Multi-Scale Homogenisation for 3D Structures

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Introduction

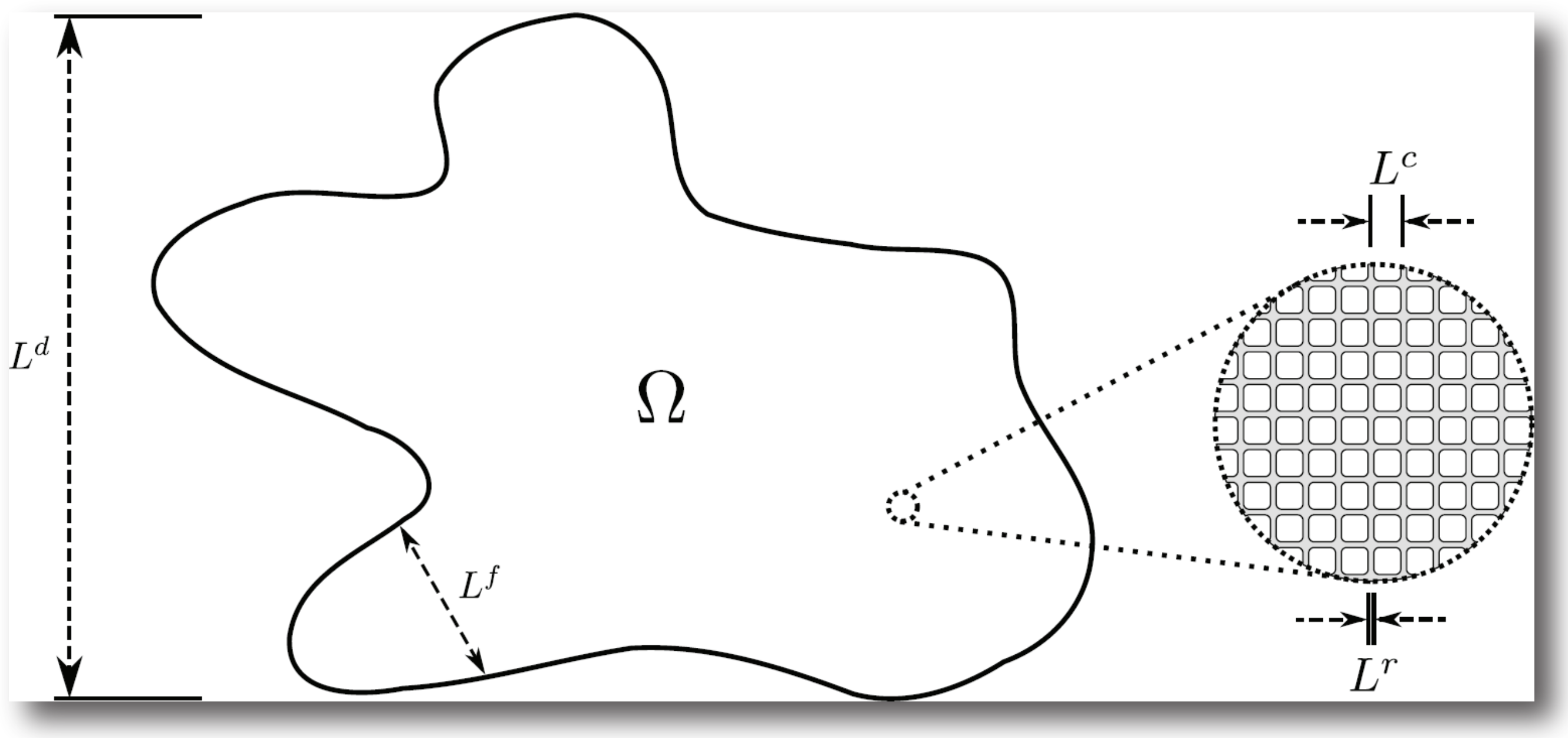

Fig 1. Lengthscales present in a multi-scale structure

When dealing with micro-architectures one issue likely to eventually arise is the analysis of the mechanical properties of macroscopically inhomogeneous multiscale structures. The bulk response of these structures can be determined by performing 'full' finite element analysis that is with the entire geometry discretized at a resolution high enough to ensure mesh independence. However, these full models may easily exceed hundreds of millions, potentially billions, of degrees of freedom.The approach taken for this work will be to treat sub-volumes of the structure as actual elements and, through a series of tests, infer appropriate effective material properties.

