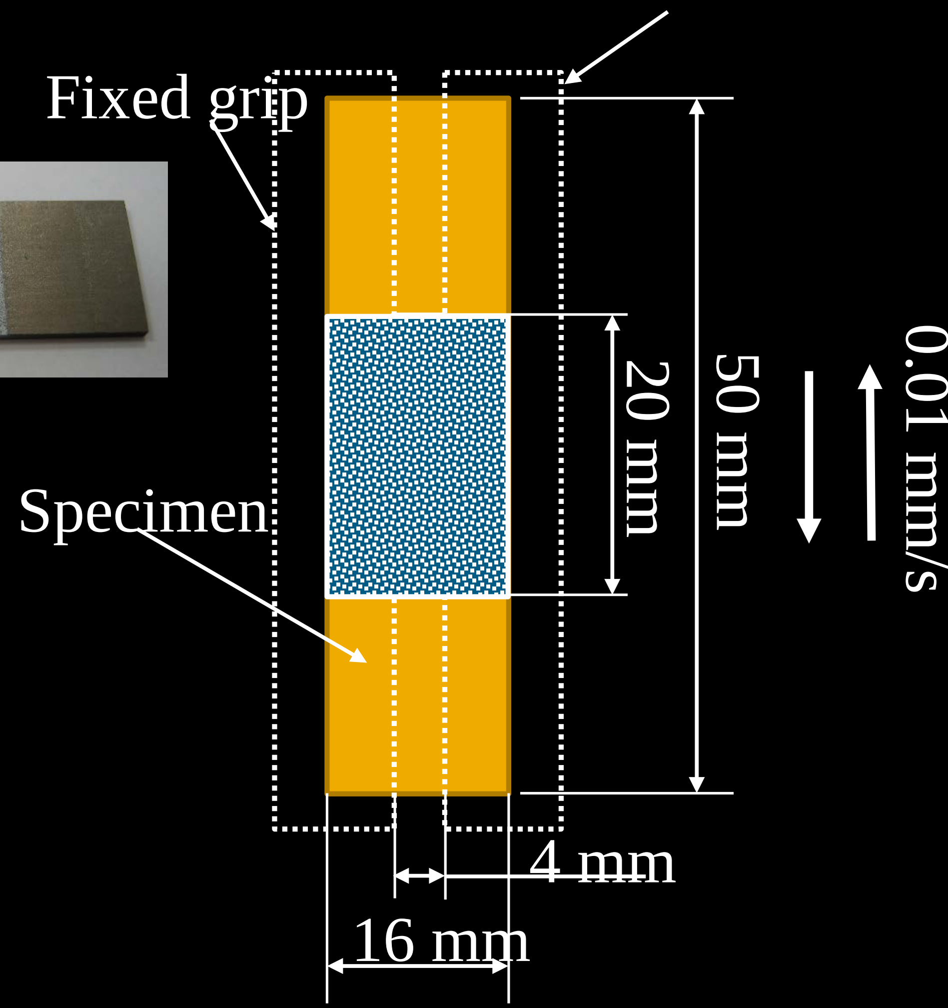
POSTECH
POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Prof. Frédéric Barlat, Dr Jiawei Fu, Dr Jin-Hwan Kim

- Fu, J., Barlat, F., Kim, J.-H., & Pierron, F. (2016). Identification of nonlinear kinematic hardening constitutive model parameters using the virtual fields method for advanced high strength steels. International Journal of Solids and Structures, 102–103, 30-43.
- Fu, J., Barlat, F., Kim, J.-H., & Pierron, F. (2017). Application of the virtual fields method to the identification of the homogeneous anisotropic hardening parameters for advanced high strength steels. International Journal of Plasticity, 93, 229-250.

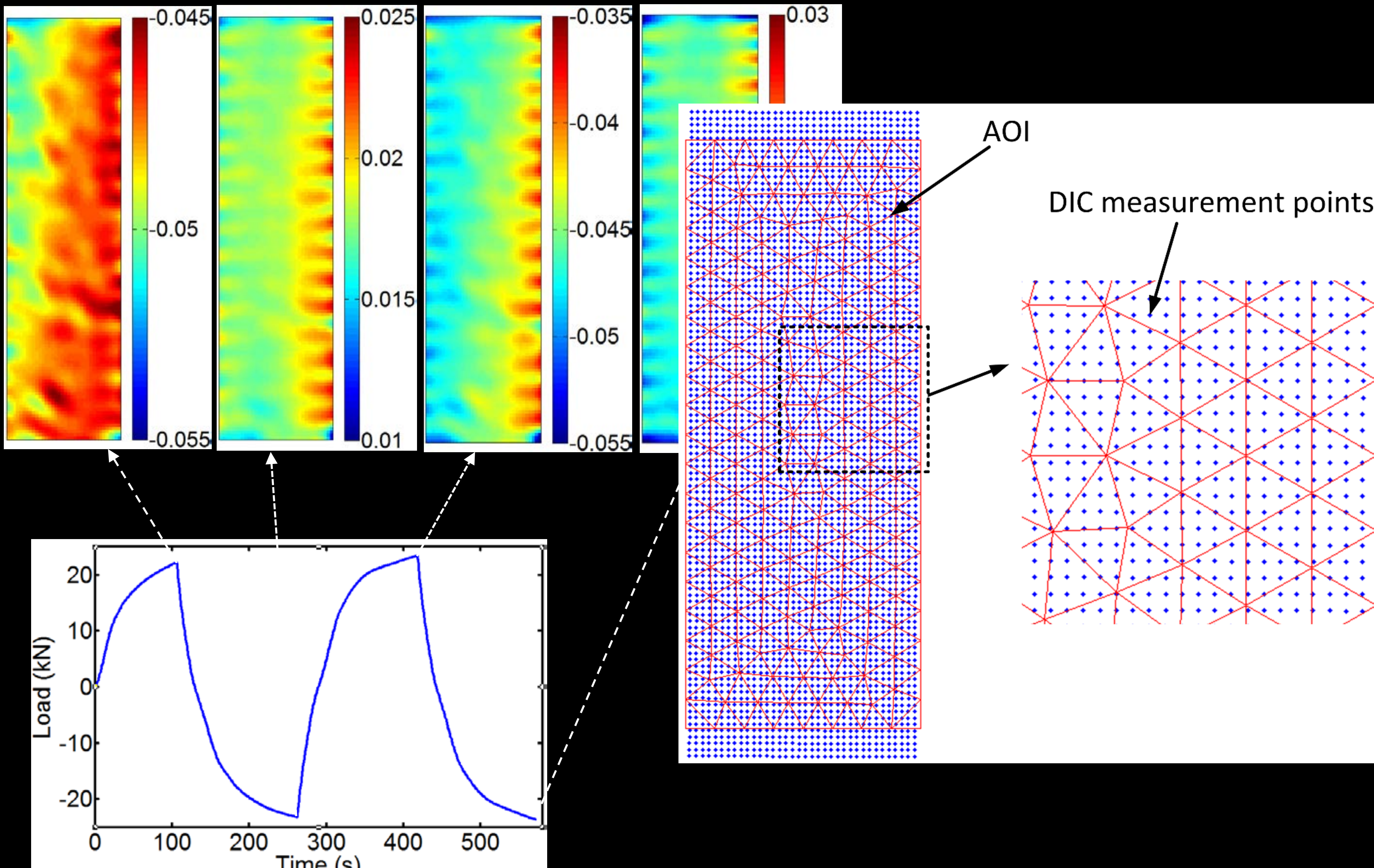
Reverse shear test (cyclic)





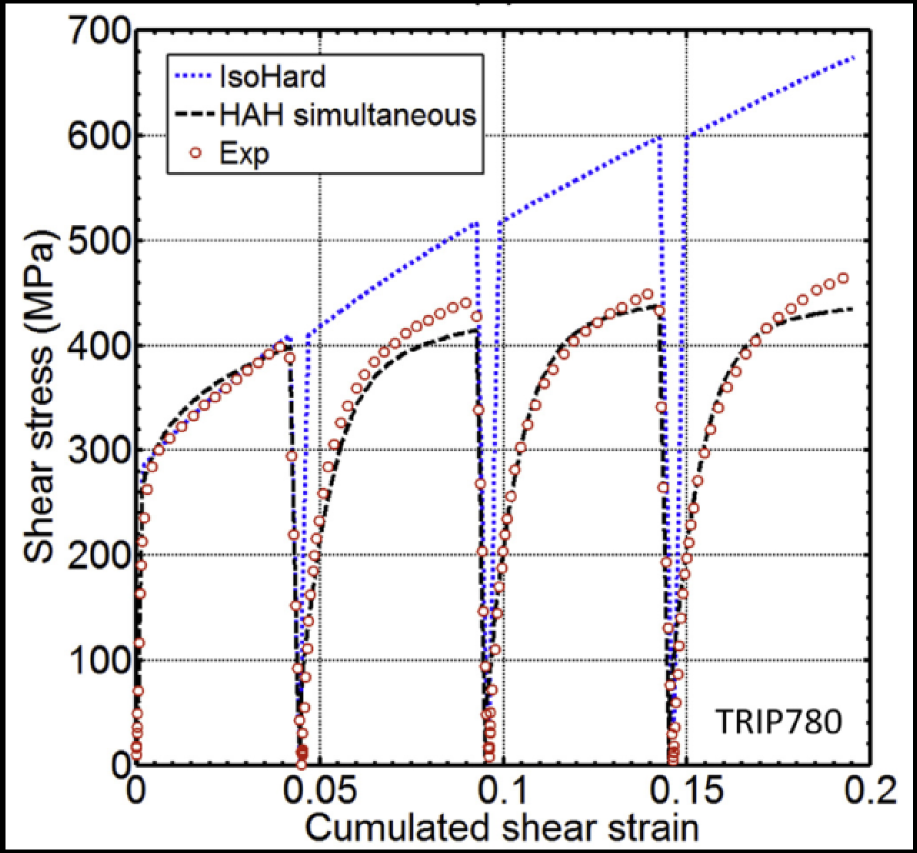
Materials	DP600	TRIP780	TWIP980

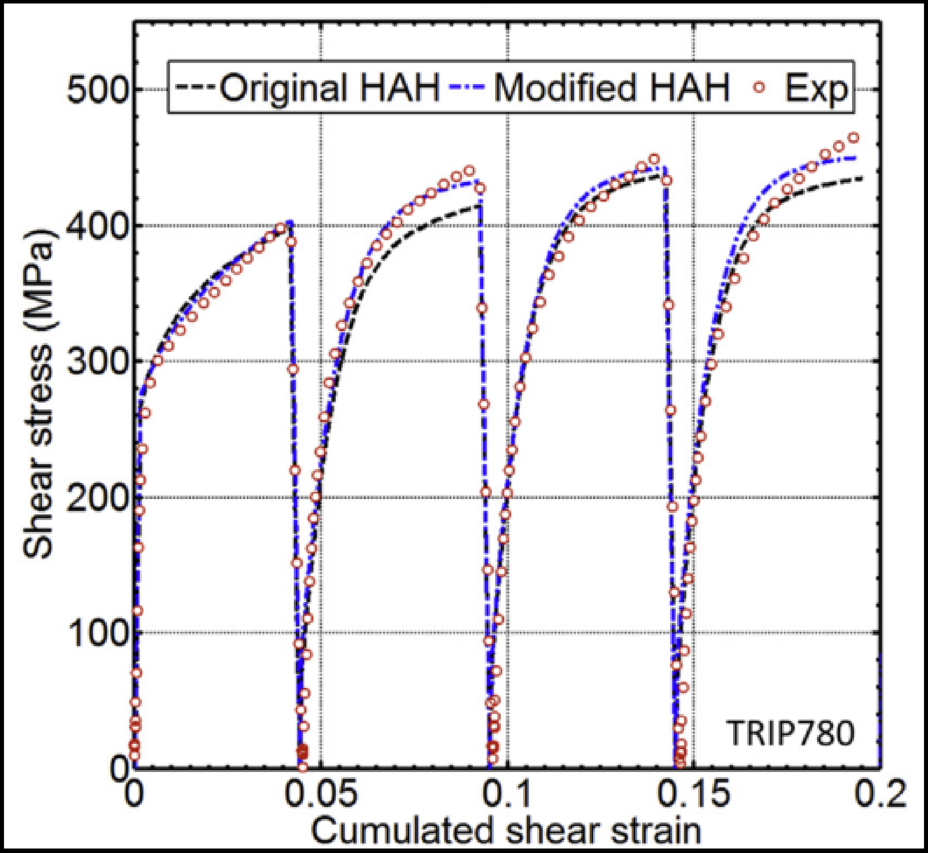
Experimental fields



Identification

- HAH distortional plasticity model (F. Barlat)





Modified HAH

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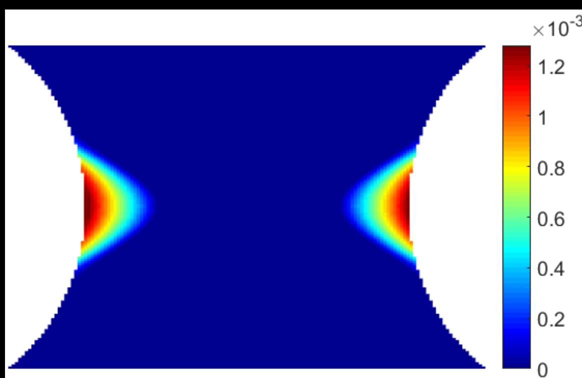
Virtual Field selection