Method

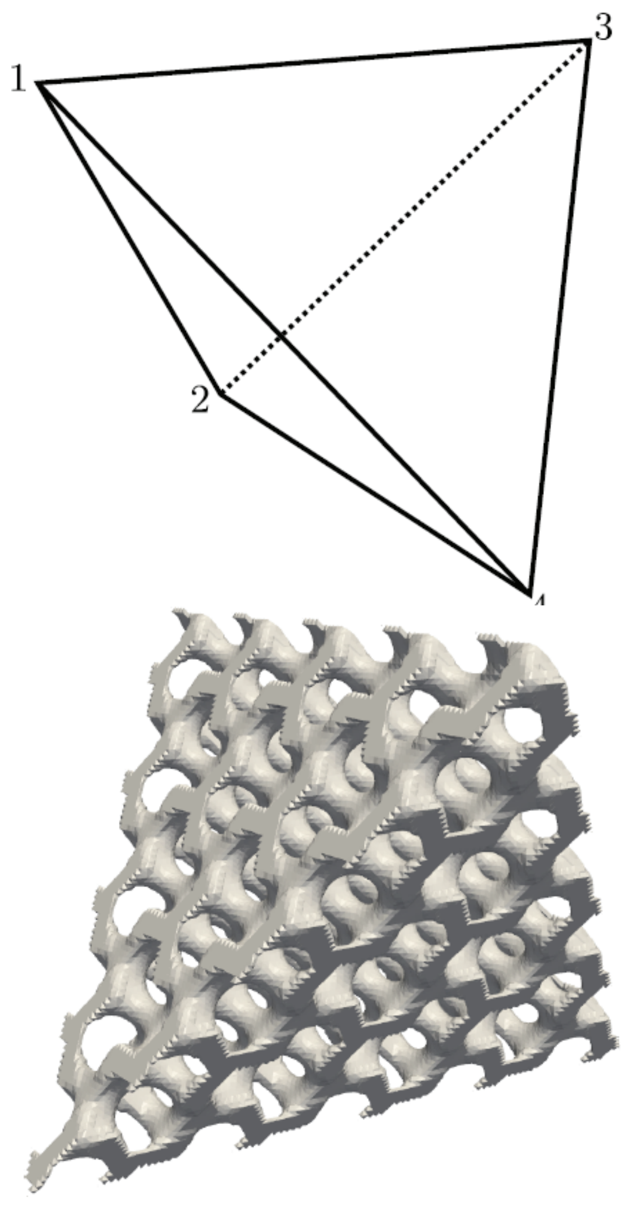

Fig 2. Example micro-architecture conforming to a tetrahedral macro element

The basic principle of the proposed homogenisation method can be demonstrated by first considering the simple case of the constant strain triangle (CST). Given a CST of an assumed unknown homogeneous material the constitutive matrix D can be accurately recovered through the use of virtual testing. We know that the forces and corresponding displacements at the element's nodes are a function of the constitutive matrix. Therefore, given a set of forces and displacements it is possible to compute D. It is known that the behaviour of the CST is defined by its stiffness matrix K, as shown in Eq 1.

(Eq 1) $K = tAB^{T}DB$

When Eq 1 is combined with Hooke's Law it becomes possible to express the forces on the element in terms of K and therefore in terms of D. Given this it becomes possible to equate the forces in terms of D to the forces measured through finite element analysis and construct a system of linear equations. In order to solve the equations it can be shown that three sets of tests are required in 2D and six in 3D.

This method can be extended to cases where the macro element bounds a microstructure by using the element's shape functions to constrain the exterior of the structure to displacements of the element. Effective macro node forces are then calculated as a weighted sum of the micro forces, using the element's shape function. It can be shown that the described method extend directly to 3D elements, as in Fig 2.