PhD Mr Alexander Marek

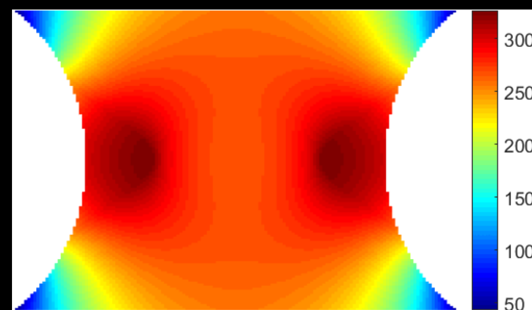
Sensitivity-based virtual fields

- New virtual fields, based on stress sensitivity

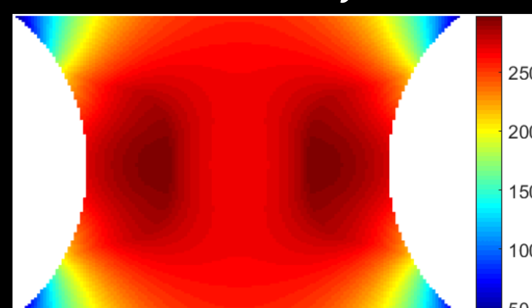


Strain

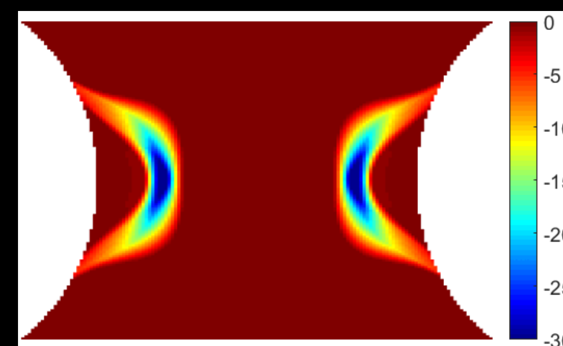
Stress $[\sigma_y, H]$



Stress $[0.9\sigma_y, H]$



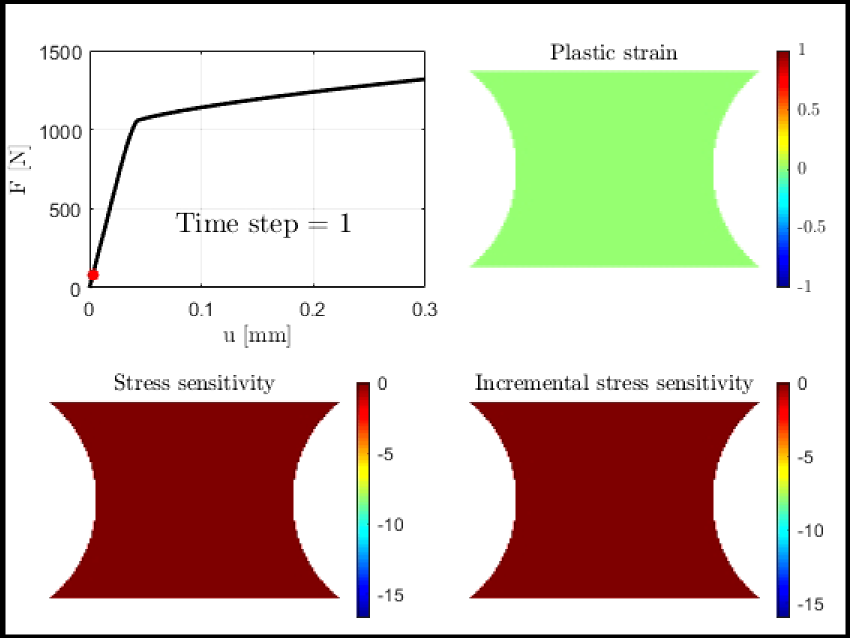
Incremental stress sensitivity



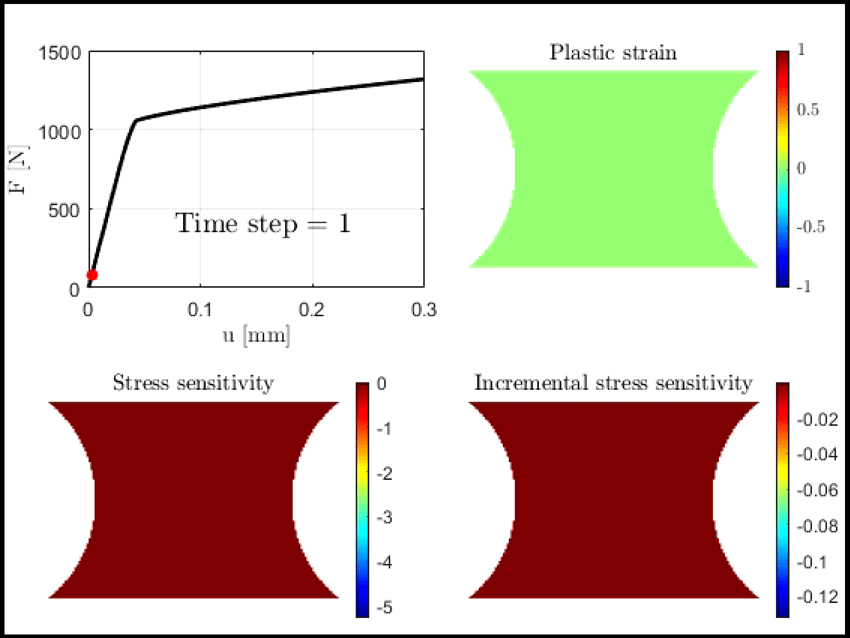
Marek A., Davis F.M., Pierron F., Computational Mechanics, in press, 2017.

Sensitivity-based optimised VFs

- Virtual fields for a linear hardening model



Yield stress related virtual field

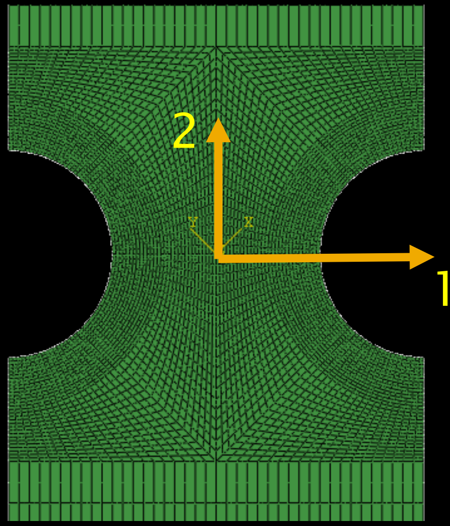


Hardening modulus related virtual field

Case study: deep notch specimen

- Steel specimen
- Anisotropic Plasticity: Yld2000-2D
  - 8 parameters defining anisotropy
  - Requires 4 standards tests for calibration
- Linear Hardening

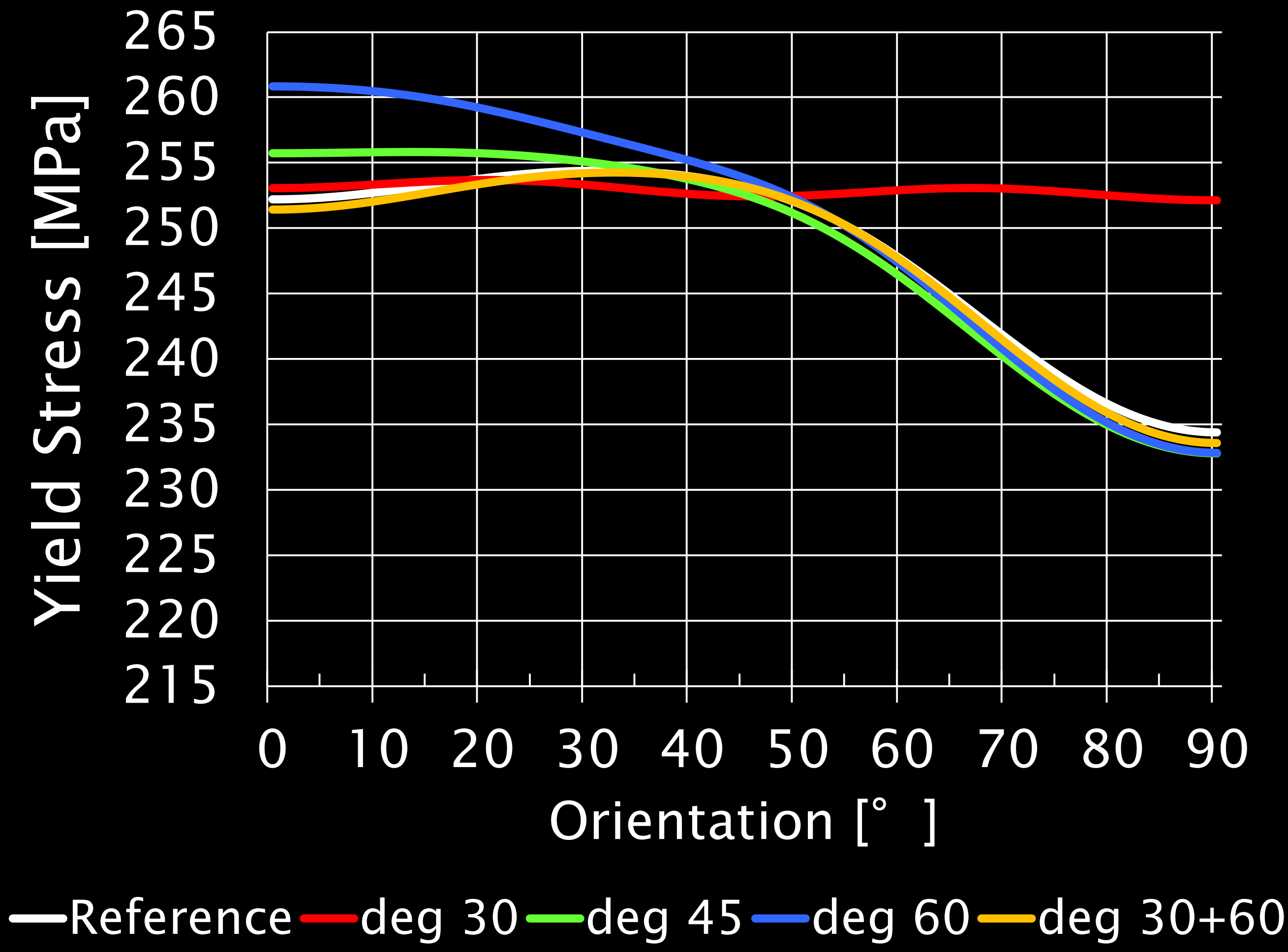
$$\sigma_y = \sigma_0 + \bar{\varepsilon}^p H$$

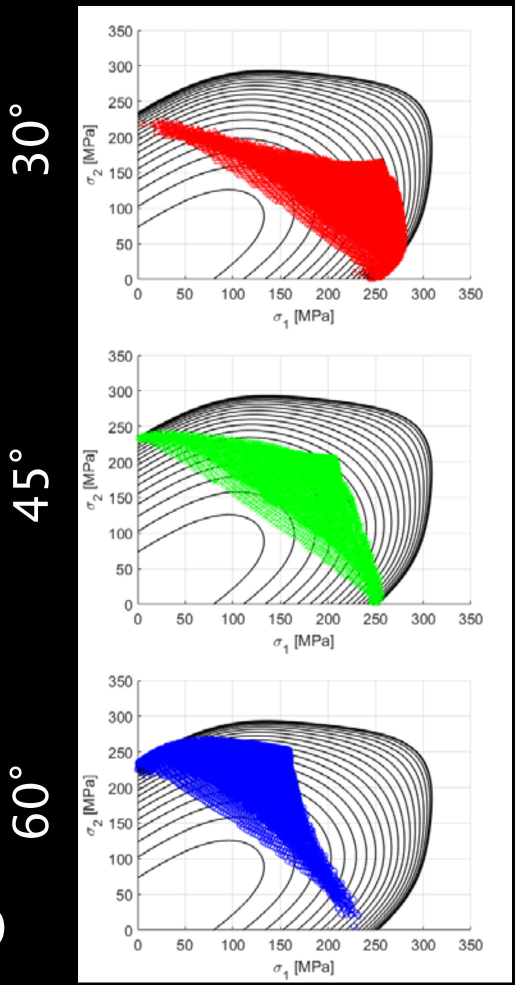


- Various material orientation
- **AIM: maximise identifiability of yield stress in range of 0° - 90° from a single test**

Results – Yld2000-2D

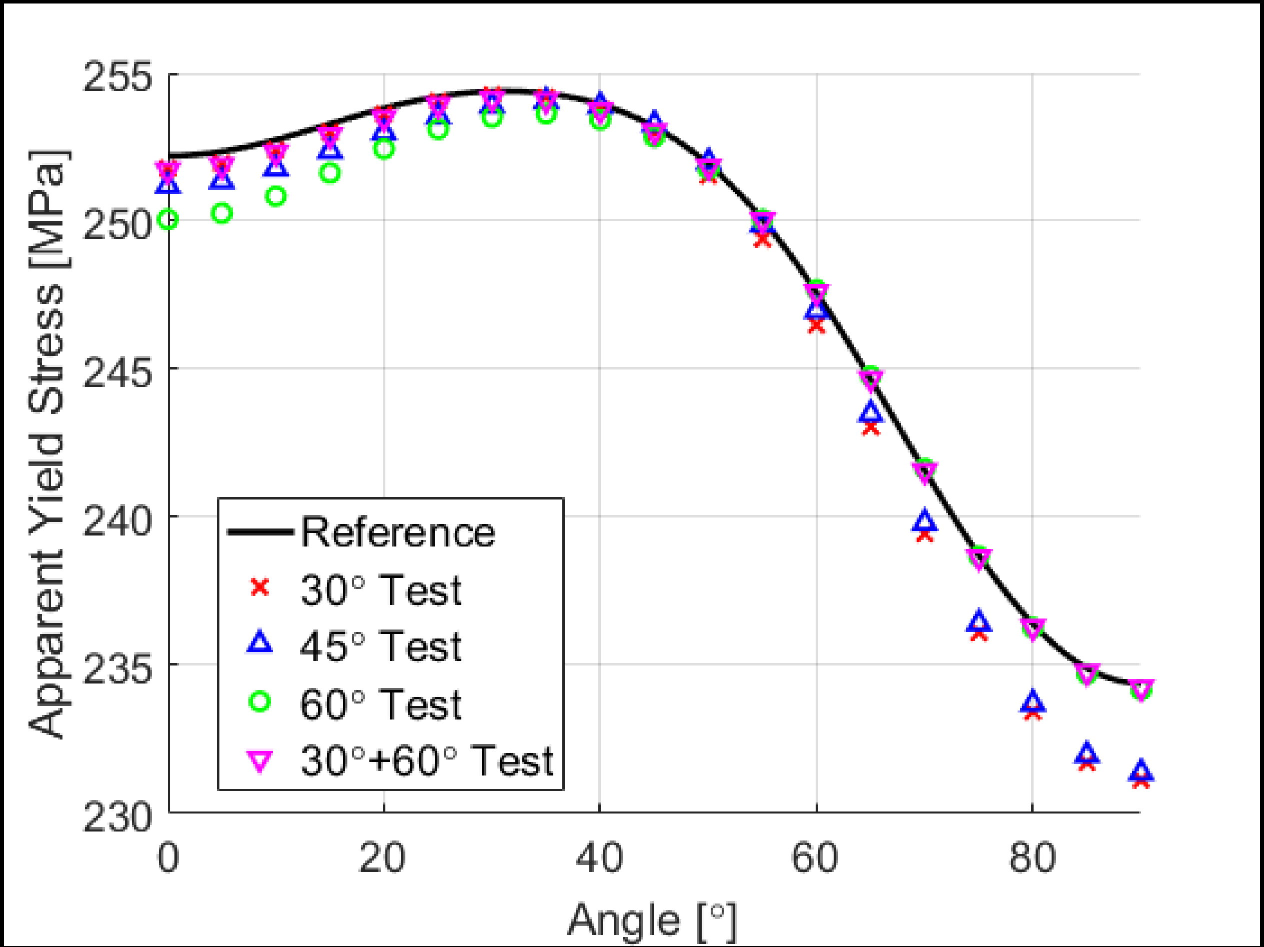
- Manual Virtual Field (uniform VF)





Results – Yld2000-2D

- New sensitivity-based VFs



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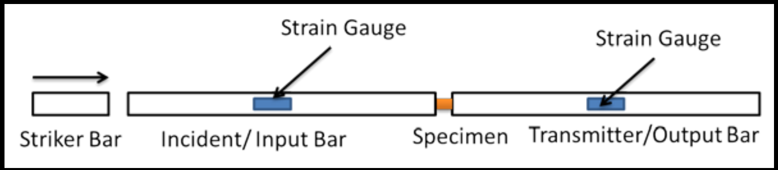
High strain rate