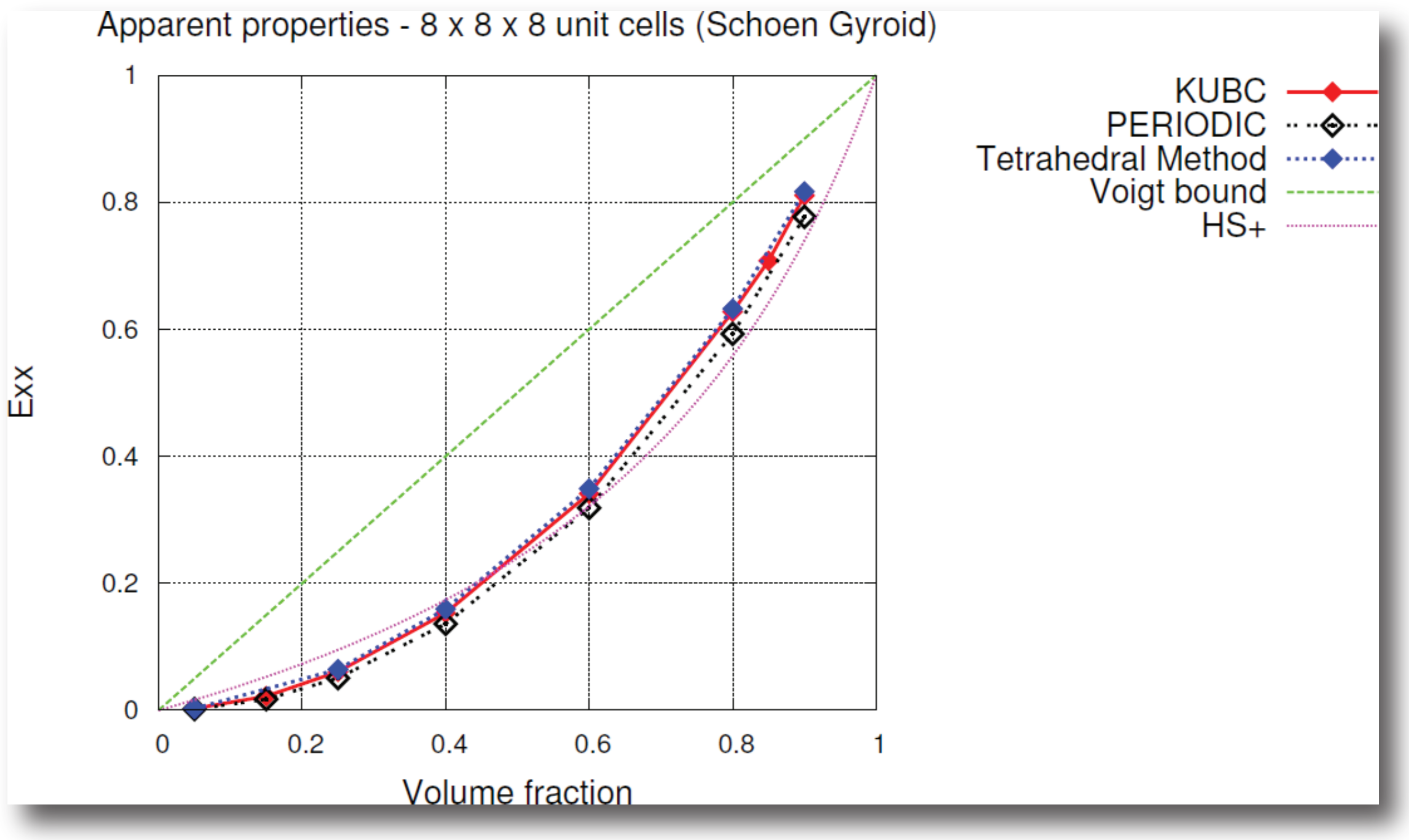

Fig 3. Comparison of presented method to establish techniques

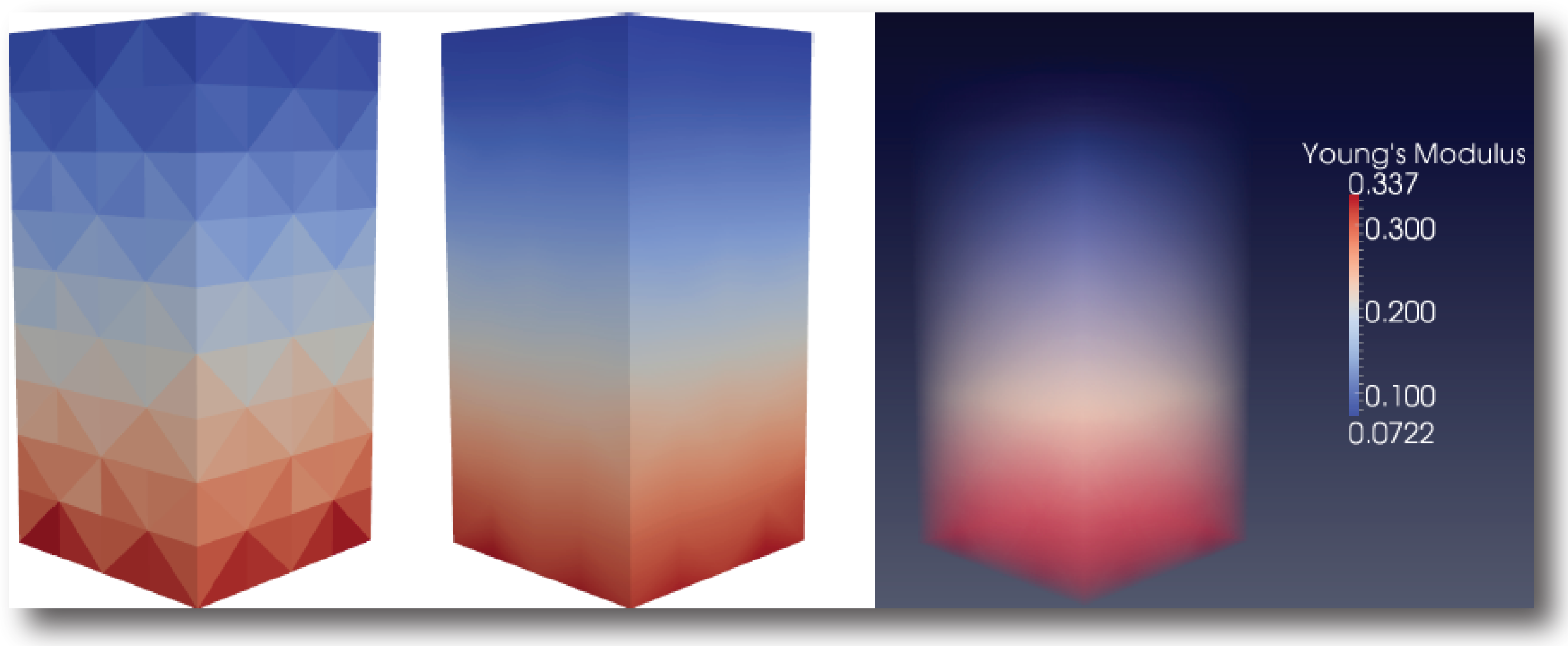

Fig 4. Visualizing the variation of Young's modulus over a functionally graded structure: original, interpolated and volumetric

Validation

In order to validate the developed homogenization method we compare it to the often used kinematic uniform boundary conditions (KUBC).The structure chosen in for this validation is a periodic micro-architecture known as the Schoen Gyroid. Results are presented in Fig 3.

Approximate Models

Fig 4 shows the results of characterizing an large functionally graded structure using the methods described in this work.

Conclusions

This work presents a novel approach to large multi-scale characterization problems in irregular domains. By dividing the domain of interest into smaller sub-volumes, based on a coarse macroscopic mesh, large problems can be processed efficiently either in parallel or series. The processing of problems using this method in series has the advantage that hardware requirements can be considerable reduced as they need only be sufficient of the largest sub-volume to be homogenized. The homogenization method itself has been shown to yield effective properties comparable to those achieved when using KUBC.