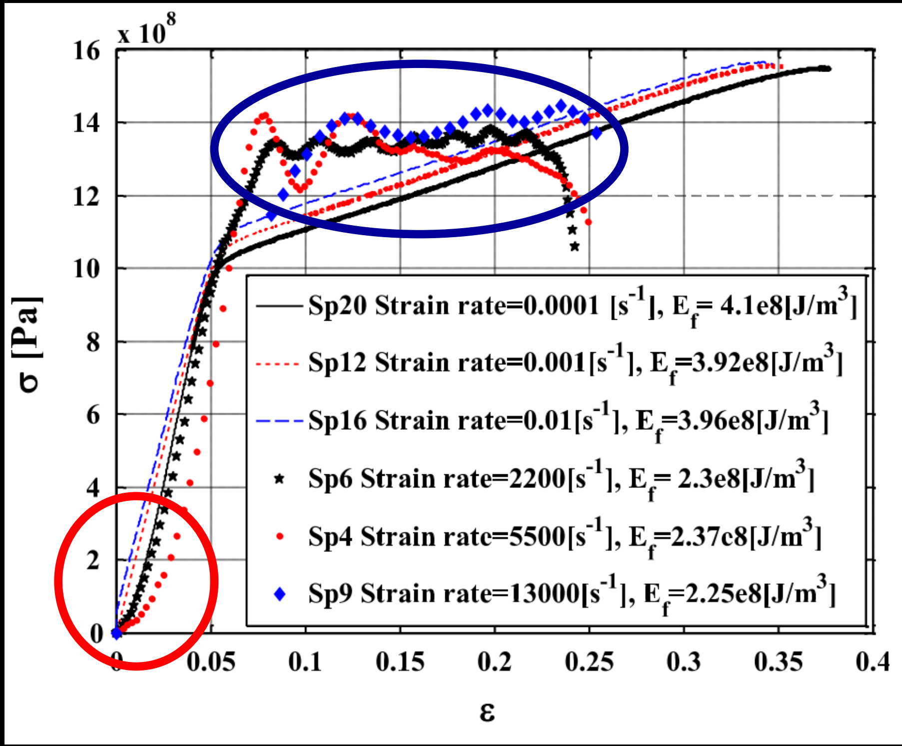
PhD Dr Sarah Dreuilhe

High strain rate

- Split Hopkinson Pressure bar
  - Limited by assumptions



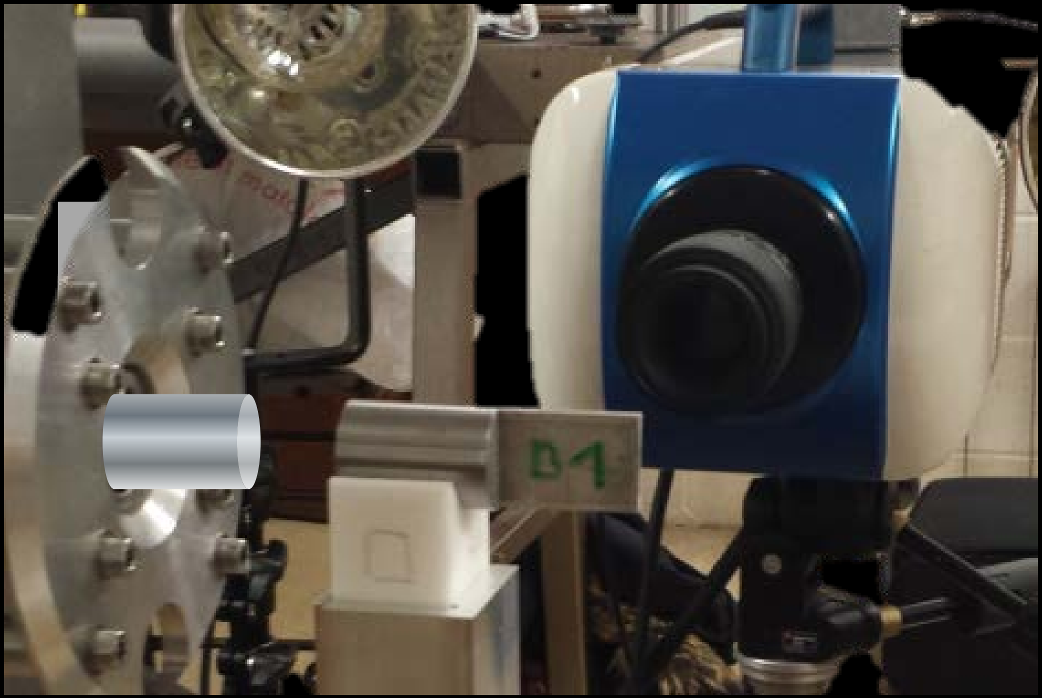
Inertia effects



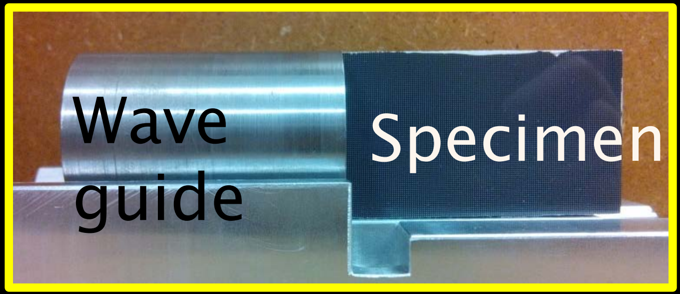
Osovski S. et al., Scripta Materialia, 2012. 67(7-8): p. 693-695.

New inertial test

- Basic idea



Grid bonded on specimen



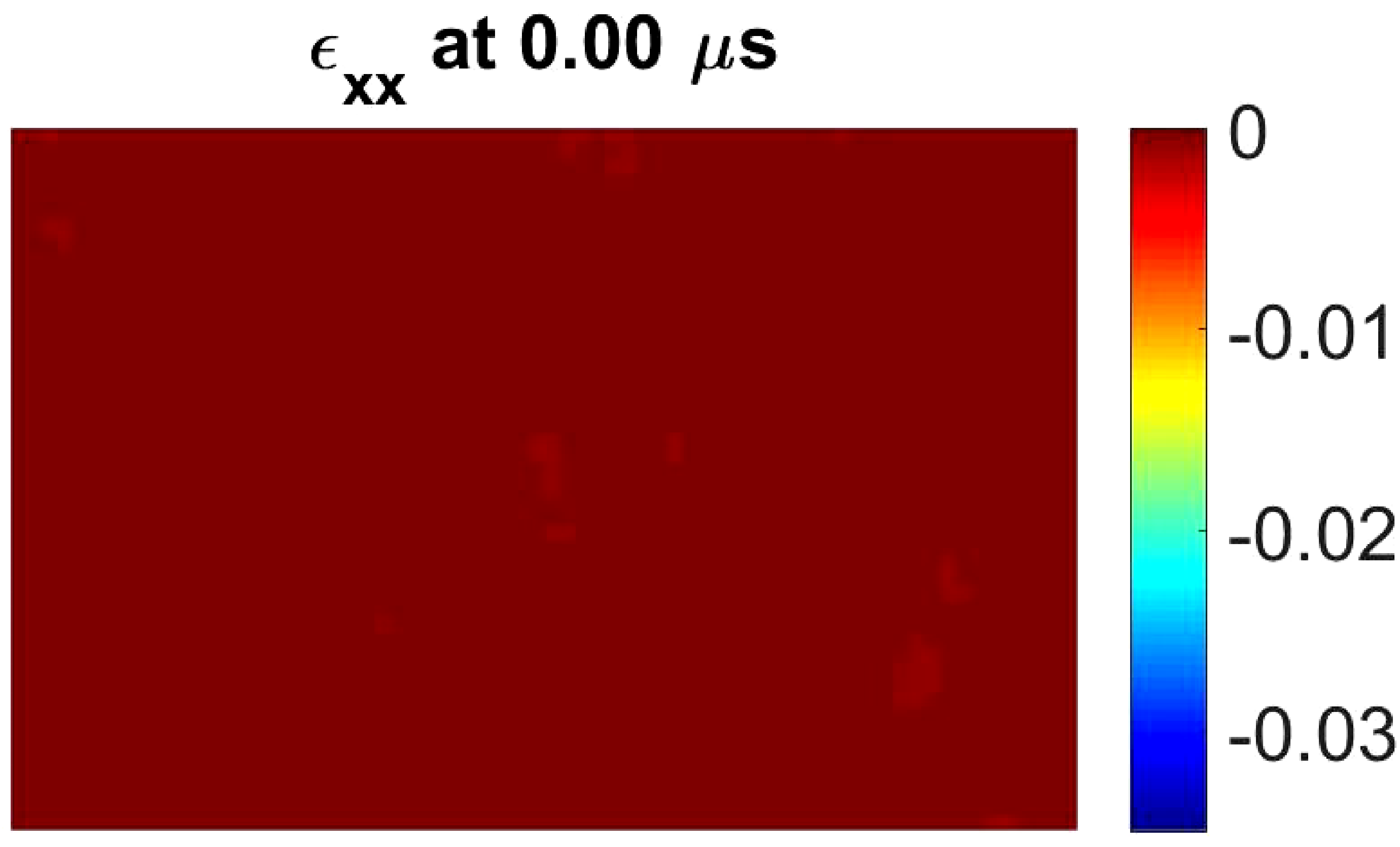
Al 6061 – T6 Aluminium
316L stainless steel

HPV-X camera, 5 Mfps, 400x250 pixels
Grid pitch: 0.6 mm, 5 pixels/period

Real time test

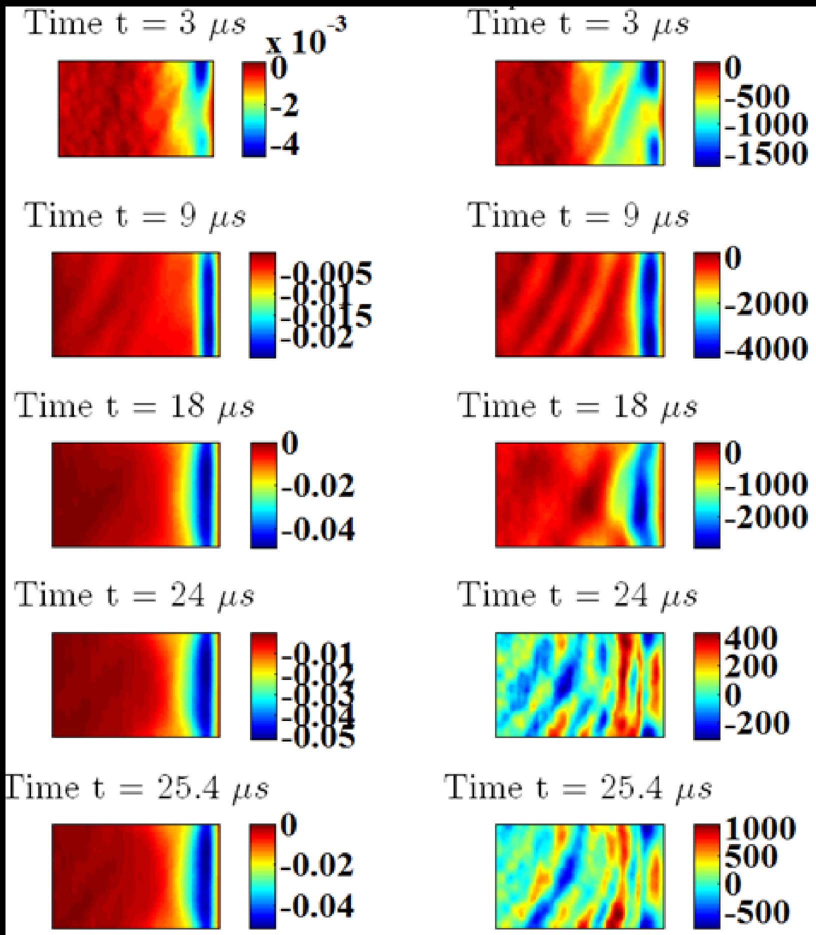
Grey level images (24 μs)

Longitudinal strain

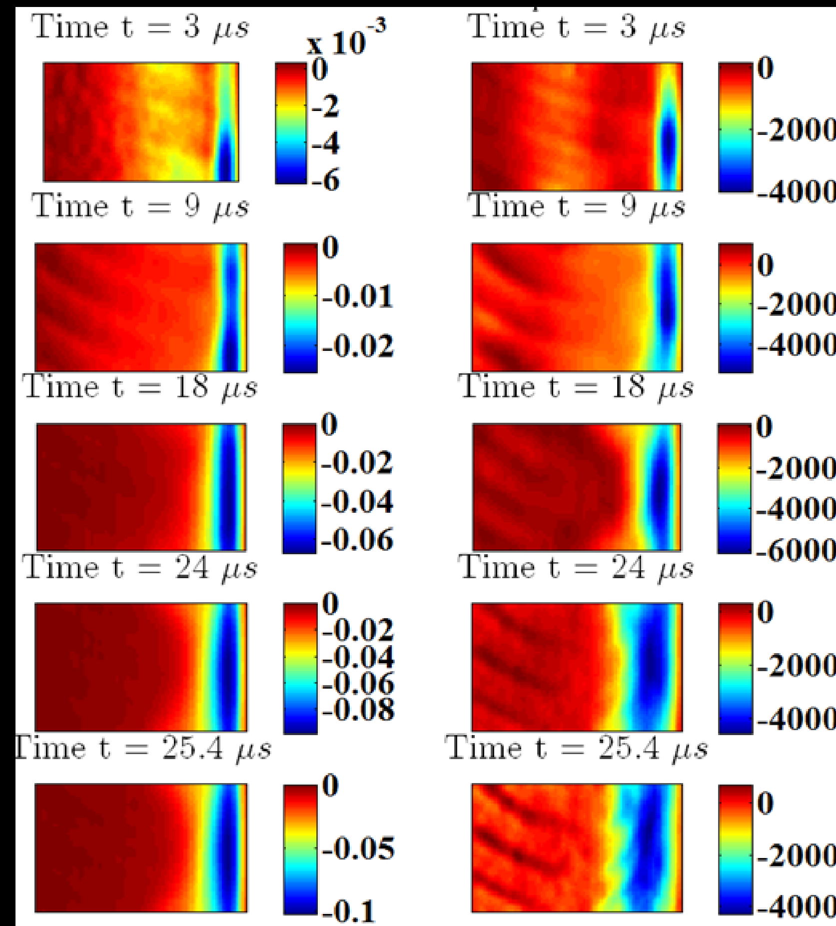


Results on steel

ε $\dot{\varepsilon}\ (s^{-1})$ ε $\dot{\varepsilon}\ (s^{-1})$

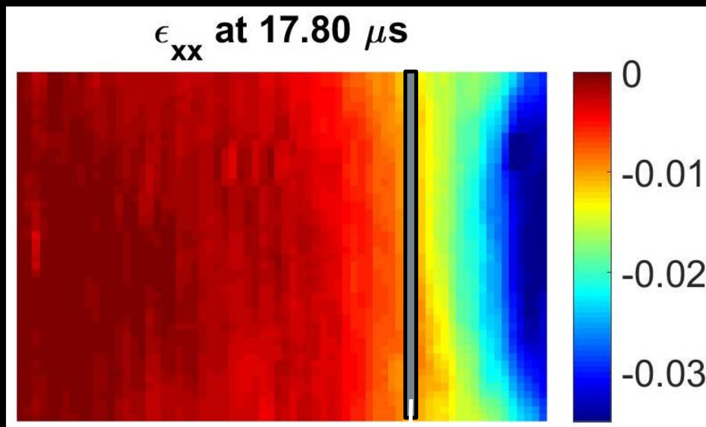


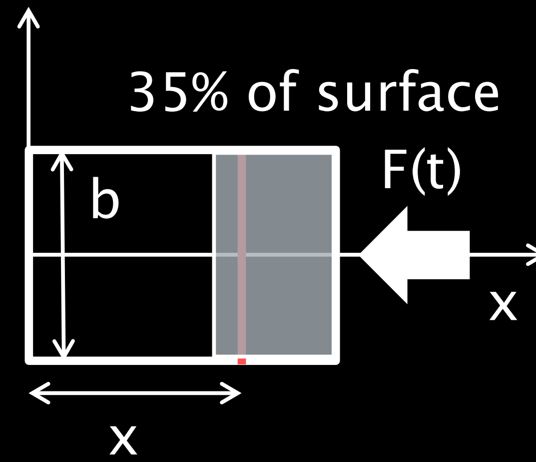
56 m.s^{-1}



121 m.s^{-1}

Identification





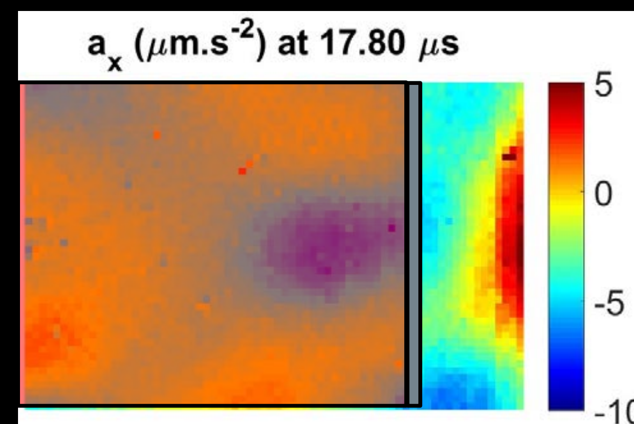
$$\sigma_Y = \sigma_Y^0 + H\varepsilon^p$$

$$\sigma_Y^{0\ init}, H^{init}$$

Return mapping algorithm

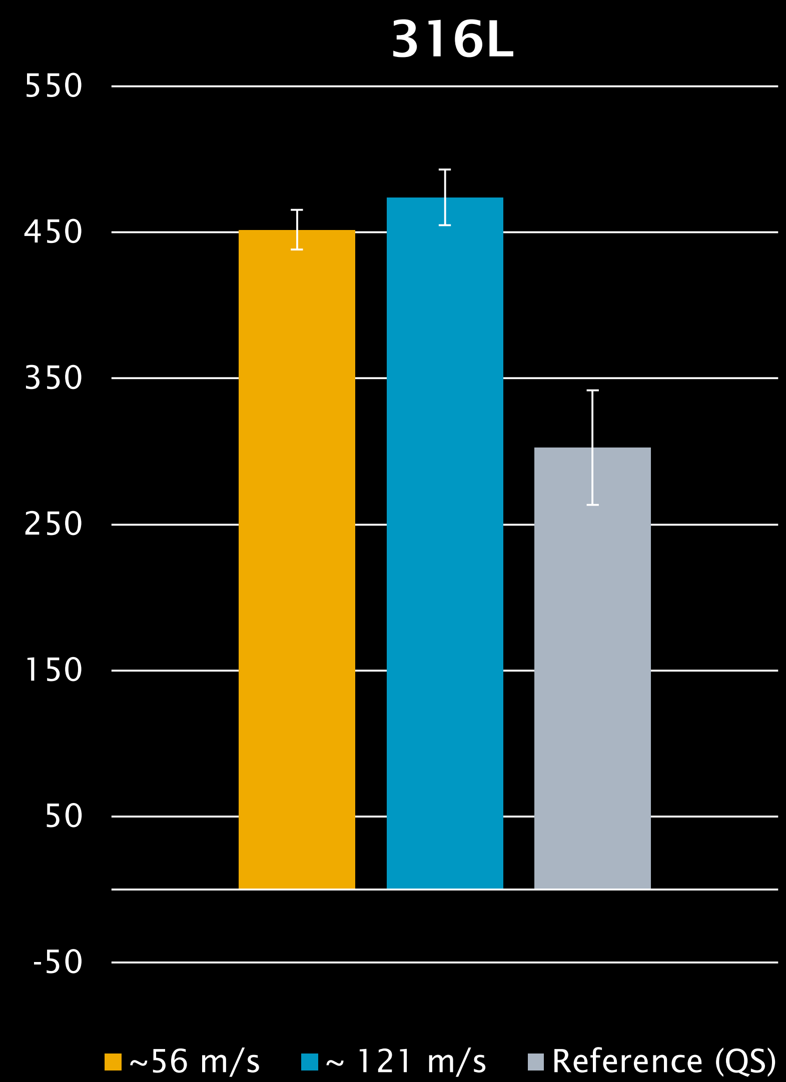
$$\overline{\sigma_x}^y(x,t)$$

$$\rho x \overline{a_x}(x,t)$$



$$\varphi(\sigma_Y^0, H) = \sum_{x=L_s}^{x=L} \sum_{t=0}^{t=t_f} \left(\overline{\sigma_x}^y(x,t) - \rho x \overline{a_x}(x,t) \right)^2$$

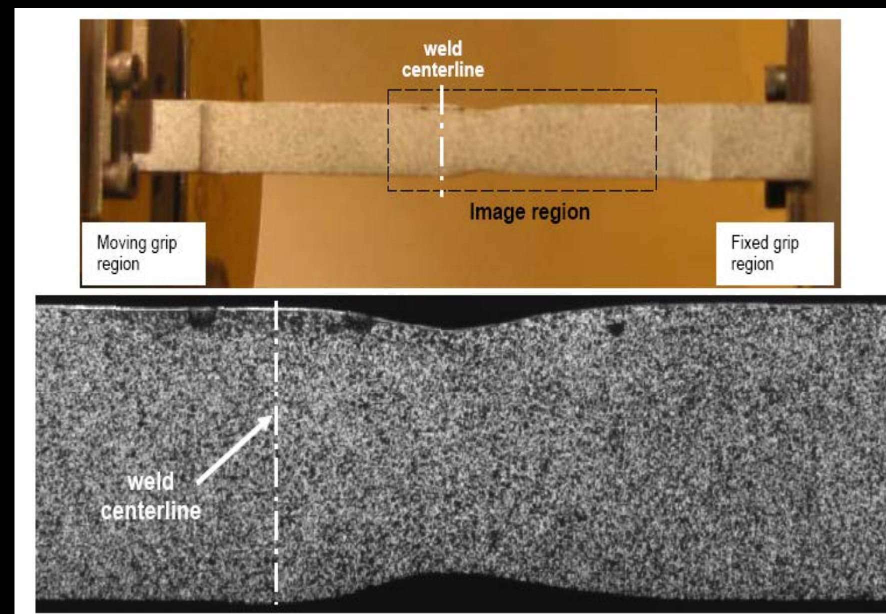
Results – Yield stress (MPa)

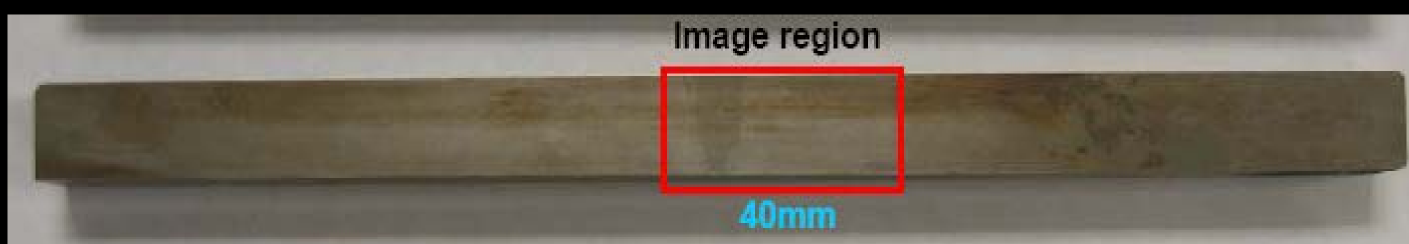


Weld

- Steel girth welds
  - Collaboration with M. Sutton

University of South Carolina

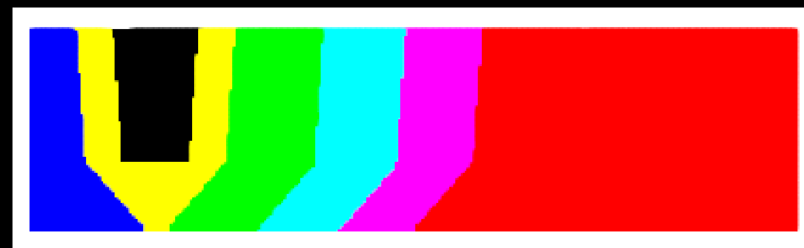




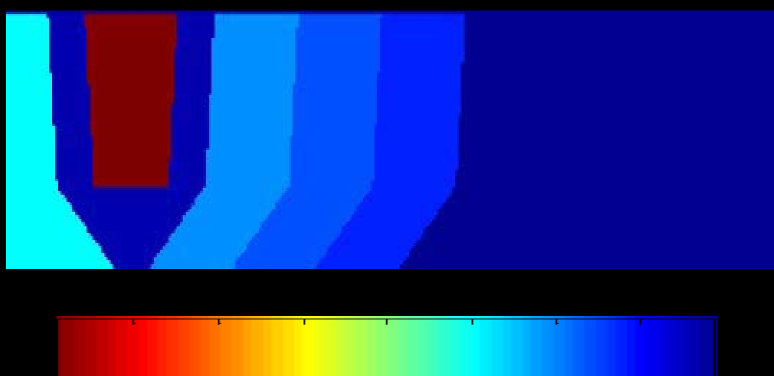
Weld

- Identification (linear isotropic hardening)

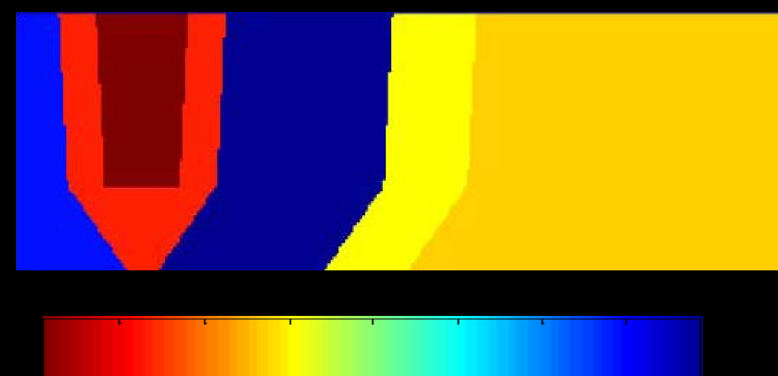
Seven zones



Yield stress (MPa)



Hardening modulus (MPa)



Sutton M.A., Yan J., Avril S., Pierron F., Adeeb S.M., Experimental Mechanics, 2008.

Conclusion

- Non-linear VFM
  - Much faster than Finite Element Model Updating, factor of 100!
- Very long identification chain: many parameters
  - Full-field technique (grid size, speckle pattern)
  - Smoothing
  - Choice of the virtual fields
  - How many time steps?
  - Stress reconstruction algorithm
  - Test configuration

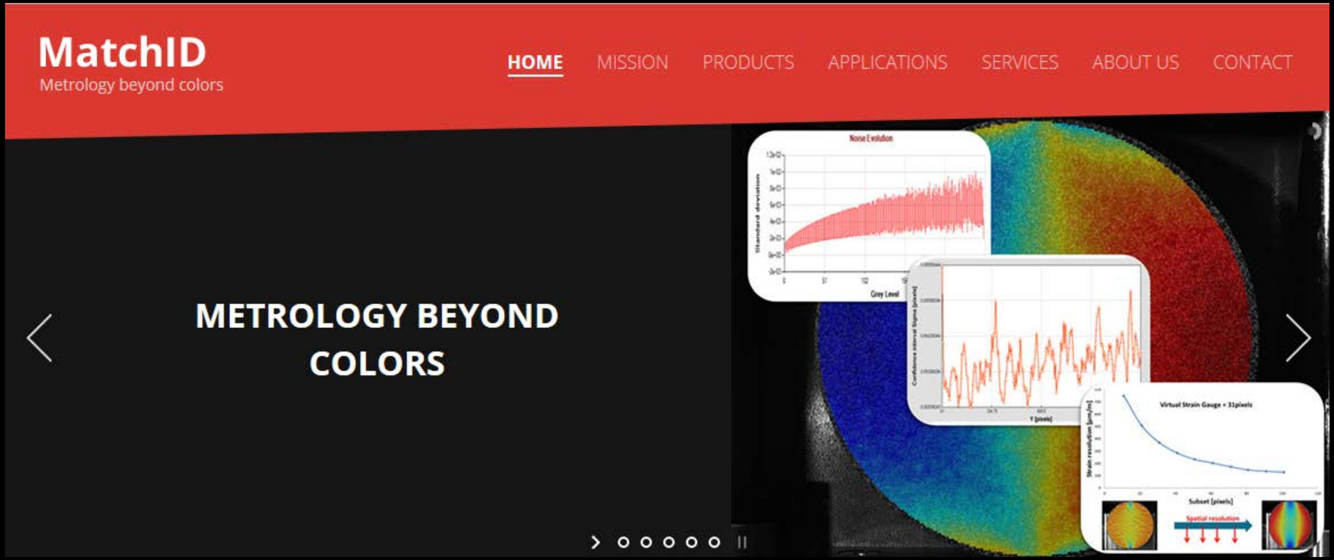
Conclusion

- Simulation required to
  - Perform uncertainty propagation*
  - Design the test (to minimize uncertainty) and maximize robustness)
- Future work
  - Test optimization
  - Welds (more complex stress states)
  - High strain rate anisotropic plasticity
  - Micro-scale plasticity imaging at high rate

*Rossi, M., & Pierron, F. (2012). On the use of simulated experiments in designing tests for material characterization from full-field measurements. *International Journal of Solids and Structures, 49(3-4), 420-435.*

Software implementation

- DIC + VFM platform
  - Simple isotropic elasto-plasticity (for now), with Bi-linear, Swift, Voce, Ludwik
  - Soon, anisotropic + more complex hardening



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